Nucleon Spin

Where does it come from?

Harald Fritzsch
Munich

QCD:

Nucleons consist of quark and gluons
1970
(glue ~ 50% of momentum, i.e.
important part of nucleon)

Gluons → spin?

Nucleon spin:

due to quarks and gluons

Simple, but very successful
quark models: spin solely due
to quarks

SU(6) →

Const. Quarks →

Quark-Spins of Hadrons

SU(6):

$$p = \frac{1}{\sqrt{18}} \,| 2 u\Uparrow u\Uparrow d\Downarrow + 2 u\Uparrow d\Downarrow u\Uparrow + 2 d\Downarrow u\Uparrow u\Uparrow$$
$$- u\Uparrow u\Downarrow d\Uparrow - u\Uparrow d\Uparrow u\Downarrow - u\Downarrow d\Uparrow u\Uparrow$$
$$- d\Uparrow u\Downarrow u\Uparrow - d\Uparrow u\Uparrow u\Downarrow - u\Downarrow u\Uparrow d\Uparrow \rangle$$

$$n = \dots \quad (u \leftrightarrow d)$$

Quark-Spin:

$$\langle p\Uparrow | \tfrac{1}{2}\sigma_q | p\Uparrow \rangle$$

s-Wave: $\vec{J} = \sum_q \frac{1}{2}\vec{\sigma}_q$

$$\langle p\Uparrow | \tfrac{1}{2}\sigma_u | p\Uparrow \rangle = \frac{1}{18}(4\cdot 1\cdot 3 + 0) = \frac{2}{3}$$

$$\langle p\Uparrow | \tfrac{1}{2}\sigma_d | p\Uparrow \rangle = \frac{1}{18}\left(4\cdot(-\tfrac{1}{2})3 + \tfrac{1}{2}\cdot 6\right) = -\frac{1}{6}$$

$$\langle \tfrac{1}{2}\sigma_u + \tfrac{1}{2}\sigma_d \rangle = \frac{1}{2}$$

Neutron: $\langle n\Uparrow | \frac{1}{2}\sigma_u | n\Uparrow \rangle = -\frac{1}{6}$

$$\langle n\Uparrow | \tfrac{1}{2}\sigma_d | n\Uparrow \rangle = \frac{2}{3}$$

Λ: $\langle \Lambda\Uparrow | \frac{1}{2}\sigma_u | \Lambda\Uparrow \rangle = \langle \Lambda\Uparrow | \frac{1}{2}\sigma_d | \Lambda\Uparrow \rangle = 0$

$$\langle \Lambda\Uparrow | \tfrac{1}{2}\sigma_s | \Lambda\Uparrow \rangle = \frac{1}{2}$$

Magnetic moments

proton: 2,793 μ_N

neutron: $-1{,}913$ μ_N

Λ : -0.613 μ_N

Σ^+: 2.458 μ_N

Σ^-: (-1.16 ± 0.03) μ_N

Expected, taking $\frac{m_s}{m_u}$ from Λ:

Σ^+: 2.68 μ_N !

Σ^-: -0.51 μ_N

SU(3)-limit:

Σ^+: 2.793 μ_N

Σ^-: -0.93 μ_N

Magnetic Moments

Constituent Quarks:

$$\vec{\mu}(p) = \sum_q \vec{\mu}(q)$$

$$= \sum_q e_q \left(\frac{e}{m_q}\right) \frac{1}{2} \vec{\sigma}_q$$

$$\mu(p) = \frac{e}{m_q}\left[\frac{2}{3}\langle\tfrac{1}{2}\sigma_u\rangle - \frac{1}{3}\langle\tfrac{1}{2}\sigma_d\rangle\right]$$

$$= \frac{e}{m_q}\left(\frac{2}{3}\cdot\frac{2}{3} - \frac{1}{3}\left(-\frac{1}{6}\right)\right) = \frac{e}{2m_q}$$

