On Heraklitean space-time

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Third International Conference on the
Nature and Ontology of Spacetime
Concordia University, Montreal, Quebec, Canada 13. June 2008

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1 Introduction : Hermann Minkowski , a somewhat distant *relative*

Please excuse me for sacrificing a *short time* to the memory of Hermann Minkowski, born 1864 in Alexoten near Kaunas, Lithuania, then Russia and deceased 1909 in Göttingen, Germany , who – as far as I know – was a cousin of my grandfather, a merchant and banker, August Minkowski, born 1849 in Minsk, Bielorus, then Russia and died 1942 [a] in Otwock near Warsaw, Poland.

H. M.'s elder daughter Lily Rüdenberg-Minkowski (now deceased), living in Boston, came to our house in Zurich several times, the first time when I was $\sim$ 12 . Unforgettable is her 'classical' way to speak german. She presented *this* copy of her fathers talk "Raum und Zeit" , 21. September 1908 in Köln, as a gift to my father. This is the nearest I can come to Hermann Minkowski.

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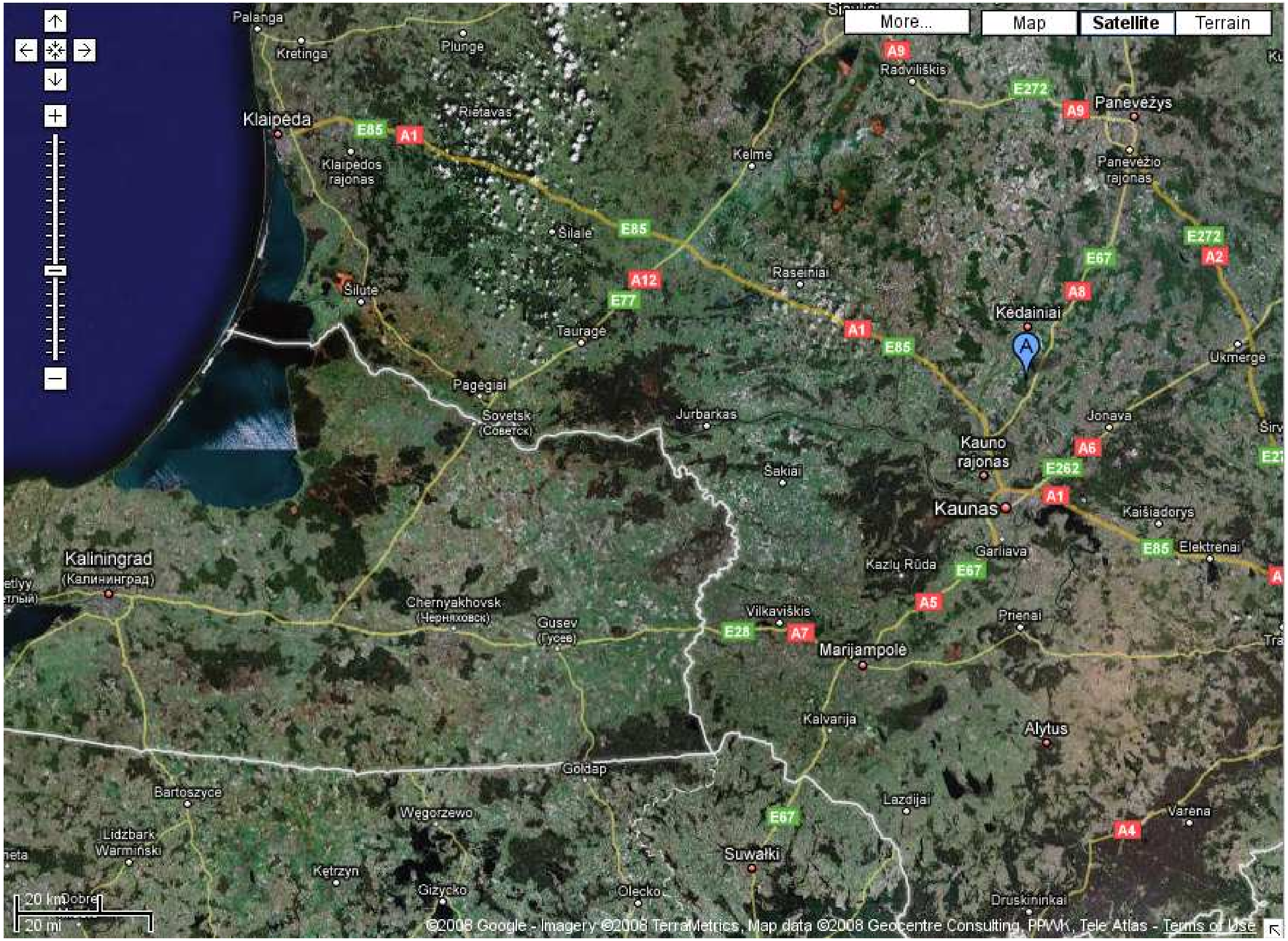
[a] August Minkowski died on 23. January 1942, not 1945 as in the original writing. This became known to the author in late 2008, [note added 25.09.2010 , PM] .



