Gauge boson binaries amidst gluonic mesons
a QCD odyssee

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Topics

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ΟΔΥΣΣΕΙΑ

Ἄνδρα μοι ἔννεπε, Μοῦσα, πολύτροπον, ὃς μάλα πολλὰ
πλάγχθη, ἐπεὶ Τροίης ἱερὸν πτολίεθρον ἔπερσε·
πολλῶν δ' ἀνθρώπων ἴδεν ἄστεα καὶ νόον ἔγνω,
πολλὰ δ' ὅ γ' ἐν πόντῳ πάθεν ἄλγεα ὃν κατὰ θυμόν,
ἀρνύμενος ἥν τε ψυχὴν καὶ νόστον ἑταίρων.
ἀλλ' οὐδ' ὣς ἑτάρους ἐρρύσατο, ἱέμενός περ·
αὐτῶν γὰρ σφετέρῃσιν ἀτασθαλίῃσιν ὄλοντο,
νήπιοι, οἳ κατὰ βοῦς Ὑπερίονος Ἠελίοιο
ἤσθιον· αὐτὰρ ὁ τοῖσιν ἀφείλετο νόστιμον ἦμαρ.

Homer’s Odyssey : the beginning in original greek .

Tell me, Muse, of that man, so ready at need, who wandered far and wide, after he had sacked the sacred citadel of Troy, and many were the men whose towns he saw and whose mind he learnt, yea, and many the woes he suffered in his heart upon the deep, striving to win his own life and the return of his company. Nay, but even so he saved not his company, though he desired it sore. For through the blindness of their own hearts they perished, fools, who devoured the oxen of Helios Hyperion: but the god took from them their day of returning.

Of these things,goddess,daughter of Zeus, whencesoever thou hast heard thereof, declare thou even unto us.

Translation into english prose : S. H. BUTCHER, M.A. and A. LANG, M.A, Project Gutenberg 1999 . →

I2

Sage mir Muse die Taten des vielgewanderten Mannes,
Welcher so weit geirrt, nach der heiligen Troia Zerstörung,
Vieler Menschen Städte gesehn und Sitte gelernt hat,
Und auf dem Meer so viel' unnennbare Leiden erduldet,
Seine Seele zu retten und seiner Freunde Zurückkunft.

Aber die Freunde rettet' er nicht, wie eifrig er strebte;
Denn sie bereiteten selbst durch Missetat ihr Verderben;
Toren! welche die Rinder des hohen Sonnenbeherrschers
Schlachteten; siehe, der Gott nahm ihnen den Tag der Zurückkunft.

Sage hievon auch uns ein weniges, Tochter Kronions.

Translation into german by Johann Heinrich Voss , Birkhäuser Verlag, Basel 1781 .

1-1 QCD – the two central anomalies : scale and chiral U1

Premises

We face the theoretical abstraction of QCD with $N_{fl} = 6$, representing strong interactions – adaptable to two or three light flavors (u , d , s) of quarks and antiquarks. $\leftrightarrow$

quarks : color is counted in $\pi^0 \to \gamma\gamma$

spin and flavor are clearly seen in $q\overline{q}$ and $3q$, $3\overline{q}$ spectroscopy.

$$\mathcal{L} = \left[\overline{q}\,{}^{\dot{c}'}_{\dot{\mathcal{B}}\,\dot{f}} \left\{ \begin{array}{c} \frac{i}{2}\,\overleftrightarrow{\partial}_{\mu}\,\delta_{c'c} \\ -\,v^{A}_{\mu}\left(\frac{1}{2}\lambda^{A}\right)_{c'\dot{c}} \end{array} \right\} \gamma^{\mu}_{\dot{\mathcal{B}}\mathcal{A}}\, q^{c}_{\mathcal{A}\,f} \; - m_{f}\,\overline{q}\,{}^{\dot{c}}_{\dot{\mathcal{A}}\,\dot{f}}\, q^{c}_{\mathcal{A}\,f} \right] - \frac{1}{4\,g^{2}}\,F^{\mu\nu\,A}\,F^{A}_{\mu\nu} + \Delta\mathcal{L} \quad (1)$$

quarks : c' , $c = 1,2,3$ color , $f = 1,\cdots,6$ flavor

$\mathcal{B},\mathcal{A} = 1,\cdots,4$ spin , m_f mass

$\longrightarrow$

gauge bosons :

(2)

$$F^{A}_{\mu\nu} = \partial_{\nu} v^{A}_{\mu} - \partial_{\mu} v^{A}_{\nu} - f_{ABC}\, v^{B}_{\nu}\, v^{C}_{\mu}$$

$$A, B, C = 1, \cdots, dim\,(G = SU3_{c}) = 8$$

Lie algebra labels, $\left[\frac{1}{2}\lambda^{A}, \frac{1}{2}\lambda^{B}\right] = i f_{ABC}\, \frac{1}{2}\lambda^{C}$

perturbative rescaling :

$$v^{A}_{\mu} = g\, v^{A}_{\mu\, pert}\,,\; F^{A}_{\mu\nu} = g\, F^{A}_{\mu\nu\, pert}$$

Degrees of freedom are seen in jets , in (e.g.) the energy momentum sum rule in deep inelastic scattering but not clearly in spectroscopy.

Completing $\Delta\,\mathcal{L}$ in Fermi gauges

(3)

$$\Delta\,\mathcal{L} = \left\{ \begin{array}{c} -\frac{1}{2\,\eta\, g^{2}} \left(\partial_{\mu} v^{\mu\, A}\right)^{2} \\ + \partial^{\mu}\, \overline{c}^{A} \left(D_{\mu}\, c\right)^{A} \end{array} \right\} \; ; \quad \eta \; : \; \textbf{gauge parameter}$$

ghost fermion fields : $c\,,\, \overline{c}\;\; ; \;\; \left(D_{\mu}\, c\right)^{A} = \partial_{\mu}\, c^{A} - f_{ABD}\, v^{B}_{\mu}\, c^{D}$

gauge fixing constraint : $C^{A} = \partial_{\mu}\, v^{\mu\, A}$

$\longrightarrow$

Gauge boson binary bilocal and adjoint string operators

One goal is, to identify – not just some candidate resonance – gluonic mesons, binary and higher modes, and to relate them to the base quantities within QCD.