Exp.: $\mu(p) \simeq 2.79\,\mu_N$

$m_q \simeq 336$ MeV

$$\mu(n) = \frac{e}{m_q}\left(\frac{2}{3}\left(-\frac{1}{6}\right) - \frac{1}{3}\cdot\frac{2}{3}\right)$$

$$= -\frac{2}{3}\cdot\frac{e}{2m_q}$$

$$\rightarrow \mu(p)/\mu(n) = -\frac{3}{2}$$

Exp: -1.47!

$$\mu(\Lambda) = \frac{e}{m_s}\left(\left(-\frac{1}{3}\right)\times\frac{1}{2}\right)$$

$$= -\frac{1}{3}\cdot\frac{e}{2m_s}$$

$$\mu(\Lambda)\,/\,\mu(p) = \left(-\frac{1}{3}\right)\cdot\left(\frac{m_{u,d}}{m_s}\right)$$

Exp.:

$$\frac{\mu(\Lambda)}{\mu(p)} \approx -0.22$$

$$\frac{m_{u,d}}{m_s} \cong 0.66$$

$m_{u,d} = 336$

$\rightarrow m_s \approx 510$ MeV

$m_s - m_{u,d} \approx 174$ MeV

($\rightarrow$ chiral dynamics)

But: $\frac{G_A}{G_V} = \langle \sigma_u - \sigma_d \rangle = 2 \cdot \left(\frac{2}{3} + \frac{1}{6} \right) = \frac{5}{3}$

$$\frac{G_A}{G_V} \cong \frac{5}{3} = 1.66\ldots$$

Observed: $G_A / G_V \cong 1.22$

Attributed to orbital effects (incl. relativistic effects)

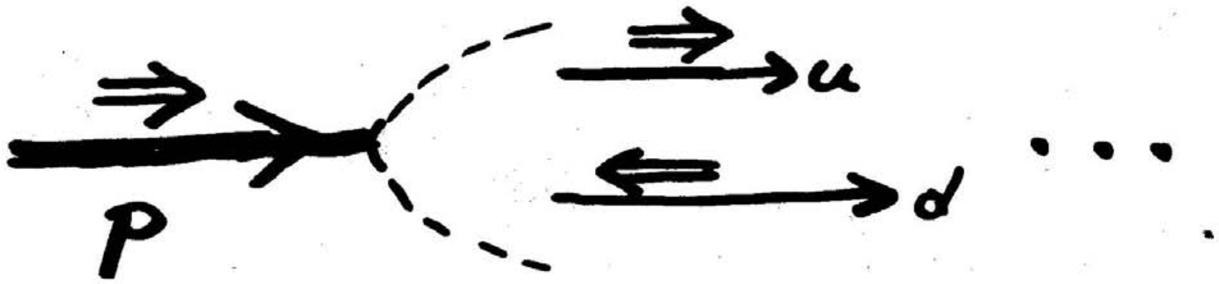
$$\frac{\text{Observation}}{\text{theory}} = \frac{1.22}{1.666\ldots} \cong 0.75$$

→ 75% of nucleon spin due to quarks

25% → orbital effects (+ relativistic effects)

But: Ratio n/p for magnetic moments remains valid!

Structure functions



$$g_1(x) = \frac{1}{2} \sum_{\text{quarks}} e_i^2 \left(q_i^+(x) - q_i^-(x) \right) + \dots$$

Deep inelastic scattering
with polarised beams (targets)

[SLAC, DESY]

Observed:

$$g_1(x) = \frac{1}{2} \sum_{\text{quarks}} e_i^2 \left(q_i^+(x) - q_i^-(x) \right) + \cdots$$

$$\Delta u = \int_0^1 dx \left[\left(u^+(x) - u^-(x) \right) + \left(\bar{u}^+(x) - \bar{u}^-(x) \right) \right]$$

$$\int_0^1 g_1(x)\, dx = \frac{1}{2} \left(\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right)$$

Exp. + SU(3):

$$\Delta \Sigma = (\Delta u + \Delta d + \Delta s) \cong 0.30 \pm 0.1$$

Expected from SU(6):

$$\Delta \Sigma \cong 0.75$$

Quarks contribute to spin
<u>only</u> ~ 30%. Why?