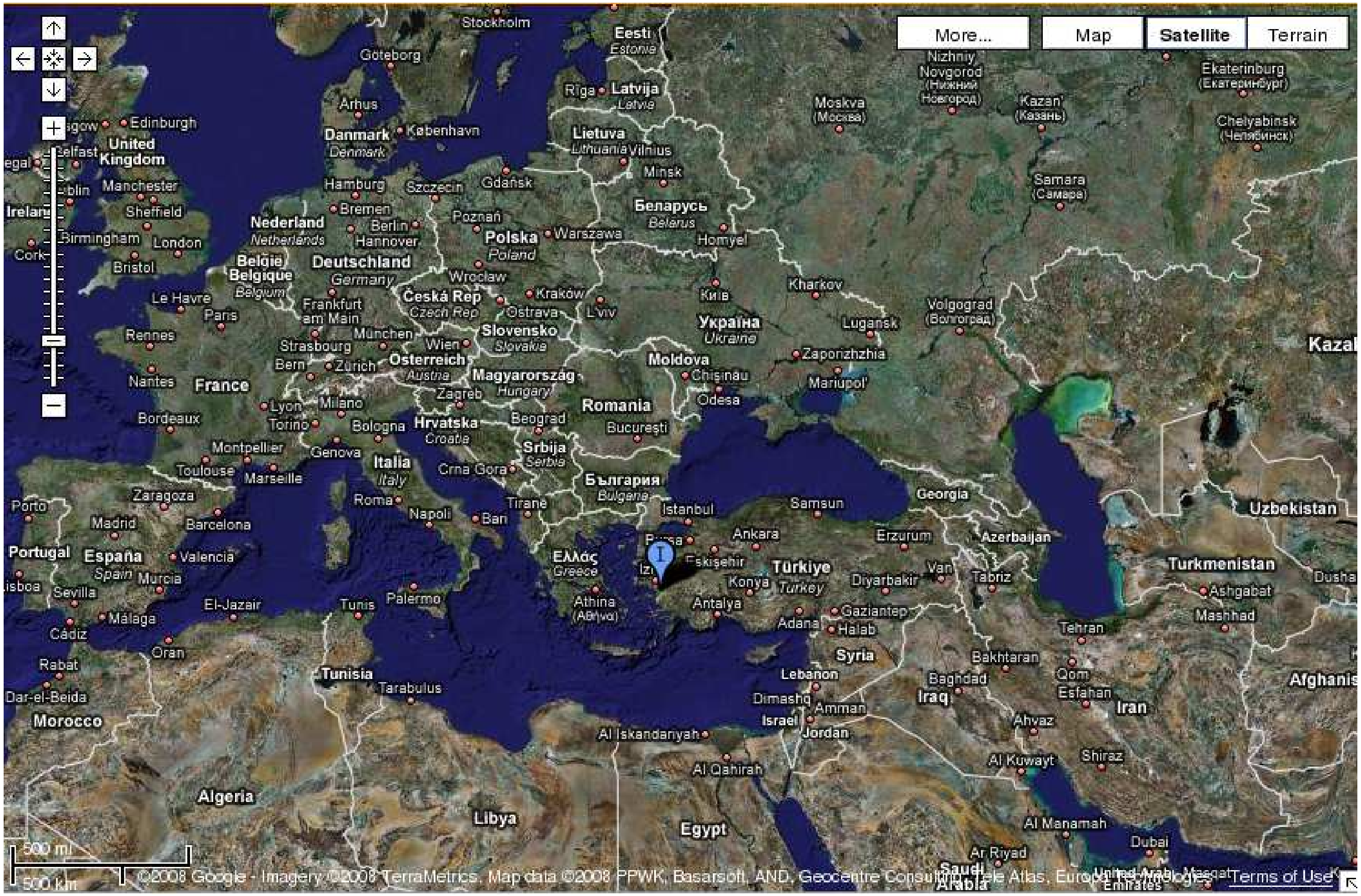
from Alexoten (Alexa) to Königsberg (Kaliningrad) →

