Towards NNLL precision in $B \to X_s \gamma$

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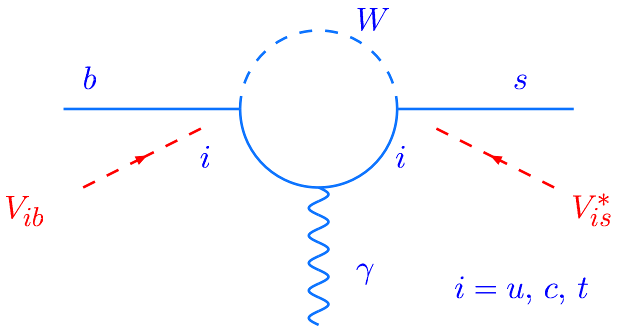
Introduction

Rare B decays, like $B \to X_s\gamma$ or $B \to X_s\ell^+\ell^-$ are induced at 1-loop level in SM,

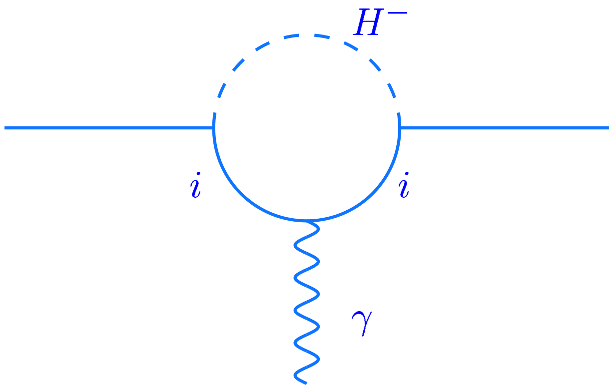
typical diagram (e.m. penguin)



-tests SM at the QT level

-sensitive to m_t

-sensitive to V_{ts}, or V_{td} in $b \to d\gamma$.



-sensitivity to ext. of the SM,

(e.g. $H^\pm$, SUSY contr., etc.)

Among the loop-induced B-decays, $B \to X_s\gamma$ has the largest BR. Good place to look for new physics! Worth calculating SM BR as precisely as possible.

Theoretical framework to calculate BR$(B \to X_s\gamma)$

$B \to X_s\gamma$ is an inclusive decay. $\to$ theoretically clean.

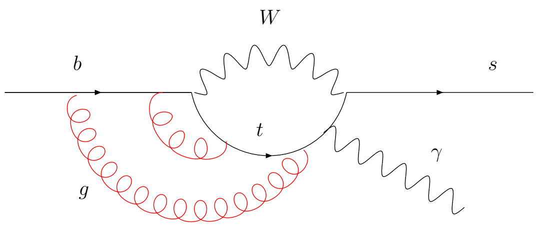
HQE: $\Gamma[B \to X_s\gamma] = \Gamma[b \to s\gamma(g)] +$ corr. in Λ_{QCD}/m_b.

- no linear corrections in Λ_{QCD}/m_b
- Corr. start at $\mathcal{O}(\Lambda_{QCD}^2/m_b^2)$ and are due to quark-gluon int. and motion of the b-quark

$\to$ consider decays at quark level!

Well-known: decay rate is significantly enhanced by QCD-effects.

There are large logs of the form (n gluons exchanged)



$$\left(\frac{\alpha_s}{\pi}\right)^n \log^n \frac{m_b^2}{M^2} \qquad M = m_t, m_W: \text{ leading logs (LL)}$$

$$\left(\frac{\alpha_s}{\pi}\right)^n \log^{n-1} \frac{m_b^2}{M^2} \qquad \text{next-to-leading logs (NLL)}$$

To get a reasonable result, one has to resum the LL and NLL terms.

Useful machinery to achieve resummation: construct effective Hamiltonian and resum logs using RGE techniques.

The effective Ham. obtained by integrating out heavy particles: In SM: top-quark, W, Z: $\rightarrow$ resulting Ham. expressible in terms of the light fields:

For $b \rightarrow s\gamma$ ($b \rightarrow s\gamma g$) one gets the following Hamiltonian $\mathcal{H}_{eff}$:

$$\mathcal{H} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(\mu) O_i(\mu) \quad .$$

The operators relevant in the following are:

$$O_1 = (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{s}_{L\alpha}\gamma_\mu c_{L\beta}) \qquad O_2 = (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{s}_{L\beta}\gamma_\mu c_{L\beta})$$

$$O_7 = \tfrac{e}{16\pi^2} m_b(\mu)\, (\bar{s}\sigma_{\mu\nu} R b)\, F^{\mu\nu} \qquad \text{phot. dipole}$$

$$O_8 = \tfrac{g_s}{16\pi^2} m_b(\mu)\, (\bar{s_\alpha}\sigma_{\mu\nu} T^A_{\alpha\beta} R b_\beta)\, G^{\mu\nu,A} \qquad \text{gluonic dipole}$$

$C_i(\mu)$ are determined through the matching procedure, i.e. one requires that certain amplitudes in the full theory are id. to those in the effective theory.