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_3$$

$$(L_3 = 0: \ \Delta G \approx 0.35)$$

Questions

Major part of spin due to (?)

a) Gluons ?

b) orbital motion ?

c) both ?

Experiments at CERN, SLAC, DESY, RHIC

→ Gluon contribution to nucleon spin

→ Flavor structure of quark polarizations

What are const. quarks ? (Internal structure ?)

Heavy quarks: b, t ?, c

quark = const. quark

u, d, s: const. quarks ≠ current quarks

Structure of "Const.-Quarks":

"Gedanken"-Experiment:

Heavy baryon: (bbu)

$U\Uparrow = (b\Uparrow b\Uparrow \underline{\underline{u}}\Uparrow)$

$D\Uparrow = (b\Uparrow b\Uparrow \underline{\underline{d}}\Uparrow)$

Spin $\frac{3}{2}$

(unobservable, but must "exist")

Have only one light constituent u or d

Mass: $m(D) - m(U) \approx 1.3$ MeV

$(m(d), m(u) + QED)$

$U, D \sim (p, n)$

(if b stable)

e.g. $D \to U + e^- + \bar{\nu}_e$

$\to$ Define matrix elements

$< U \mid \bar{u}\gamma_\mu d \mid D >$ and $< U \mid \bar{u}\gamma_\mu\gamma_5 d \mid D >$

~ 1

$U - D$–doublet: strong interaction
e. g. $\pi^- U \to \pi^0 D$

($\to$ Goldberger–Treiman or Adler–Weissberger-Relation)

U or D as targets:

Measurement of the distribution function of u or d in U or D.

$$U \Uparrow = \frac{1}{\sqrt{3}} (b \Uparrow b \Uparrow u \Uparrow + b \Uparrow u \Uparrow b \Uparrow + u \Uparrow b \Uparrow b \Uparrow)$$

b-content of U, D neglected
($e(b) = 0$?)

Sum rules:

$$\int_0^1 (u_+ + u_- - \bar{u}_+ - \bar{u}_-)\, dx = 1$$

$$\int_0^1 (d_+ + d_- - \bar{d}_+ - \bar{d}_-)\, dx = 0$$

($\rightarrow$ Adler sum rule)

$$\int_0^1 \{[u_+ + \bar{u}_+ - (u_- + \bar{u}_-)] - [(d_+ + \bar{d}_+) - (d_- + \bar{d}_-)]\}\, dx = g_a$$

$$(\rightarrow < U \mid \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d \mid U >$$

Isosinglet:

$- d_-$

$$\int_0^1 [(u_+ + \bar{u}_+ - u_- - \bar{u}_-) + (d_- + \bar{d}_+ - \bar{d}_-)]\, dx = \Sigma$$

$\sim \langle u \mid \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d \mid u \rangle$

Σ: Contribution of u– and d–quarks

to U–spin

Naïvely:

$$\int_0^1 u_+ dx = 1$$

$$d_+ = d_- = \bar{d}_+ = \bar{d}_- = \cancel{U}_- \, u_- = \bar{u}_+ = \bar{u}_- = 0$$

$$g_a = \Sigma = 1 \qquad \left(\rightarrow \frac{G_A}{G_V} = \frac{5}{3}\right)$$

But:

Anomaly:

$$\partial^\mu \left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d\right) =\sim G_{\mu\nu}\tilde{G}^{\mu\nu} \cdot \frac{g^2}{4\pi^2} \quad (\sim \vec{E} \times \vec{B})$$

Gluonic anomaly: Violation of Zweig rule in 0^{-+}–channel

GM, F., B.
GM., F., L.