2 Heraklitos from Ephesos 'the obscure' – $\sim$ 540 - 480 b.c.

An alternative characterization of $H\varrho\alpha\kappa\lambda\varepsilon\iota\tau o\zeta\ \acute{o}\ E\varphi\varepsilon\sigma\iota o\zeta$ **could be :**

"There is nothing that *it as well as the contrary* **another has not thought before** *or after* **."**

According to 'Brockhaus' Heraklit deposited a treatise in the temple of Ephesos near the Aegean Sea in Turkey, of which only 120 sentences were recovered through modern times. The philosopher Georg Wilhelm Friedrich Hegel (1770-1831) has taken this treatise as an antique basis and developed his 'systematic ontology' from its foundations.



Ephesos, Turkey the temple $\longrightarrow$

Heraklit ’the obscure’

$\sim$ 540 - 480 b.c.

painted $\sim$ 1510

by Michelangelo, in the

’Stanza della Segnatura’

in the Vatican

’panta rhei’ – all flows $\longrightarrow$

3 The notion of Heraklitean space-time for quantized local fields pertaining to gravity

Preamble : the following is at most a sketch of irreducible problems inherent to space-time within gravity or general relativity , whence quantized maintaining locality . The notions of classical, i.e. non-Heraklitean, space-time – which this meeting commemorates – continue to be *the* consistent basis. [a]

3a gauging orientation on a differentiable manifold – affine connection and parallel transport in $d = 1 + (d - 1)$ dimensions

Quantized quantities shall be distinguished furtheron from classical ones. The former shall be *underlined*, the latter not.

We consider the parallel transport of a (classical) contravariant vector field $v^{\varrho} (x)$ **, x denoting (classical) space-time variables in** $d = 1 + (d - 1)$

$$
\begin{array}{l}
\delta_{\parallel} v^{\varrho} = - d x^{\kappa} (\Gamma_{\kappa})^{\varrho}_{\ \sigma} (x) v^{\sigma} \\
\left(\Gamma^{(1)} = d x^{\kappa} \Gamma_{\kappa} \right)^{\varrho}_{\ \sigma} \quad : \text{matrix valued 1-form}
\end{array}
\tag{1}
$$

$\longrightarrow$

[a] 'natures way ... our way' a seminar given in Beijing, China, September 2007 served as base material (in naturesway2007.pdf) .

and along a curve C from x to y , giving rise to the (curve associated) parallel transport matrix , denoted $T\,(\,y \overset{C}{\leftarrow} x\,)$

(2)

$$\left\{ T\,(\,y \overset{C}{\leftarrow} x\,) \;=\; P\,\exp - \int_x^y \Gamma^{\,(1)} \right\}^{\varrho}_{\;\sigma}$$

$$v_{\,||}\,(\,y \overset{C}{\leftarrow} x\,) \;=\; T\,(\,y \overset{C}{\leftarrow} x\,)\,v$$

matrix notation

In eq. (2) P denotes ordering from left (further along) to right along the path C .

Now we imagine the same parallel transport done using *transformed* local coordinates

(3)

$$x^{\,\prime\,\varrho} \;=\; x^{\,\prime\,\varrho}\,(\,x\,) \;\;\rightarrow\;\; \left\{ M^{\,\varrho}_{\;\sigma} \;=\; \partial_{\,\sigma}\,x^{\,\prime\,\varrho} \right\}\,(\,x\,)$$

Eq. (2) takes the (trans-) form

(4)

$$\left\{ T^{\,\prime}\,(\,y^{\,\prime} \overset{C}{\leftarrow} x^{\,\prime}\,) \;=\; P\,\exp - \int_{x^{\,\prime}}^{y^{\,\prime}} \Gamma^{\,\prime\,(1)} \right\}^{\varrho}_{\;\sigma}$$

$$v^{\,\prime}_{\,||}\,(\,y^{\,\prime} \overset{C}{\leftarrow} x^{\,\prime}\,) \;=\; T^{\,\prime}\,(\,y^{\,\prime} \overset{C}{\leftarrow} x^{\,\prime}\,)\,v^{\,\prime}$$

$$v^{\,\prime}_{\,||} \;=\; M\,(\,y\,)\,v_{\,||} \;\;,\;\; v^{\,\prime} \;=\; M\,(\,x\,)\,v$$