(4)
$$\begin{aligned} & B_{[\mu_1 \nu_1], [\mu_2 \nu_2]} (x_1, x_2) = \\ & = F_{[\mu_1 \nu_1]} (x_1 ; A) \, U (x_1, A ; x_2, B) \, F_{[\mu_2 \nu_2]} (x_2 ; B) \\ & A, B, \cdots = 1, \cdots, 8 \ ; \quad \text{no flavor but spin} \end{aligned}$$

$F_{[\mu \nu]} (x ; A)$ **denote the color octet of field strengths.**
The quantity $U (x, A ; y, B)$ **in eq. (4) denotes the octet string operator, i. e. the path ordered exponential over a straight line path** $\mathcal{C}$ **from y to x**

(5)
$$\begin{aligned} & U (x, A ; y, B) = P \exp \left(\int_y^x \Big|_{\mathcal{C}} d z^{\mu} \, \tfrac{1}{i} \, v_{\mu} (z, D) \, \mathcal{F}_D \right)_{A B} \\ & (\mathcal{F}_D)_{AB} = i \, f_{A D B} \end{aligned}$$

with the local limit $\longrightarrow$

(6) $$B_{[\mu_1 \nu_1],[\mu_2 \nu_2]}(x_1 = x_2 = x) = (:) F^A_{[\mu_1 \nu_1]}(x) F^A_{[\mu_2 \nu_2]}(x)(:)$$

no flavor but spin

The same procedure involving a triplet string applies to $\overline{q}\, q$ bilinears

$$B^q_{[\dot{\mathcal{A}}\, \dot{f}_1,\, \mathcal{B}\, f_2]}(x_1, x_2) = \overline{q}^{\,\dot{c}_1}_{\mathcal{B}\, \dot{f}_2}(x_1)\, U(x_1, c_1; x_2, \dot{c}_2)\, q^c_{\mathcal{A}\, f_1}(x_2)$$

(7) $$U(x_1, c_1; x_2, \dot{c}_2) = P \exp\left(\int_y^x \Big|_{\mathcal{C}} d z^\mu \tfrac{1}{i} v_\mu(z, D) \tfrac{1}{2} \lambda_D \right)_{c_1 \dot{c}_2}$$

flavor *and* spin

with the local limit

(8) $$B^q_{[\dot{\mathcal{B}}\, \dot{f}_2,\, \mathcal{A}\, f_1]}(x_1 = x_2 = x) = (:) \overline{q}^{\,\dot{c}}_{\mathcal{B}\, \dot{f}_2}(x)\, q^c_{\mathcal{A}\, f_1}(x)\,(:)$$

The symbols $(:)$ in eqs. 6 and 8 should indicate that normal ordering of regulating the local limits is required and further that such normal ordering is *not* unique , and dependent on quark masses in the case of the $\overline{q}\, q$ bilinears . $\longrightarrow$

The U1-axial central anomaly involves the local chiral current projections from $B^{\,q}_{\,[\,\dot{\mathcal{B}}\,\dot{f}\,_2\,,\,\mathcal{A}\,f\,_1\,]}\,(x\,)$ **in eq. 8**

$$\left(j^{\,\pm}_{\,\mu}\right)_{\dot{f}\,_2\,f\,_1}\,(\,x\,) \;=\; B^{\,q}_{\,[\,\dot{\mathcal{B}}\,\dot{f}\,_2\,,\,\mathcal{A}\,f\,_1\,]}\,(x\,)\,\left(\gamma_{\,\mu}\,\tfrac{1}{2}\,(\,\P\,\pm\,\gamma_{\,5}\,)\right)_{\mathcal{B}\,\dot{\mathcal{A}}}$$

(9)

$$=\;(:)\,\overline{q}^{\,\dot{c}}_{\,\dot{f}\,_2}\,\gamma^{\,\pm}_{\,\mu}\,q^{\,c}_{\,f\,_1}\,(\,x\,)\,(:)$$

$$\gamma_{\,5} \;=\; \gamma_{\,5\,R} \;=\; \tfrac{1}{i}\,\gamma_{\,0}\,\gamma_{\,1}\,\gamma_{\,2}\,\gamma_{\,3} \;\;;\;\; \gamma^{\,\pm}_{\,\mu} \;=\; \gamma_{\,\mu}\,\tfrac{1}{2}\,(\,\P\,\pm\,\gamma_{\,5}\,)$$

The equations of motion for the fermion fields are *and superficially imply* **(upon** $f\,_1 \;\leftrightarrow\; f\,_2$ **)**

$$\begin{array}{lll} \partial\!\!\!/\;q^{\,c}_{\,f\,_2} & = & \tfrac{1}{i}\,\left(v\!\!\!/^{\,c\,\dot{c}'} + \delta^{\,c\,\dot{c}'}\,m_{\,f\,_2}\right)\,q^{\,c'}_{\,f\,_2} \\ \overline{q}^{\,\dot{c}}_{\,\dot{f}\,_1}\;\overleftarrow{\partial\!\!\!/} & = & \overline{q}^{\,\dot{c}'}_{\,\dot{f}\,_1}\,\tfrac{1}{i}\,\left(-\,v\!\!\!/^{\,c'\,\dot{c}} - \delta^{\,c'\,\dot{c}}\,m_{\,f\,_1}\right) \end{array} \quad ; \quad \text{no sums over } \dot{f}\,_1\,,\,f\,_2 \;\longrightarrow$$

$$\partial^{\,\mu}\,\left(j^{\,\pm}_{\,\mu}\right)_{\dot{f}\,_1\,f\,_2} = \frac{1}{2\,i}\,\left(\,(\,m_{\,f\,_2} - m_{\,f\,_1}\,)\;S_{\,\dot{f}\,_1\,f\,_2} \mp (\,m_{\,f\,_2} + m_{\,f\,_1}\,)\;P_{\,\dot{f}\,_1\,f\,_2}\right)$$

$$S_{\,\dot{f}\,_1\,f\,_2} = (:)\,\overline{q}^{\,\dot{c}}_{\,\dot{f}\,_1}\,q^{\,c}_{\,f\,_2}\,(:) \;\;,\;\; P_{\,\dot{f}\,_1\,f\,_2} = (:)\,\overline{q}^{\,\dot{c}}_{\,\dot{f}\,_1}\,\gamma_{\,5}\;\;q^{\,c}_{\,f\,_2}\,(:)$$

(10)

In eq. 10 $m_{\,f}$ **denotes the** *real* , *nonnegative* **quark mass for flavor f.** $\longrightarrow$

From eq. 10 the relations for vector and axial vector currents *superficially* follow

(11)

$$\begin{aligned}
(j_{\mu})_{\dot{f}_1 f_2} &= \left(j^{+}_{\mu}\right)_{\dot{f}_1 f_2} + \left(j^{-}_{\mu}\right)_{\dot{f}_1 f_2} \\
(j^{5}_{\mu})_{\dot{f}_1 f_2} &= \left(j^{+}_{\mu}\right)_{\dot{f}_1 f_2} - \left(j^{-}_{\mu}\right)_{\dot{f}_1 f_2} \\
\partial^{\mu} (j_{\mu})_{\dot{f}_1 f_2} &= \tfrac{1}{i} \left(m_{f_2} - m_{f_1}\right) S_{\dot{f}_1 f_2} \\
\partial^{\mu} (j^{5}_{\mu})_{\dot{f}_1 f_2} &= \left(m_{f_2} + m_{f_1}\right) i\, P_{\dot{f}_1 f_2}
\end{aligned}$$