Anomaly: $\Sigma \neq g_a$

$$2\int_0^1 (d_+ + \bar{d}_+ - d_- - \bar{d}_-) = \Sigma - g_a$$

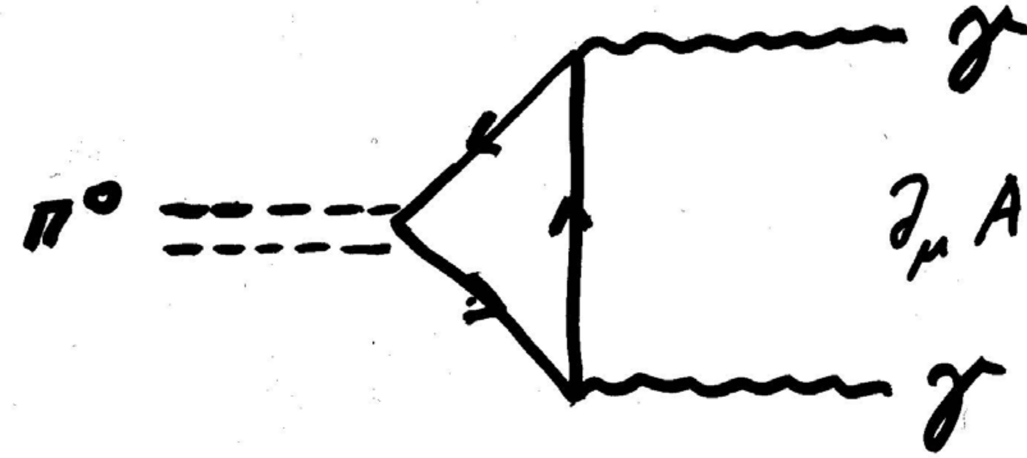
$$< U | \, \bar{d}\gamma_\mu\gamma_5 d \, | U > = \frac{1}{2} < U | \, \bar{d}\gamma_\mu\gamma_5 d - \bar{u}\gamma_\mu\gamma_5 u \, | U >$$

$$+ \frac{1}{2} < U | \, \bar{d}\gamma_\mu\gamma_5 d + \bar{u}\gamma_\mu\gamma_5 u \, | U >$$

U(1) - Anomaly (electric)

Adler
Bardeen

π^0 - decay:

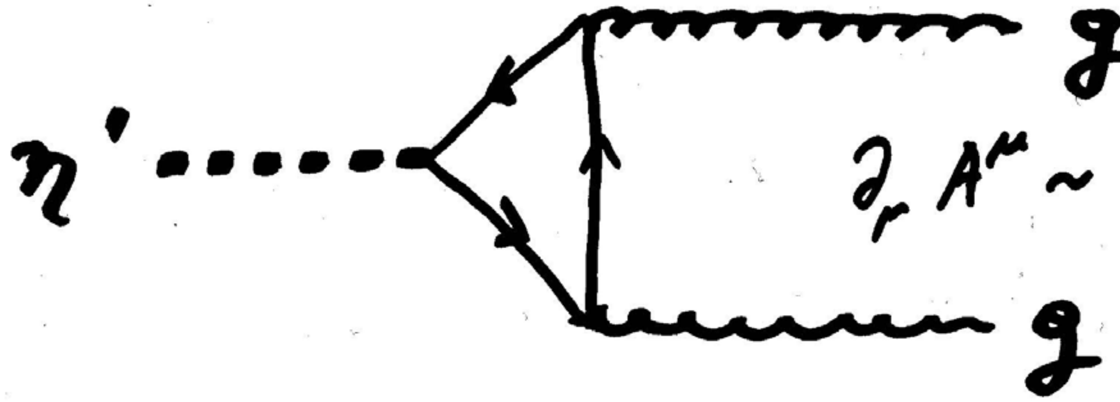


$$\partial_\mu A^\mu \sim \frac{\alpha}{\pi} \vec{E} \cdot \vec{B}$$

Agrees very well with observation
(higher orders do not contribute)