$\longrightarrow$

and substituting one (coordinate-) system relative to the other

(5)
$$M(y)\,T(y \overset{C}{\leftarrow} x)\,v = T'(y' \overset{C}{\leftarrow} x')\,M(x)\,v \quad \rightarrow$$

(6)
$$T'(y' \overset{C}{\leftarrow} x') = M(y)\,T(y \overset{C}{\leftarrow} x)\,(M(x))^{-1}$$

In eqs. (2 - 6)

(7)
$$\{\, M(z) \,|\, \forall z \,\}$$

forms the family of local transformations , gauging orientation [a] .
The role of the entire set of parallel transport matrices $T(y \overset{C}{\leftarrow} x)$ is clear and perfectly covariant, while the local connection $\Gamma^{(1)}$ transforms inhomogeneously.

---

a They form the group $GL(d, R)$, where d is the (real) dimension of the manifold.

The parallel transported vectors along the path C, using a path parameter s

(8) $$C \, : \, \{ \, z(s) \, | \, z(1) = y \, ; \, z(0) = x \, \}$$

satisfy the differential equation ($\dot{} = d/ds$)

(9) $$\begin{aligned} \dot{v}(s) &= - \dot{z}^{k}(s) \, \Gamma_{k}(s) \, v(s) \\ v(s) &= T(z(s) \overset{C}{\leftarrow} x) \, v \end{aligned}$$

Comparing with the coordinate transformed equation and using $M(s) = M[z(s)]$

(10) $$\begin{aligned} \dot{v}(s) &= - \dot{z}^{k}(s) \, \Gamma_{k}(s) \, v(s) \\ \dot{v}'(s) &= - \dot{z}'^{k}(s) \, \Gamma'_{k}(s) \, v'(s) \\ \begin{pmatrix} v'(s) \\ \dot{z}'(s) \end{pmatrix} &= M(s) \begin{pmatrix} v(s) \\ \dot{z}(s) \end{pmatrix} \end{aligned}$$

The second relation in eq. (10) thus becomes

(11) $$M \, \dot{v} + \dot{M} \, v = - \dot{z}^{k} \, M^{r}_{k} \, \Gamma'_{r} \, M \, v$$

$\rightarrow$

and substituting the first one

(12)
$$\dot{z}^{k} \Gamma_{k} v = \dot{z}^{k} M^{r}_{k} M^{-1} \Gamma'_{r} M v + M^{-1} \dot{M} v$$

$$\dot{M} = \dot{z}^{k} \partial_{z\,k} M \qquad \longrightarrow$$

$$M^{r}_{k} \Gamma'_{r} = M \Gamma_{k} M^{-1} + M \partial_{z\,k} M^{-1}$$

$$\Gamma'_{r} = \left\{ M \Gamma_{k} M^{-1} + M \partial_{z\,k} M^{-1} \right\} \left(M^{-1} \right)^{k}_{r}$$

From eq. (12) the transformation of the one-form $\Gamma^{(1)} = d x^{\kappa} \Gamma_{\kappa}$ (eq. 1) follows

(13)
$$\Gamma'^{(1)} = d x'^{\kappa} \Gamma'_{\kappa}$$

$$\Gamma'^{(1)} = M \Gamma^{(1)} M^{-1} + M d M^{-1}$$

$$d F = d x^{\kappa} \partial_{x\,\kappa} F \quad ; \quad F \; : \; \text{matrix valued}$$

[a]

[a] In d = 3 orientation refers to a top-compass, e.g. in Cardano’s suspension, whreas in general d it is more difficult to draw and was called ’repère mobile’ by Élie Cartan .

Cardano’s suspension →

We proceed noting the one special feature of the connection transformation (eq. 12) , written in full , upon using $M\, d\, M^{-1} = -(d\, M)\, M^{-1}$

(14)

$$\Gamma'_{r'}{}^{u'}{}_{t'} = \left[\begin{array}{c} M^{u'}_{u}\, \Gamma_{r}{}^{u}{}_{t}\, (M^{-1})^{r}{}_{r'}\, (M^{-1})^{t}{}_{t'} + \\ + I'_{r'}{}^{u'}{}_{t'} \end{array} \right]$$

$$I'_{r'}{}^{u'}{}_{t'} = -(\partial_r\, M^{u'}_{u})\, (M^{-1})^{r}{}_{r'}\, (M^{-1})^{u}{}_{t'}$$

$$\partial_r\, M^{u'}_{u} = \partial_r\, \partial_u\, x'^{u'}(x) = \partial_u\, M^{u'}_{r}$$