As it follows from the original derivation by Adler and Bell and Jackiw [1-1-1969] in QED, the vecor current Ward identities in eq. 11 can be implemented also in QCD , leaving the axial current ones reduced to the flavor non-singlet case , leaving the U1 axial current divergent anomalous

$$\partial^{\mu} (j_{\mu})_{\dot{f}_1 f_2} = \tfrac{1}{i} \left(m_{f_2} - m_{f_1}\right) S_{\dot{f}_1 f_2} \quad \surd$$

(12)

$$\left\{ \begin{array}{c} j^{5}_{\mu} \\ P \end{array} \right\}^{NS}_{\dot{f}_1 f_2} = \left\{ \begin{array}{c} j^{5}_{\mu} \\ P \end{array} \right\}_{\dot{f}_1 f_2} - \frac{1}{N_{fl}} \delta_{\dot{f}_1 f_2} \sum_f \left\{ \begin{array}{c} j^{5}_{\mu} \\ P \end{array} \right\}_{\dot{f} f}$$

$\longrightarrow$

and similarly

(13) $$\left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}^S_{\dot{f}_1 f_2} = \sum_f \left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}_{\dot{f} f}$$

Quark masses and splittings : m_f and $\Delta m_f = m_f - \langle m \rangle$

In the subtitle above $\langle m \rangle$ stands for the mean quark mass

(14) $$\langle m \rangle = \frac{1}{N_{fl}} \sum_f m_f$$

The identities for vector currents in eqs. 11 and 12 can be extended separating the contributions proportional to Δm_f and $\langle m \rangle$

(15)
$$\partial^\mu (j_\mu)_{\dot{f}_1 f_2} = \frac{1}{i} (\Delta m_{f_2} - \Delta m_{f_1}) S_{\dot{f}_1 f_2} \quad \surd$$
$$\partial^\mu (j_\mu^5)^{NS}_{\dot{f}_1 f_2} = (\Delta m_{f_2} + \Delta m_{f_1}) \, i P^{NS}_{\dot{f}_1 f_2} \quad \surd$$
$$\partial^\mu (j_\mu^5)^S_{\dot{f}_1 f_2} = 2 \langle m \rangle \, i P^S \quad \surd\!\!\!/ \; [\longrightarrow + \delta_5]$$
$$\delta_5 = (2 N_{fl}) \frac{1}{32\pi^2} F^A_{\mu\nu} \widetilde{F}^{\mu\nu A} \Big|_{\rightarrow ren.gr.inv} \; ; \; \widetilde{F}^A_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\sigma\tau} F^{\sigma\tau A}$$
[a]

$\longrightarrow$

[a] δ_5 was – as far as I know – introduced by Murray Gell-Mann in lectures $\sim$ 1970 in Hawaii .

The singlet axial current anomaly

We shall return to the question of how the local operator $ch_2 (F) \equiv \frac{1}{32\pi^2} (:) F^{A}_{\mu\nu} \widetilde{F}^{\mu\nu A} (:)$ **is to be normalized and rendered renormalization group invariant [1-2-1991] . Here we just assume this to have been achieved and denote the U1-axial anomaly, the first of the central two, in its general form (eq. 15)**

(16)
$$\left\{ \partial^{\mu} \left(j^{5}_{\mu} \right)^{S} = 2 \langle m \rangle \, i P^{S} + \delta_{5} \right\} (x)$$
$$\delta_{5} = \left(2 N_{fl} \right) \frac{1}{32\pi^2} \, (:) F^{A}_{\mu\nu} \widetilde{F}^{\mu\nu A} (:) \Big|_{\rightarrow ren.gr.inv}$$

From here it is conceptually clear how the scale- (or trace-) anomaly arises but strictly within QCD . The renormalizability of a field theory in the limit of uncurved space-time gives rise to a local , symmetric and *conserved* **energy momentum tensor , implying exact Poincaré invariance**

(17)
$$\{ \vartheta_{\mu\nu} = \vartheta_{\nu\mu} \} (x)$$
$$\partial^{\nu} \vartheta_{\mu\nu} = 0$$

In connection with the normal ordering questions it is important to admit in the precise form of the energy momentum tensor a nontrivial vacuum expected value , which →

in view of exact Poincaré invariance must be of the form

$$\langle \Omega | \vartheta_{\mu\nu}(x) | \Omega \rangle = \frac{1}{4} \eta_{\mu\nu} \tau$$

$$\left\{ \begin{array}{c} \eta_{\mu\nu} = diag(1, -1, -1, -1) \\ \tau \end{array} \right\}$$ independent of x $\longrightarrow$

(18)

$$\Delta \vartheta_{\mu\nu}(x) = \vartheta_{\mu\nu}(x) - \langle \Omega | \vartheta_{\mu\nu}(x) | \Omega \rangle \times \left\{ \begin{array}{l} \widehat{\P} \\ \text{or} \quad | \Omega \rangle \langle \Omega | \end{array} \right.$$

with $\partial^{\nu} \Delta \vartheta_{\mu\nu}(x) = 0 \;\; ; \;\; \langle \Omega | \Delta \vartheta_{\mu\nu}(x) | \Omega \rangle = 0$

In eq. 18 $\widehat{\P}$ denotes the unit operator in the entire Hilbert space of states , while $P_{\Omega} = | \Omega \rangle \langle \Omega |$ stands for the projector on the ground state .

Furthermore from the two local , *conserved* tensors in eq. 18 only $\Delta \vartheta_{\mu\nu}(x)$ with vanishing vacuum expected value is acceptable as representing the conserved 4 momentum *operators* in the integral form

(19) $$\widehat{P}_{\mu} = \int_t d^3 x \, \Delta \vartheta_{\mu 0}(t, \vec{x})$$

$\longrightarrow$

All these arguments ***notwithstanding*** **to subtract any eventual vacuum expected values of local operators , often put forward as mathematical prerequisites , it is wise** ***not to do so*** **in the presence of spontaneous parameters , the dynamical origin of spontaneous symmetry breaking, e.g. chiral symmetries in the limit or neihbourhood of some** $m_f \rightarrow 0$ **.**

Using the (classical) equations of motion pertaining to the Lagrangean in eqs. 1 - 3

$$(D_\nu F^{\mu\nu})^A = j^{\mu\ A}(\overline{q}, q) \ ; \ F \rightarrow F_{pert}$$

$$(D_\varrho F^{\mu\ \nu})^A = \partial_\varrho F^{\mu\ \nu\ A} - f_{ABD}\, v_\varrho^B\, F^{\mu\ \nu\ D}$$