QCD

Gluonic Anomaly



$$\partial_\mu A^\mu \sim \frac{\alpha_s}{\pi} \vec{E}^c \times \vec{B}^c$$

$8_c \times 8_c = 1 + \ldots$

$\begin{pmatrix} \text{F.} \\ \text{GM.} \\ \text{B} \end{pmatrix}$

$\rightarrow \eta'$ has mass even in chiral limit ($m_q = 0$)

$\Delta\Sigma$ (nucleon) $= \Sigma\,(U)$

Isospin Triplet: $\Rightarrow$ pion pole.

Singlet without anomaly $\rightarrow$ η–pole.

$$\Sigma = g_a$$

$g_a \cong 0.75$
($\rightarrow G_A/G_V$)

Anomaly: $\Sigma \neq g_a$

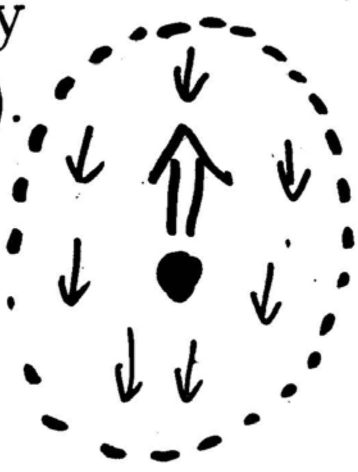
Reality: $\Sigma \approx 0.30$

$\rightarrow$ const. quarks
$\rightarrow$ U-D-system

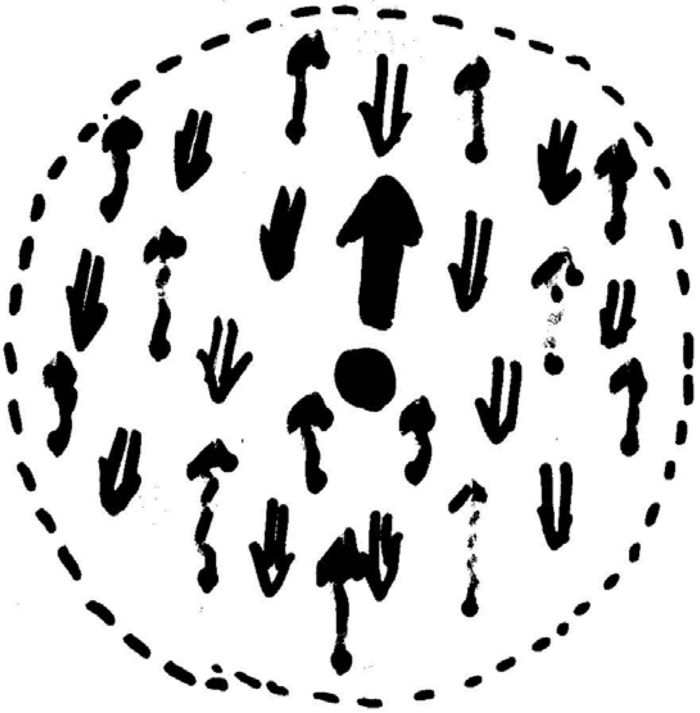
$$\int_0^1 dx\,(d_+ + \bar{d}_+ - d_- - \bar{d}_-) \;=\; \frac{1}{2}(\Sigma - g_a) \cong -0.22$$

$$\int_0^1 dx\,(u_+ + \bar{u}_+ - u_- - \bar{u}_-) \;=\; \frac{1}{2}(\Sigma + g_a) \cong +0.53$$

QCD–anomaly:
$\bar{q}q$–pairs inside U are generated. Pairs are polarized. They cancel partially the spin of the u–quark ($\Sigma - g_a < 0$).