Let's look at the structure of the eff. Hamiltonian:

$$\mathcal{H}_{eff} \quad \sim \quad \sum_i C_i(\mu) O_i(\mu)$$

$\mathcal{H}_{eff}$ independent of μ, while C_i and O_i depend on μ:

$\longrightarrow$ RGE for $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij}^T C_j(\mu)\,; \qquad \gamma_{ij} \text{ : anomalous dim. matrix}$$

Matching usually done at high scale μ_W, i.e. $\mu_W \sim O(m_W)$:

μ_W: full theory and mat. el. of op. have same large log's:

Corr. to $C_i(\mu_W)$ rel. small.

$\downarrow$ RGE

$\mu_b = O(m_b)$: mat. el. of op. don't have large log's: They are contained in the $C_i(\mu_b)$.

The dependence of the heavy degrees of freedom is contained in the Wilson coefficients.

Calculation of BR$(B \to X_s \gamma)$ consists of three steps:

	LL	NLL	NNLL
-matching at $\mu = \mu_W$: $\to C_i(\mu_W)$	α_s^0	α_s^1	α_s^2
-RGE: $\to C_i(\mu_b)$ [with $\mu_b = O(m_b)$]	α_s^1	α_s^2	α_s^3
-calc. of matrix element $\langle X_s \gamma \| O_i(\mu_b) \| B \rangle$	α_s^0	α_s^1	α_s^2

NLL results for BR$(B \to X_s \gamma)$

The NLL QCD results were completed in 1997. This was a common effort of many groups.

Somewhat later, also certain classes of electro-weak corrections were calculated (Czarnecki, Marciano; Neubert, Kagan; Baranowski, Misiak; Gambino, Haisch).

Combining the two, one obtains the following branching ratio:

$$\mathrm{BR}(\mathrm{B} \to \mathrm{X_s}\gamma) = (3.32 \pm 0.14[\mu_\mathrm{b}] \pm 0.26[\mathrm{pars.}]) \times 10^{-4}.$$

This BR depends (among other things) on m_c/m_b.

One should note that this ratio was tacitly interpreted to be pole ratio

$m_c^{\mathrm{pole}}/m_b^{\mathrm{pole}}$.

Motivation for going to NNLL precision

In 2001, Gambino and Misiak pointed that one should use MS-bar mass $\overline{m}_c$ instead of m_c^{pole}, c-quark only appears in loops.

This is, however a purely intuitive argument.

One cannot really decide which choice is better, because the difference is simply a NNLL effect.

Numerically, however, the difference between using m_c^{pole} or $\overline{m}_c$ is large:

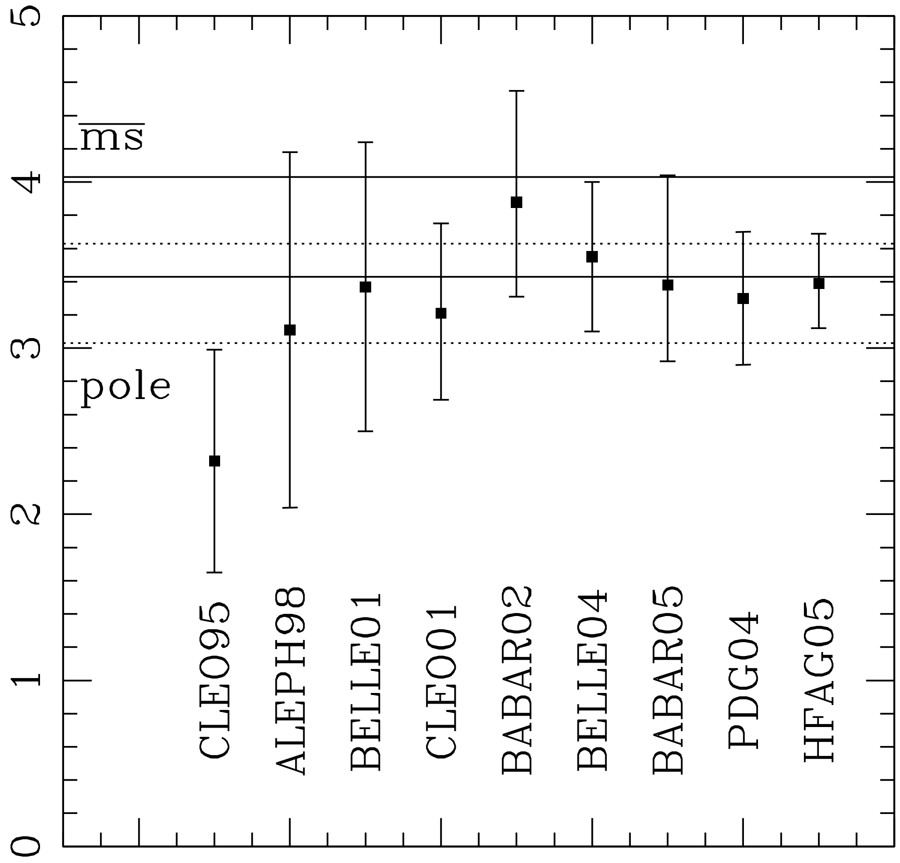
Using pole-interpretation [with $m_c/m_b = 0.29 \pm 0.02$] $\longrightarrow \text{BR} = (3.32 \pm 0.30) \times 10^{-4}$