(20) $$j_\mu^A(\overline{q}, q) = g\,\overline{q}^{\dot{c}}_{\dot{\mathcal{A}}\ \dot{f}}\,(\gamma_\mu)_{\dot{\mathcal{A}}\ \mathcal{B}}\ \frac{1}{2}\left(\lambda^A\right)_{c\dot{c}'}\, q^{c'}_{\mathcal{A}\ f}$$

$$i\,(\gamma^\mu D_\mu q)^c_{\mathcal{A}\ f} = m_f\, q^c_{\mathcal{A}\ f} \quad \textbf{and} \quad q \rightarrow \overline{q}$$

$$(D_\mu q)^c_{\mathcal{A}\ f} = \left[\partial_\mu \delta_{c\dot{c}'} + i\,g\,v^D_\mu\ \frac{1}{2}\left(\lambda^D\right)_{c\dot{c}'}\right] q^{c'}_{\mathcal{A}\ f}$$

the associated form of the energy momentum becomes $\longrightarrow$

(21)
$$\vartheta_{\mu\,\nu}^{\,(cl)} \;=\; \left[\begin{array}{l} F_{\mu\,\varrho}^{\,A}\,F^{\,\varrho\,A}_{\;\;\nu} \;-\; \frac{1}{4}\,\eta_{\,\mu\,\nu}\,F_{\sigma\,\varrho}^{\,A}\,F^{\,\varrho\,\sigma\,A} \;+ \\ +\;\frac{1}{2}\,\left[\overline{q}_{\,\dot{f}}\,\gamma_{\,\mu}\,\frac{i}{2}\,\overleftrightarrow{D}_{\nu}\;q_{\,f}\;+\;\mu\;\leftrightarrow\;\nu\right] \end{array}\right]$$

and using once more the fermion part of the equations of motion the trace of the classical energy momentum tensor becomes

(22)
$$\vartheta^{\,\mu\;(cl)}_{\;\;\mu} \;=\; \sum_{\,f}\,m_{\,f}\,S_{\,\dot{f}\;f}$$
$$S_{\,\dot{f}_{\,1}\;f_{\,2}} \;=\; (:)\,\overline{q}^{\;\dot{c}}_{\;\dot{f}_{\,1}}\;q^{\;c}_{\;f_{\,2}}\;(:)$$

The scale- or trace- anomaly

From the classical soft fermionic contribution to the trace of the energy momentum tensor there is a clear conjecture , also by Murray Gell-Mann , of the anomalous contribution , which subsequently became the scale- or trace- anomaly within QCD

(23)
$$\vartheta^{\,\mu}_{\;\;\;\mu} \;=\; \sum_{\,f}\,m_{\,f}\,S_{\,\dot{f}\;f} \;+\; \delta_{\,0}$$
$$\delta_{\,0} \;=\; -\;\left(\,-2\,\beta\,(\,g\,)\,/\,g^{\,3}\,\right)\;\left[\,\frac{1}{4}\,(:)\,F^{\,A}_{\mu\,\nu}\,F^{\,\mu\,\nu\,A}\,(:)\,\right]_{\;\rightarrow\;ren.gr.inv}$$

$\longrightarrow$

The two central anomalies alongside : scale- or trace- and U1-axial anomaly

We collect the two anomalous identities in eqs. 23 and 16

$$\left\{ \vartheta^{\mu}{}_{\mu} = \sum_{f} m_{f} S_{\dot{f}\, f} + \delta_{0} \right\} (x)$$

$$\left\{ \partial^{\mu} \left(j^{5}_{\mu} \right)^{S} = 2 \langle m \rangle \, i P^{S} + \delta_{5} \right\} (x)$$

(24)

$$\delta_{0} = - \left(-2 \beta(g) / g^{3} \right) \left[\tfrac{1}{4} (:) F^{A}_{\mu\nu} F^{\mu\nu A} (:) \right]_{\rightarrow ren.gr.inv}$$

$$\delta_{5} = \left(2 N_{fl} \right) \tfrac{1}{8\pi^{2}} \left[\tfrac{1}{4} (:) F^{A}_{\mu\nu} \widetilde{F}^{\mu\nu A} (:) \right]_{\rightarrow ren.gr.inv}$$

$$-\beta / g^{3} = \tfrac{1}{16\pi^{2}} b_{0} + O(Y) \; ; \; Y = g^{2} / (16 \pi^{2}$$

$\beta(g)$: **Callan-Symanzik rescaling function in QCD**

The qualification 'central' for the anomalies in eq. 24 stands for the property that in rendering the square coupling constant and the associated $\vartheta -$ **parameter in the gauge boson** *renormalized* **Lagrangean density x dependent**

(25)

$$\mathcal{L}_{g.b.} = - \tfrac{1}{g^{2}} \tfrac{1}{4} (:) F^{A}_{\mu\nu} F^{\mu\nu A} (:) + \vartheta \tfrac{1}{8\pi^{2}} \tfrac{1}{4} (:) F^{A}_{\mu\nu} \widetilde{F}^{\mu\nu A} \longrightarrow$$

$$g^{2} \rightarrow g^{2}(x) \; ; \; \vartheta \rightarrow \vartheta(x)$$

maintains perturbative renormalizability and acts together with suitable boundary conditions $\rightarrow$

as external sources for the scalar and pseudoscalar local field strength bilinears

(26) $$\frac{1}{4}\,(:)\,F^{\,A}_{\mu\,\nu}\,F^{\,\mu\,\nu\,A}\,(:)\quad,\quad\frac{1}{4}\,(:)\,F^{\,A}_{\mu\,\nu}\,\widetilde{F}^{\,\mu\,\nu\,A}$$

We will use the following definitions relative to the rescaling function β

$$-\,\beta\,/\,g\;=\;X\,B\,(\,X\,)\quad;\quad B\,(\,X\,)\;=\;b_{\,0}\,A\,(\,X\,)$$

$$B\,(\,X\,)\;\sim\;\sum_{n=0}^{\infty}\,b_{\,n}\,X^{\,n}\quad,\quad A\,(\,X\,)\;\sim\;\sum_{n=0}^{\infty}\,a_{\,n}\,X^{\,n}$$

$\kappa\;=\;g^{\,2}\,/\,(\,16\,\pi^{\,2}\,)$ generic $X\,,\,Y$

(27) $$b_{\,0}\;=\;\frac{1}{3}\,\big(\,33\;-\;2\,N_{\,fl}\,\big)\quad,\quad a_{\,0}\;=\;1\quad,\quad a_{\,n}\;=\;b_{\,n}\,/\,b_{\,0}$$

$$b_{\,1}\;=\;\frac{2}{3}\,\big(\,153\;-\;19\,N_{\,fl}\,\big)$$

$$b_{\,2}\;=\;\frac{1}{54}\,\Big(\,77139\;-\;15099\,N_{\,fl}\;+\;325\,N_{\,fl}^{\,2}\,\Big)$$

$$b_{\,3}\;\sim\;29243\;-\;6946.3\,N_{\,fl}\;+\;405.089\,N_{\,fl}^{\,2}\;+\;1.49931\,N_{\,fl}^{\,3}$$

2-1 Consolidation and development of (new) methods adapted to the perturbatively accessible region – in a nutshell [a]

Three figures subsumming the work of many authors including those quoted below shall be *shortly* discussed →

[a] see the talks at this conference by D. Gross, J. Stirling, S. Frixione, W. Vogelsang, S. Bethke, N. Glover, J. Vermaseren, J. Kühn, K. Chetyrkin, S.-O. Moch, M. Steinhauser, M. Czakon, P. Uwer, Z. Bern, L. Dixon, T. Gehrmann and M. Neubert .