Gluons must balance the spin



→ Constituent Quarks → internal structure

Anomaly: $(\bar{u}u)$ $(\bar{d}d)$ $(\bar{s}s)$ - pairs
(opposite spin)
gluons (polarised in same direction)

Expect:

$$d_- = \bar{d}_- = \bar{u}_-$$
$$d_+ = \bar{d}_+ = 0\,(?)$$

Interesting case: $\bar{u}_+ = d_+ = \bar{d}_+ = 0$

$$\int_0^1 (d_- + \bar{d}_-)\, dx = -\frac{1}{2}(\Sigma - g_a) \cong 0.22$$

$$\int_0^1 \bar{u}_- dx = \cancel{\text{[illegible]}} \cong 0.11$$

$$\frac{1}{2}(\Sigma + g_a) = \int_0^1 (u_+ - \overset{u_-}{\cancel{d}}_- - \bar{u}_-)\, dx \overset{+}{\cong} 0.53$$

Naively: 0.75

s–quarks included:

$$SU(3) \sim m_u = m_d = m_s$$

$$\int_0^1 [(u_+ + \bar{u}_+ - u_- - \bar{u}_-) + (u \to d) + (u \to s)]\, dx = \Sigma$$

$$\int_0^1 [(d_+ + \bar{d}_+ - d_- - \bar{d}_-)\, 2 + (s_+ + \bar{s}_+ - s_- - \bar{s}_-)]\, dx = \Sigma - g_a$$

Expect:

$$\int_0^1 (d_- + \bar{d}_-)\, dx = \int_0^1 (s_- + \bar{s}_-)\, dx \cong 0.15$$

$$\int \bar{u}_- dx \cong 0.075$$

$SU(3)$–Breaking:

