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# Extremal Black-Holes and Attractors

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On the occasion of P. Minkowski retirement.

In his long and lasting career  
 Peter has often worked on  
 applications of group-theoretical  
 techniques to the theory  
 of particle interactions and  
 in particular the role that  
 "exceptional groups" may  
 have in this context

(for example  $G_2, E_6 \dots$ ).

Such groups, and in particular  
 their non compact forms, remarkably  
 appear in supergravity and  
 superstring theory and they play  
 an important role in some aspects  
 of the physics of extremal black holes

# SYMMETRIC SPACES BASED ON EXCEPTIONAL GROUPS

(WHICH OCCUR IN SUPERGRAVITY)

d		N. COMPACT form <i>M type</i>	$\Lambda_{\text{min}}$	H
14	$\mathfrak{g}_2$	$\mathbb{Q}$ $\mathfrak{g}_{2,2}$	2	$SU(2) \times SU(2)$
51	$\mathfrak{f}_4$	$\mathbb{Q}$ $\mathfrak{f}_{4,4}$	4	$USp(6) \times SU(2)$
		$N=9, d=3$ $\mathfrak{f}_{4,-20}$	1	$SO(9)$
		$N=8, d=5$ $\mathfrak{e}_6(6)$	6	$Usp(8)$
78	$\mathfrak{e}_6$	$N=10, d=3$ $\mathfrak{e}_6(-14)$	2	$SO(10) \times SO(2)$
		$\mathbb{Q}$ $\mathfrak{e}_6(2)$	4	$SU(6) \times SU(2)$
		$\mathfrak{e}_6(-26)$	2	$\mathfrak{f}_4$
		$N=8, d=4$ $\mathfrak{e}_7(7)$	7	$SU(8)$
133	$\mathfrak{e}_7$	$\mathbb{Q}$ $\mathfrak{e}_7(-5)$	4	$SO(12) \times SU(2)$
		SK $\mathfrak{e}_7(-25)$	3	$\mathfrak{e}_6 \times SO(2)$
		$N=16, d=3$ $\mathfrak{e}_8(8)$	8	$SO(16)$
248	$\mathfrak{e}_8$	$\mathbb{Q}$ $\mathfrak{e}_8(-24)$	4	$\mathfrak{e}_7 \times SU(2)$

## CLASSIFICATIONS

TABLE V

IRREDUCIBLE RIEMANNIAN GLOBALLY SYMMETRIC SPACES OF TYPE I AND TYPE III

	Noncompact	Compact	Rank	Dimension
<i>A I</i>	$SL(n, R)/SO(n)$	$SU(n)/SO(n)$	$n - 1$	$\frac{1}{2}(n - 1)(n + 2)$
<i>A II</i>	$SU^*(2n)/Sp(n)$	$SU(2n)/Sp(n)$	$n - 1$	$(n - 1)(2n + 1)$
<i>A III</i>	$SU(p, q)/S(U_p \times U_q)$	$SU(p + q)/S(U_p \times U_q)$	$\min(p, q)$	$2pq$
<i>BD I</i>	$SO_0(p, q)/SO(p) \times SO(q)$	$SO(p + q)/SO(p) \times SO(q)$	$\min(p, q)$	$pq$
<i>D III</i>	$SO^*(2n)/U(n)$	$SO(2n)/U(n)$	$[\frac{1}{2}n]$	$n(n - 1)$
<i>C I</i>	$Sp(n, R)/U(n)$	$Sp(n)/U(n)$	$n$	$n(n + 1)$
<i>C II</i>	$Sp(p, q)/Sp(p) \times Sp(q)$	$Sp(p + q)/Sp(p) + Sp(q)$	$\min(p, q)$	$4pq$
<i>E I</i>	$(e_{6(6)}, sp(4))$	$(e_{6(-78)}, sp(4))$	6	42
<i>E II</i>	$(e_{6(2)}, su(6) + su(2))$	$(e_{6(-78)}, su(6) + su(2))$	4	40
<i>E III</i>	$(e_{6(-14)}, so(10) + R)$	$(e_{6(-78)}, so(10) + R)$	2	32
<i>E IV</i>	$(e_{6(-26)}, f_4)$	$(e_{6(-78)}, f_4)$	2	26
<i>E V</i>	$(e_{7(7)}, su(8))$	$(e_{7(-133)}, su(8))$	7	70
<i>E VI</i>	$(e_{7(-5)}, so(12) + su(2))$	$(e_{7(-133)}, so(12) + su(2))$	4	64
<i>E VII</i>	$(e_{7(-25)}, e_6 + R)$	$(e_{7(-133)}, e_6 + R)$	3	54
<i>E VIII</i>	$(e_{8(8)}, so(16))$	$(e_{8(-248)}, so(16))$	8	128
<i>E IX</i>	$(e_{8(-24)}, e_7 + su(2))$	$(e_{8(-248)}, e_7 + su(2))$	4	112
<i>F I</i>	$(f_{4(4)}, sp(3) + su(2))$	$(f_{4(-52)}, sp(3) + su(2))$	4	28
<i>F II</i>	$(f_{4(-20)}, so(9))$	$(f_{4(-52)}, so(9))$	1	16
<i>G</i>	$(g_{2(2)}, su(2) + su(2))$	$(g_{2(-14)}, su(2) + su(2))$	2	8