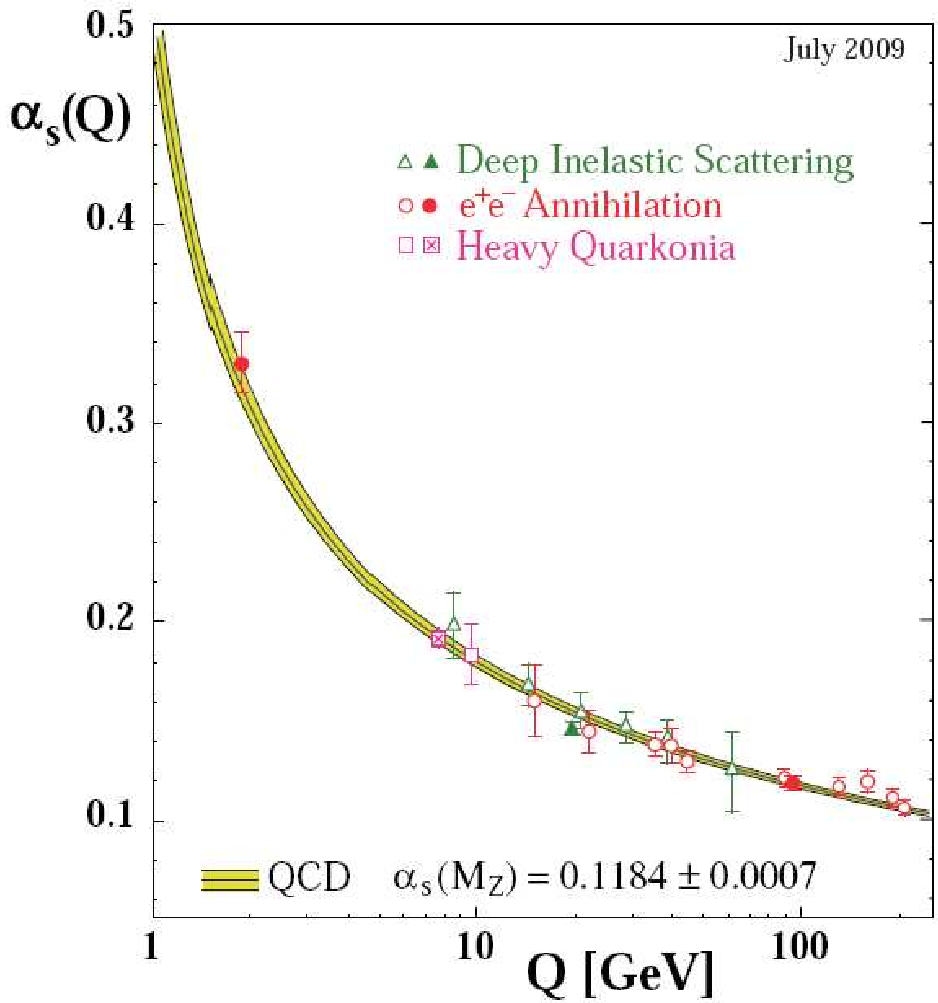

Fig A21 : $\alpha_s(Q) = 4\pi\,\kappa_{\overline{\mu}=Q}$ **from ref. [A215-2009] .**

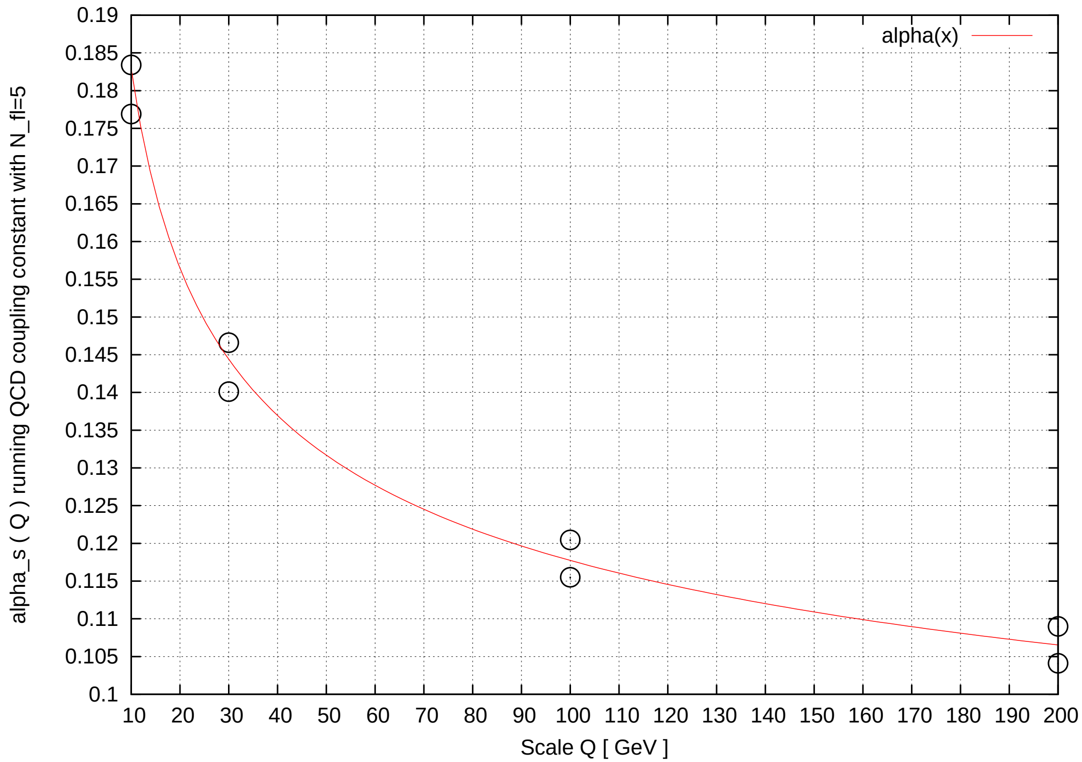

Fig A22 : $\alpha_s(Q) = 4\pi\,\kappa_{\overline{\mu}=Q}$ **compared with Fig. A21 .**