$$\eta \cong \frac{1}{2}\left(\bar{u}u + \bar{d}d - \sqrt{2}\bar{s}s\right)$$

Feynman work

$$\eta' \cong \frac{1}{2}\left(\bar{u}u + \bar{d}d + \sqrt{2}\bar{s}s\right)$$

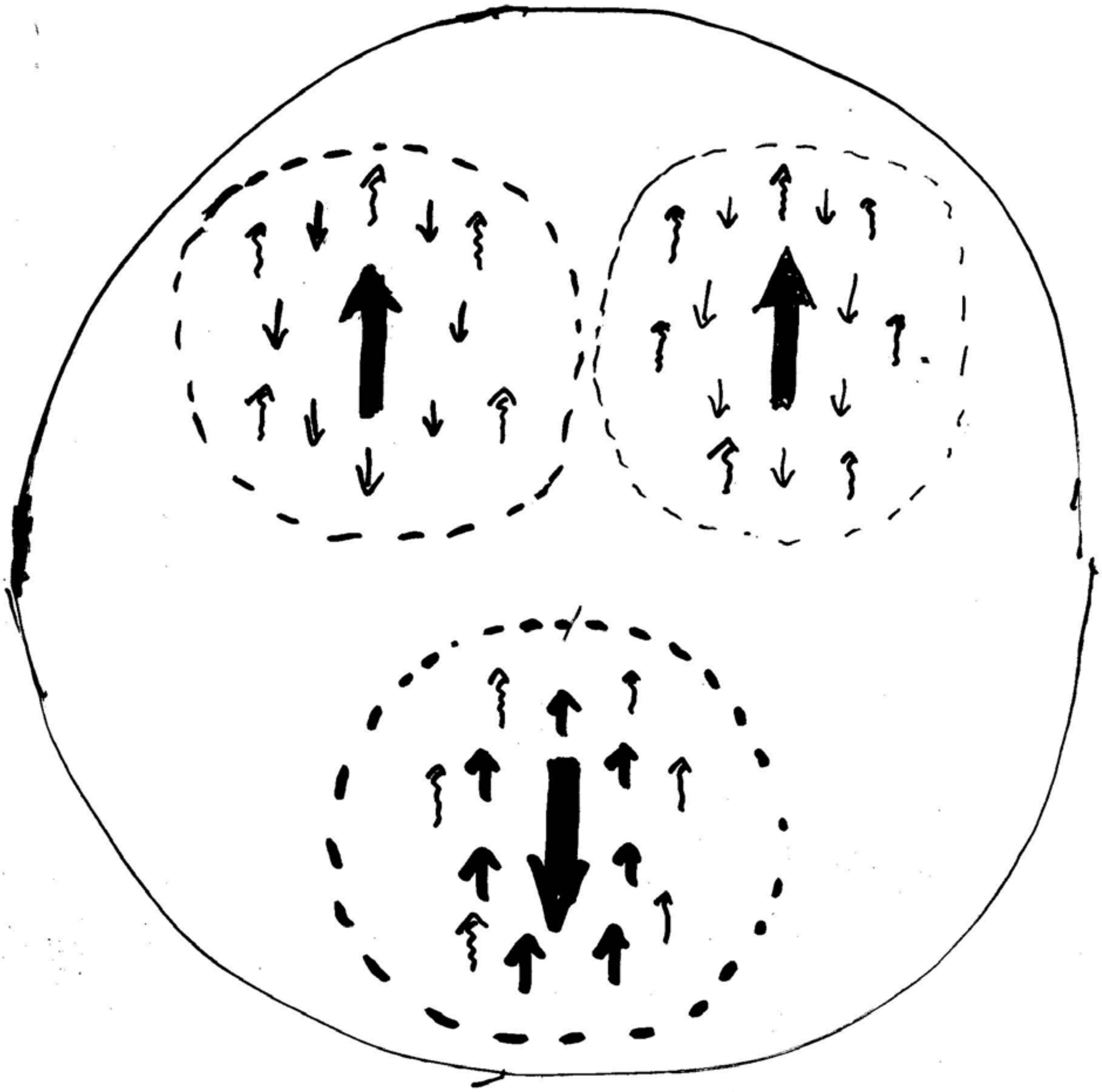
$$\int_0^1 (d_- + \bar{d}_-)\, dx \cong 0.18$$

$$\int_0^1 \bar{u}_- dx \cong 0.09$$

$$\int_0^1 (s_- + \bar{s}_-)\, dx \cong 0.09$$

(~ 50 % of d)

Nucleon



$\rightarrow$ Internal structure of const. quarks ($\sim 2\%$)

$$\Delta\Sigma \cong 0.30$$

$$\Delta G \cong \underline{0.45}\ (?)$$

$$\underline{0.75}\quad \text{(naive expectation)}$$

$$\frac{1}{2} = \frac{1}{2}\left(0.45 + 0.30 + 0.25 \right)$$

Gluons (?) — Quarks — orbital motion

15

QCD-anomaly:

spin reduced by $(\bar{q}q)$-pairs (polarised)

Nucleon:

30% of spin

$\longrightarrow$ valence quarks + pairs

25% "Orbital effects"

$$1 = 0.3 + 0.45 + 0.25$$

45% : Gluons + L_z?

$$\Delta G \cong 0.45 \quad (?)$$

$\cancel{\Delta G \approx 3}$

Compare: $\Delta G \approx 0.4 \pm 0.15 (?)$
(consistent)

Polarised pairs: $(\bar{d}_-, d_-)$ $(\bar{u}_-, u_-)$ $(\bar{s}_-, s_-)$

(HERA?)

$(\bar{s}s)$: no strong effect

Conclude

Nucleon spin:
strongly affected by gluon anomaly

→ $\bar{u}u$, $\bar{d}d$, $\bar{s}s$- pairs polarized
in opposite direction

Gluons: polarized in same direction

Pairs: seen at HERA?

Gluons: Expect $\Delta G \doteq 0.45$ (?)

Compass: $\Delta G \approx 0.4 \pm 0.15$

Dynamics: How are pairs
generated dynamically ???

(→ lattice theory?)