It follows that the inhomogeneous orientation gauging part is symmetric

(15)

$$\Gamma'_{r'}{}^{u'}{}_{t'} = \left[\begin{array}{c} M^{u'}_{u}\, \Gamma_{r}{}^{u}{}_{t}\, (M^{-1})^{r}{}_{r'}\, (M^{-1})^{t}{}_{t'} + \\ + I'_{r'}{}^{u'}{}_{t'} \end{array} \right]$$

$$I'_{r'}{}^{u'}{}_{t'} = I'_{t'}{}^{u'}{}_{r'}$$

→

Two things emerge

a) the antisymmetric part of the connection defines a 3-tensor $T_{[r \;\; t]}^{\;\;\; u}$ **: torsion** √

(16) $$T_{[r \;\; t]}^{\;\;\; u} = \frac{1}{2} \left(\Gamma_{r \;\; t}^{\; u} - \Gamma_{t \;\; r}^{\; u} \right)$$

c) a symmetric metric yields a symmetric Riemannian (minimal) connection

(17) $$\overset{o}{\Gamma}_{\{r \;\; t\}}^{\;\;\; u} = \\ = \frac{1}{2} g^{uv} \left[\partial_r g_{vt} + \partial_t g_{vr} - \partial_v g_{rt} \right] \\ \gamma_{\{r \;\; t\}}^{\;\;\; u} = \frac{1}{2} \left(\Gamma_{r \;\; t}^{\; u} + \Gamma_{t \;\; r}^{\; u} \right) - \overset{o}{\Gamma}_{\{r \;\; t\}}^{\;\;\; u}$$

$\gamma_{\{r \;\; t\}}^{\;\;\; u}$ **defined in eq. (17) – if not vanishing – defines a symmetric 3-tensor, in addition to torsion .**

→

Notwithstanding the eventual presence of 3-tensors $T_{[\,r \quad t\,]}^{\quad\; u}$ *and* $\gamma_{\{\,r \quad t\,\}}^{\quad\; u}$ the general 1-form , defined in eq. (1) with transformation properties given in eq. (13) (repeated below for clarity)

(18)
$$\left(\Gamma^{(1)} = d x^{\kappa} \Gamma_{\kappa} \right)^{\varrho}_{\ \sigma} \quad : \text{matrix valued 1-form}$$
$$\Gamma^{\prime\,(1)} = d x^{\prime\,\kappa} \Gamma^{\prime}_{\kappa}$$
$$\Gamma^{\prime\,(1)} = M \, \Gamma^{(1)} M^{-1} + M \, d \, M^{-1}$$

generate a matrix curvature 2-form , a 4-tensor

(19)
$$R^{(2)} = d \, \Gamma^{(1)} + \left(\Gamma^{(1)} \right)^{2}$$
$$\rightarrow \frac{1}{2} \left(R_{[\,\sigma \, \tau\,]} \right)^{u}_{\ v} \, d x^{\sigma} \wedge d x^{\tau}$$

as follows from the transformation properties (eq. 18) [a]

(20)
$$R^{\prime\,(2)} = M \, R^{(2)} M^{-1}$$

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[a] **... well known yet remarkable ...**

3b gauging a Lie tranformation group acting on a distinct fibre – and neglecting gravity on the $d_B = 1 + 3$ base space

In section 3a) we did not introduce a special name for the manifold considered. Meanwhile the notation of fibre bundles distinguishes a base manifold B –

here we take B to represent *classical* $d_B = 1 + 3$ - dimensional Poincaré invariant 'Minkowski' space-time *in the limit of vanishing gravitational interactions*

– and a *compact* d_F - dimensional fibre F , combining their direct product to an extended manifold E

$$\begin{array}{c} (B , d_B ; F , d_F) \rightarrow \\ E (B ; F , d_B + d_F) \sim B \times F \end{array} \tag{21}$$

The fibre F shall be a homogeneous space : right coset $F = G / H$ of a *compact* Lie group G modulo a Lie subgroup H . [a] We consider G to be semi-simple *for simplicity* here . $\longrightarrow$

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[a] An analogy of the fibre space can be seen with the powder method of Debye and Scherrer [1] for the study of crystalline structure. Therein the property of the basic powder crystals to be 'invisible' directly is the common ingredient.