$\longrightarrow$

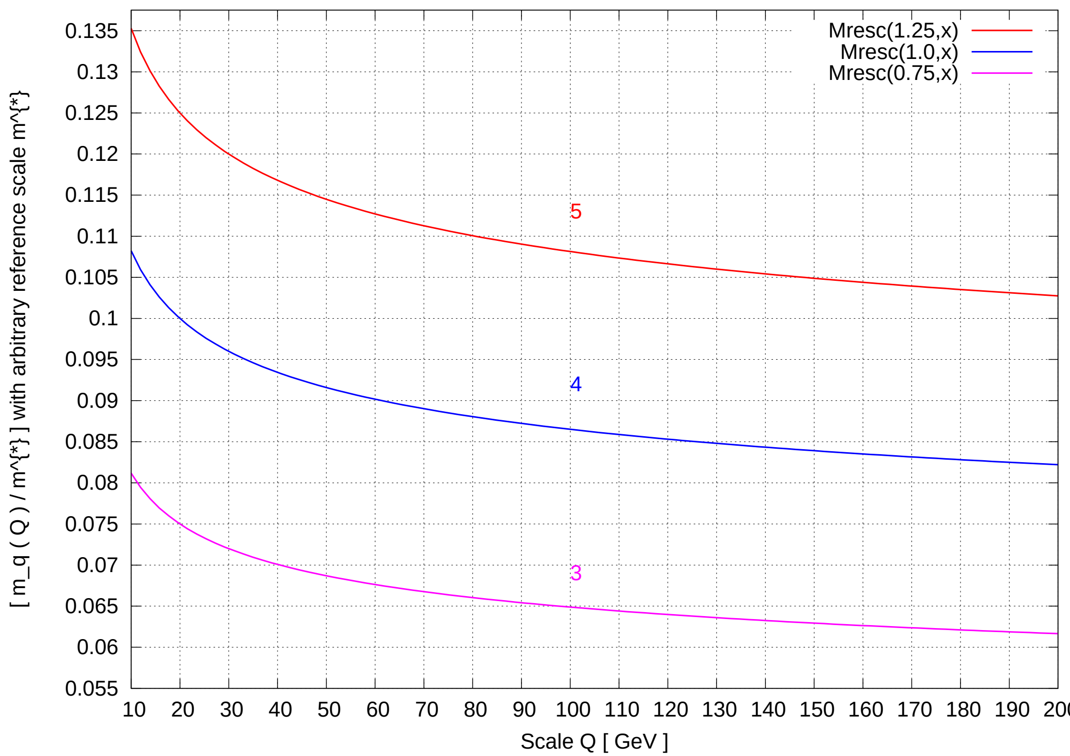

Fig A23 : $m_q(Q)/m^*$ **with fixed ratio of rescaled quark masses**

$$m_u : \frac{1}{2}(m_d + m_u) : m_d = 3 : 4 : 5 \, .$$

→

To Fig A21 : In the four loop evaluation of the running coupling constant in ref. [A215-2009] the renormalization group invariant quantity is obtained in the $\overline{MS}$ renormalization scheme

$$\Lambda_5^{(4)} = 213 \pm 9 \text{ MeV} \quad \longleftrightarrow \quad \alpha_s(m_Z) = 0.1184 \pm 0.0007$$

(28)
$$\Lambda_5^{(4)} = 213 \text{ MeV} \rightarrow \begin{cases} \Lambda_4^{(4)} = 296 \text{ MeV} \\ \Lambda_3^{(4)} = 338 \text{ MeV} \end{cases}$$

The matching between $N_{fl} = 5 \rightarrow 4 \rightarrow 3$ in ref. [A215-2009] involves the modeling of the b- and c-flavor associated thresholds through the perturbatively assigned b- and c-quark ***pole-*****masses $m_b = 4.7$ GeV , $m_c = 1.5$ GeV . This is a nonuniversal way to rescale quark masses , and thus does not follow the strict quark mass rescaling at zero quark mass , used here . As a comparison in determining up and down quark masses at an $\overline{MS}$ scale of 2 GeV , Dominguez, Nasrallah, Röntsch and Schilcher [A223-2008] use**

(29)
$$\Lambda_3^{(4)} = 381 \pm 16 \text{ MeV} \leftrightarrow \alpha_s(m_\tau) = 0.344 \pm 0.009$$

and adopting the scheme of quark mass rescaling at zero mass obtain for the u,d,s quark mass ratios

(30)
$$\begin{array}{ccccccc} m_u & : & \frac{1}{2}(m_d + m_u) & : & m_d & : & m_s \\ 2.9 \pm 0.2 & : & 4.1 \pm 0.2 & : & 5.3 \pm 0.5 & : & 102 \pm 8 \end{array}$$

→

To Fig A22 : The following value was used : $\alpha_s(M_Z) = 0.184$ corresponding – for the two loop running – to $\Lambda_5^{(2)} \sim 408$ MeV . The so determined running coupling constant is compared with the 1 - σ limits of the same quantity as determined in 4 loop order in ref. [A215-2009] confirming the validity of the two loop approximation in the range 10 GeV $\leq Q \leq 200$ GeV within the accuracy claimed in ref. [A215-2009] .

To Fig A23 : Here the strength and weakness of the mass rescaling at zero mass within the perturbatively accessible region is illustrated using as a guide *only* the ratio of u,d quark masses

(31)
$$\begin{array}{ccccc} m_u & : & \frac{1}{2}(m_d + m_u) & : & m_d \\ 3 & : & 4 & : & 5 \end{array}$$

It seems appropriate to me to refer to the *in principle* approach of rescaling in a universal way the coupling constant and quark masses *initially* restricting all analysis to the perturbatively accessible region , citing (adapting) the pertinent comment by Murray Gell-Mann :
’Rising when last (first) seen .’
On the other hand the progress achieved in transgressing the perturbatively accessible region , using universal mass rescaling , in refs. [A223-2008] , [A221-2006] and references cited therein, is significant, based on improved treatment of finite energy sum rules pioneered by Shifman , Vainshtain and Zakharov [A224-1979] . $\longrightarrow$

To Fig A23 *continued* **: It is worth noting that the value of the gauge boson condensate, found in ref. [A223-2008] , approximated as**

(32) $$\langle \Omega | \frac{\alpha_s}{\pi} : F^A_{\mu\nu} F^{\mu\nu\, A} : | \Omega \rangle \rightarrow 0.06 \text{ GeV}^4$$

is 5 times larger , than its original estimate in ref. [A224-1979] .