Using $\overline{\text{MS}}$ interpretation [with $m_c/m_b = 0.22 \pm 0.04$] $\longrightarrow \text{BR} = (3.70 \pm 0.30) \times 10^{-4}$

The difference of the central values is about 11%.

To really fix this issue, a NNLL calc. was started and is now close to completion!

Compilation of branching ratio measurements for $B \to X_s \gamma$ (in units 10^{-4})



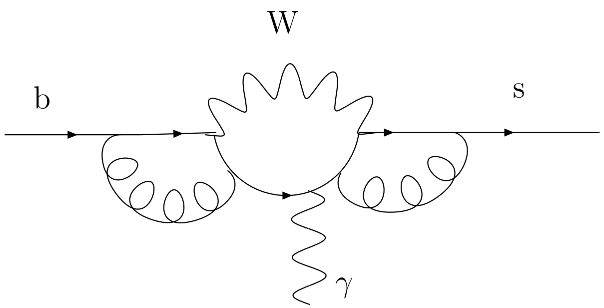
Present world average from HFAG 2005 (based on CLEO, BELLE and BABAR):

$$\mathrm{BR}(\mathrm{B} \to \mathrm{X_s}\gamma) = (3.39^{+0.30}_{-0.27}) \times 10^{-4}$$

Status of the NNLL calculation

Matching: needed to α_s^2 precision.

This means in particular a 3-loop calculation in the full theory to fix $C_7(\mu_W)$ and $C_8(\mu_W)$ [$O(10^3)$ diagrams]:



→ done by Misiak and Steinhauser, hep-ph/0401041.

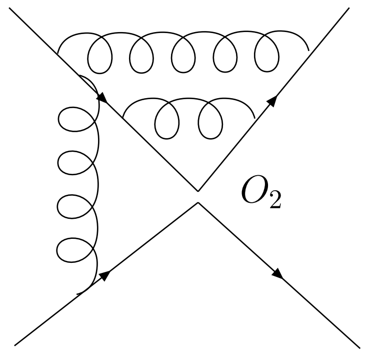
For other operators $O(\alpha_s^2)$ means two-loop. Done some time ago.

→ matching complete for NNLL precision!

Anomalous dimensions: needed up to α_s^3 precision.

• $(O_1 - O_6)$-sector

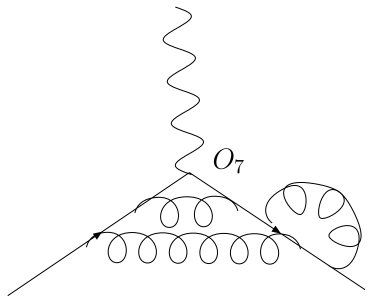
done by Gorbahn and Haisch, hep-ph/0411071.



• (O_7, O_8)-sector

was finished last year by

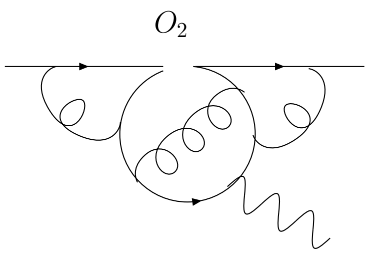
Gorbahn, Haisch and Misiak, hep-ph/0504194.



• most difficult: mixing $O_2 \to O_7$, $O_2 \to O_8$:

4-loop!.