The basics of chiral expansions in assessing ratios of the u,d,s quark masses continue to provide additional benchmarks at low hadron energies [A225-2001] and references cited therein, while fine details of these ratios can be subject to improvement . Finally the validity of chiral expansions as guidelines for lattice calculations present another *strategy in principle* **[A226-2008] .**

We add here a few representative determination of $\alpha_s(m_Z)$

	$\alpha_s(m_Z)$	processes	source	authors
(33)	0.1176 ± 0.0020	average	[A227-2008]	PDG
	0.1172 ± 0.0022	thrust distributions at LEP	[A228-2008]	Becher , Schwartz
	0.1184 ± 0.0007	average	[A215-2009]	Bethke

2-2 The Cartan subalgebra injection and vacuum condensates

The two representative Regge trajectories and their challenge

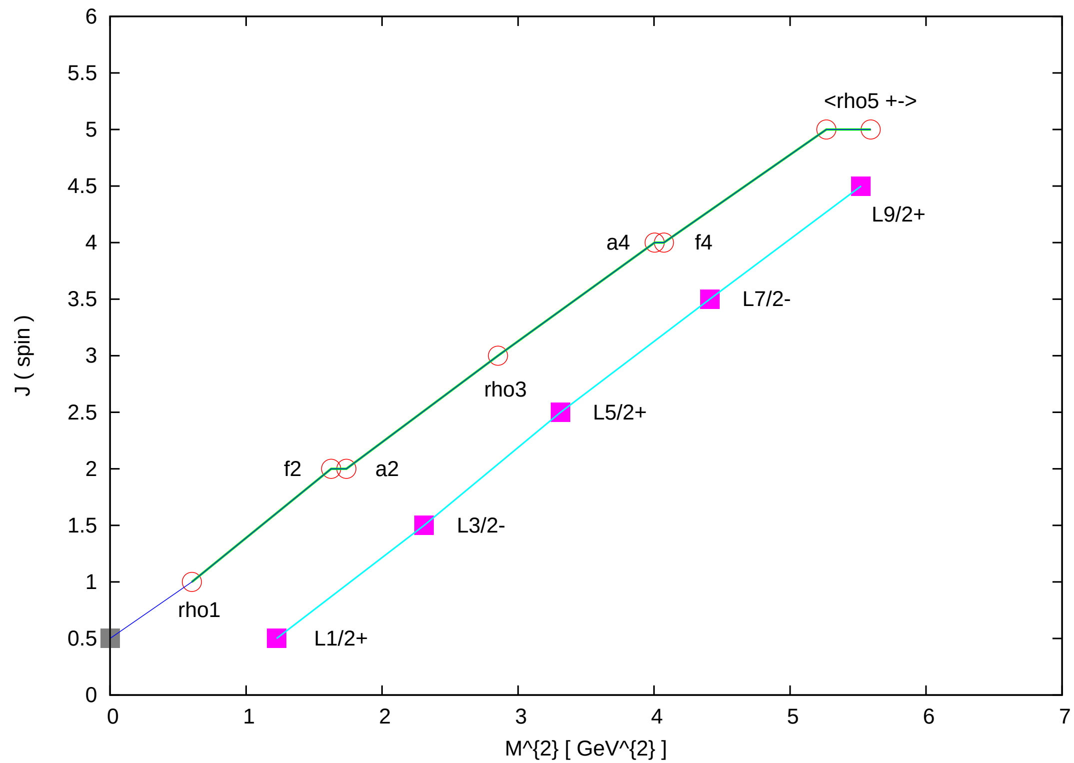

Fig R22 : The two representative Regge trajectories : $\varrho - f \,/\, a \,;\; J^{\,PC} \;(\,1, 3, 5\,)^{\,--}$ **and** $J^{\,PC} \;(\,2, 4\,)^{\,++}$ **together with** $\Lambda \,;\; J^{\,P} \;(\,\frac{1}{2}, \frac{5}{2}\,, \frac{9}{2}\,)^{\,+}$ **and** $J^{\,P} \;(\,\frac{3}{2}, \frac{7}{2}\,)^{\,-}$ **.**

$\longleftrightarrow$

The challenge has been met and invokes the search for 'harmonic analysis' in the framework of QCD and pertaining to $q\,\overline{q}$ *and* $3\,q$ configurations . These configurations shall con-form with the (apparently) universal spectrospic relation as shown in figure R22 and idealized to

$$A\,(\,s\,)\;=\;A'\,s\;+\;A_{\,0}\quad\leftrightarrow\quad s\;=\;(\,A'\,)^{\,-1}\,[\,A\,(s)\;-\;A_{\,0}\,]$$

(34) $$A\,(\,s\,)\;=\;J\quad\rightarrow\quad m^{\,2}_{\,J}\;=\;(\,A'\,)^{\,-1}\;J\;+\;m^{\,2}_{\,0}\;\;;\;\;=\;M^{\,2}\,J\;+\;m^{\,2}_{\,0}$$

$M^{\,2}\;=\;(\,A'\,)^{\,-1}\;\sim\;1.12$ **GeV** ^{2} **for** $q_{\,1}\,\overline{q}_{\,2}$ **and** $q_{\,1}\,q_{\,2}\,q_{\,3}$ $m_{\,q}\;\rightarrow\;0$ **configurations**

Fig 3 : 'the Pythagoreans'

$\longleftrightarrow$

Let me repeat the central anomalies here (eq. 24) and transform to euclidean space time

$$\left\{ \vartheta^{\,\mu}{}_{\mu} = \sum_f m_f \, S_{\dot{f}\, f} + \delta_0 \right\} (x)$$

$$\left\{ \partial^{\,\mu} \left(j^{\,5}_{\mu} \right)^{S} = 2 \, \langle m \rangle \, i \, P^{\,S} + \delta_5 \right\} (x)$$

$$\delta_0 = - \left(-2\,\beta(g) / g^3 \right) \left[\tfrac{1}{4} (:) \, F^{\,A}_{\mu\,\nu} \, F^{\,\mu\,\nu\,A} (:) \right]_{\rightarrow \, ren.gr.inv}$$

(35) $$\left[\tfrac{1}{4} (:) \, F^{\,A}_{\mu\,\nu} \, F^{\,\mu\,\nu\,A} (:) \right]_{\rightarrow \, ren.gr.inv} \equiv \tfrac{1}{2} (:) \left[\left(\vec{\mathcal{B}}^{\,A} \right)^2 + \left(\vec{\mathcal{E}}^{\,A} \right)^2 \right] (:) \Big|_{eucl}$$

$$\left[\tfrac{1}{4} (:) \, F^{\,A}_{\mu\,\nu} \, \widetilde{F}^{\,\mu\,\nu\,A} (:) \right]_{\rightarrow ren.gr.inv} \equiv (:) \left[\vec{\mathcal{B}}^{\,A} \, \vec{\mathcal{E}}^{\,A} \right] (:) \Big|_{eucl}$$

$\left(\vec{\mathcal{B}}^{\,A} \, , \, \vec{\mathcal{E}}^{\,A} \right)_{eucl}$: **euclidean magnetic- and electric field strengths**

$(:) \, , \; _{eucl}$: **to be suppressed in the following**

This brings up the question – for me long 'undecidable' until the seminal work of Shifman, Vainshtein and Zakharov [A224-1979] induced a new try *to the affirmative* **[in refs. [2-21-1978]** *and* **[2-22-1980]] – whether the positive parity operator $\frac{1}{4} F^{\,A}_{\mu\,\nu} \, F^{\,\mu\,\nu\,A}$ could develop a vacuum expected value .** $\longrightarrow$