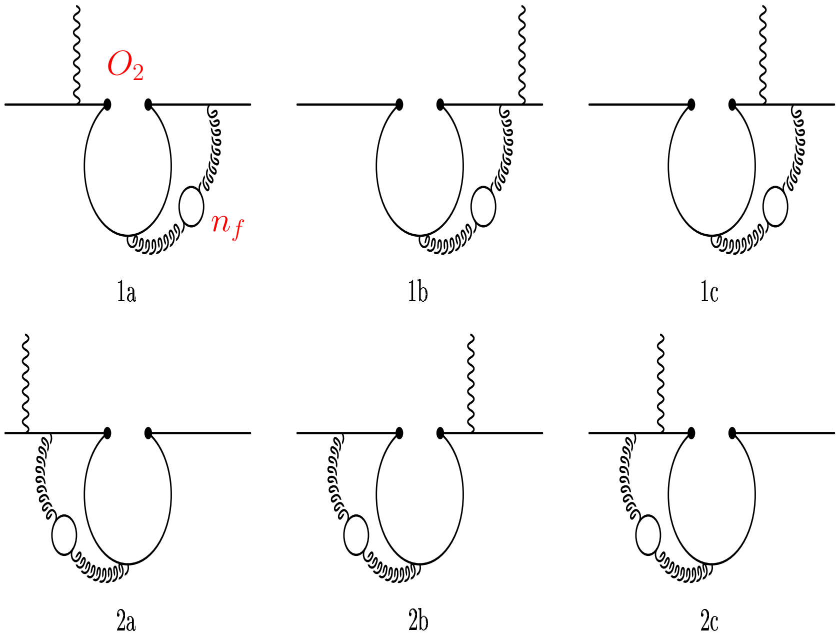
Cal. finished but not published yet, Czakon et al.

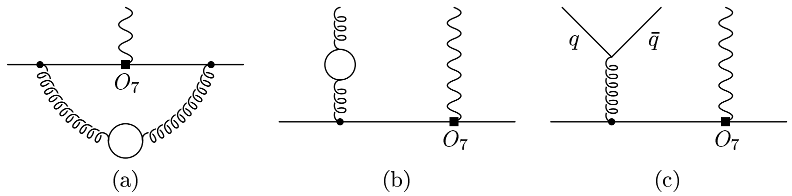


Matrix elements: needed up to α_s^2 precision [mention only published results]

1) NNLL terms prop. to n_f

In 2003 K. Bieri, C.G., M. Steinhauser worked out $\mathcal{O}(\alpha_s^2 n_f)$ corrections to the matrix elements of O_1, O_2, O_7 and O_8. Diagrammatically: Take the $O(\alpha_s)$ diagrams and dress the gluon-prop. with light quark bubbles (n_f), e.g.:





In many cases source of large corrections. In $\Gamma(b \to c l \nu_l)$ 80% of the complete $\mathcal{O}(\alpha_s^2)$ corrections are covered.

2) Complete result for the (O_7, O_7)-interference

At order α_s^2 the following subprocesses are involved:

$$b \to s\gamma \; ; \; b \to s\gamma g \; ; \; b \to s\gamma gg \; ; \; b \to s\gamma q\bar{q}$$

2 independent calculations exist, using different methods. Results are in full agreement.

First method: Blokland et al., hep-ph/0506055

They used that the contribution to the decay width of these subprocesses is related to the absorptive part of b-quark self energy (optical theorem). They worked out these absorptive parts by loop-techniques.

Second method: Asatrian,Hovhannisyan,Poghosyan,Ewerth,Greub,Hurth,hep-ph/0605009

We calculated the $2-$, $3-$ and $4-$particle particle processes individually.

This method allows to implement a kinematical cut on the photon energy in a straightforward way (only events with $E_\gamma > 1.8$ GeV are measured).

Also the extension to $O(\alpha_s^2)$ corr. to the (O_7, O_8) and (O_8, O_8)-interferences is straightforward.

Summary

At NLL precision the theory error related to the def. of m_c is rather large. To reduce it a NNLL calc. was started.

Many ingredients for NNLL results are already available:

- NNLL matching complete
- $O(\alpha_s^3)$ anomalous dimensions known. Those related to 4-loop mixing of $O_2 \to O_{7,8}$ not published yet.
- Concerning $O(\alpha_s^2)$ matrix elements: n_f-terms known for the operators $O_{1,2}$,$O_{7,8}$
  (O_7, O_7)-terms completely calculated
  contributions related to $O_{1,2}$ in far advanced status; also the (O_7, O_8) and (O_8, O_8)-terms on the way.

Many groups contributed to the NNLL business for $B \to X_s\gamma$; The results will be published soon!