Indeed

$$_{B^*}\langle\, \Omega \,|\; \tfrac{1}{4}\, F^{\,A}_{\mu\,\nu}\, F^{\,\mu\,\nu\,A} \;|\, \Omega\, \rangle_{B^*} \;=\; p \;\equiv\; (\, B^* \,)^{\,2} \;>\; 0$$

(36) $$B^* \;=\; \tfrac{1}{2}\, M^{\,2} \;\sim\; m^{\,2}_{\varrho} \;\sim\; 0.60 \;\pm\; 0.007 \text{ GeV}^{\,2} \qquad \longrightarrow$$

$$p \;=\; \tfrac{1}{4}\, M^{\,4} \;\sim\; m^{\,4}_{\varrho} \;\sim\; 0.36 \;\pm 0.01 \text{ GeV}^{\,4}$$

The estimate from charmonium sum rules (ref. [A224-1979]) was $p \;\sim\; 0.12$ GeV ^{4} , while in a recent evaluation of the u-d flavor currents and (pseudo-) scalar densities $S_{\,\dot{f}_{\,1}\, f_{\,2}}$ and $P_{\,\dot{f}_{\,1}\, f_{\,2}}$ by Dominguez, Nasrallah, Röntsch and Schilcher in ref. [A223-2008] approximated as (eq. 32)

(37) $$\langle\, \Omega \,|\; \frac{\alpha_{\,s}}{\pi} \;:\; F^{\,A}_{\mu\nu}\, F^{\,\mu\nu\;A} \;:|\, \Omega\, \rangle \;=\; \frac{1}{\pi^{\,2}}\, p \;\longrightarrow\; 0.06 \text{ GeV}^{\,4}$$

yields $p \;\sim\; 0.59$ GeV ^{4} , (almost) 5 times larger than the original estimate .

The fields $(V^a_\mu)_\infty$, $(V^a_{\mu\nu})_\infty$ in eq. (1) are extrema of the classical action subject to the constraint

$$\int \left[\tfrac{1}{4} V^a_{\mu\nu} V^a_{\mu\nu} - p \right] \mathrm{d}^4 x = \text{finite},$$

$$p = \text{constant} = 4(R_\infty)^2 . \tag{4}$$

The constant p in eq. (4) with the dimension of pressure [i.e., (mass)^4] is related to the gluon pairing strength—a spontaneous parameter of the ground state:

$$\left\langle \Omega \middle| \left(: V^a_{\mu\nu}(x) V^b_{\alpha\beta}(x) : \right)_{\mathrm{ph}} \middle| \Omega \right\rangle = \tfrac{1}{3}\delta_{ab} \left\{ \left[g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha} \right] \tfrac{1}{3} p - \varepsilon_{\mu\nu\alpha\beta} \tfrac{1}{3} q \right\} . \tag{5}$$

Fig 4 : boundary conditions are important (from ref. [2-22-1980] with $V\,;\;^{a,b} \;\longrightarrow\; F\,;\;^{A,B}$)

and gauge group $SU2_{\,c} \;\longrightarrow$

Consequences from the gauge boson bilinear vacuum expected value (eq. 36)

(38) $$_{B^*}\langle \Omega | \ \tfrac{1}{4} F^{\ A}_{\mu\ \nu} F^{\mu\ \nu\ A} \ | \Omega \rangle_{B^*} = p \equiv (B^*)^2 > 0$$

1) $p > 0$ **in eq. 38 represents a local , gauge invariant Bose** *field pair-* **condensation , coupled with the full Poincaré invariance pertaining to the ground state.**

2) *single field quantum-* **Bose condensation is forbidden by exact gauge invariance.**

3) the symmetries including C , P , T of the ground state in 1) and 2) enforce the spin structure

$$_{B^*}\langle \Omega | \ \tfrac{1}{4} F^{\ A}_{\mu\nu} F^{\sigma\tau\ A} \ | \Omega \rangle_{B^*} = X^{\ \sigma\ \tau}_{\mu\ \nu} = \tfrac{1}{3} \left(\delta^{\ \sigma}_{\mu} \delta^{\ \tau}_{\nu} - \delta^{\ \tau}_{\mu} \delta^{\ \sigma}_{\nu} \right) p$$

$$Y^{\ \sigma}_{\mu} = X^{\ \sigma\ \varrho}_{\mu\ \varrho} = \delta^{\ \tau}_{\mu} p \ , \ Z = Y^{\ \mu}_{\mu} = 4 p \ ; \ q = 0 \ \rightarrow$$

$$\vartheta^{\ \sigma}_{\mu} = \left(\vartheta^{\ \sigma}_{\mu} - \tfrac{1}{4} \delta^{\ \sigma}_{\mu} \vartheta^{\ \varrho}_{\varrho} \right) + \tfrac{1}{4} \delta^{\ \sigma}_{\mu} \vartheta^{\ \varrho}_{\varrho}$$

$$_{B^*}\langle \Omega | \ \vartheta^{\ \sigma}_{\mu} - \tfrac{1}{4} \delta^{\ \sigma}_{\mu} \vartheta^{\ \varrho}_{\varrho} \ | \Omega \rangle_{B^*} = 0$$

(39) $$\vartheta^{\ \sigma}_{\mu} \Big|_{spin\ 0} = \tfrac{1}{4} \delta^{\ \sigma}_{\mu} \vartheta^{\ \varrho}_{\varrho} = - \frac{1}{\pi^2} \frac{b_0}{8} \delta^{\ \sigma}_{\mu} \left[\tfrac{1}{4} F^{\ A}_{\alpha\beta} F^{\alpha\beta\ A} \right] \ \rightarrow$$

$$_{B^*}\langle \Omega | \ \vartheta_{\mu\nu} \ | \Omega \rangle_{B^*} = (- \eta_{\mu\ \nu}) \frac{1}{4 \pi^2} \frac{b_0}{8} p = \begin{cases} \varepsilon \ \textbf{for} \ \mu = \nu = 0 \\ \tilde{p} \ \textbf{for} \ \mu = \nu = 1, 2, 3 \end{cases}$$

with $\tilde{p} = - \varepsilon > 0$ $\longrightarrow$

2-2-5

3) (continued) – the last two relations in eq. 39 constitute a cosmological term with *positive pressure and negative energy density* **,** *defying* **a response from gravity , whence restricted to four space-time dimensions , yet consistent with the ground state belonging to** *the* **minimum of energy density .**

4) the embedding of chiral symmetry depends in a nontrivial way on the strength of the *field pair-* **Bose condensate (in eq. 39) as does the phase structure of QCD** *and* **the excitation of binary and higher gauge boson compounds ('glueballs') . The experimental verification of their spectral properties however is reminiscent of the 'odyssey' – and thus shall be continued in due time .**

— Thank you —

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