With transformations $a \in G$

$$a : f \rightarrow a\,.\,f \;\; ; \;\; f = \left(f^{\,k} ; \, k = 1 \cdots dim\, F \right)$$

(22) $$a = (a^{\,\varrho} ; \, \varrho = 1 \cdots dim\, G)$$

$$(a\,.\,f)^{\,k} = \Omega^{\,k} (a ; \, f)$$

The Killing fields correspond to the infinitesimal transformations

(23) $$h^{\,k}_{\;\varrho} (f) = \partial_{\,b\,\varrho} \; \Omega^{\,k} (b ; \, f) \,\Big|_{\, b = 0}$$

The transformation $a : f \rightarrow a\,.\,f$ on F allows to associate $a \rightarrow Ad (a)$, where $Ad (a)$ denotes the adjoint ($dim\, G \times dim\, G$) representation of G . [a] →

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[a] The case $F = S_{\,1}$ has been considered by Theodor Kaluza [2] and Oskar Klein [3] (1921,1926) , the general case by Élie Cartan [4] ($\sim$ 1937) and $F = S_{\,2} = SU2 \,/\, U1$ by Wolfgang Pauli [5] (1953) .

The association $a \longrightarrow Ad(a)$ is established through the relation

(24)
$$h^{k}{}_{\varrho}(a.f) = \psi^{k}{}_{l}(a;f)\,h^{l}{}_{\sigma}(f)\,\left(Ad(a^{-1})\right)^{\sigma}{}_{\varrho}$$
$$\psi^{k}{}_{l}(a;f) = \partial_{f^{l}}\Omega^{k}(a;f)$$

$\psi^{k}{}_{l}(a;f)$ defined in eq. (24) is the Jacobian of the coordinate transformation in F

(25)
$$a : f \longrightarrow a.f$$

The group property follows from the matrix form of eq. (24)

(26)
$$h(a.f) = \psi(a;f)\,h(f)\,Ad(a^{-1})$$
$$h(b.a.f) = \begin{bmatrix} \psi(b;a.f)\,\psi(a;f)\,h(f)\times \\ \times\, Ad(a^{-1})\,Ad(b^{-1}) \end{bmatrix}$$
$$= \psi(b.a;f)\,h(f)\,Ad((b.a)^{-1})$$
$$Ad(a^{-1})\,Ad(b^{-1}) = Ad((b.a)^{-1})$$
$$\psi(b;a.f)\,\psi(a;f) = \psi(b.a;f)$$

→

projecting on the adjoint connection

The base space B shall be described by coordinates

(27) $$B \; : \; (\, x^{\,\mu} \, ; \; \mu \; = \; 1 \cdots dim \, B \,)$$

Now we consider x-dependent group transformations from G on $F \; = \; G \, / \, H$

(28) $$a \; \rightarrow \; a \, (\, x \,) \; : \; f \; \rightarrow \; a \, (\, x \,) \, . \, f$$

As a consequence of eqs. (24 - 26) we project on the (adjoint Lie algebra -) matrix valued connection on the base space B

(29)
$$\begin{array}{l}
(\, \mathcal{W}_{\,\mu} \,)^{\,\sigma}_{\quad \varrho} \; = \; W^{\,\kappa}_{\,\mu} \; (\, ad_{\,\kappa} \,)^{\,\sigma}_{\quad \varrho} \\
(\, ad_{\,\kappa} \,)^{\,\sigma}_{\quad \varrho} \; = \; f_{\,\sigma \, \kappa \, \varrho} \quad \rightarrow \quad \left[\, ad_{\,\alpha} \, , \, ad_{\,\beta} \, \right] \; = \; f_{\,\alpha \, \beta \, \gamma} \, ad_{\,\gamma} \\
\mathcal{W}_{\,\mu} \; = \; \mathcal{W}_{\,\mu} \, (\, x \,) \; \leftrightarrow \; \Gamma_{\,\kappa} \\
\mathcal{W}^{\,(1)} \; = \; d \, x^{\,\mu} \, \mathcal{W}_{\,\mu} \; \leftrightarrow \; \Gamma^{\,(1)} \; = \; d \, x^{\,\kappa} \, \Gamma_{\,\kappa}
\end{array}$$

in analogy or relation with eqs. (1 - 2) , *as long as* $\mathcal{W}_{\,\mu} \, (\, x \,)$ *is a classical field*.

Then **we proceed to construct the parallel transports as in eq. (2)** $\rightarrow$

(30)

$$\left\{ U(y \overset{C}{\leftarrow} x) = P \exp - \int_x^y \mathcal{W}^{(1)} \right\}_\sigma^\varrho$$

$$\left\{ T(y \overset{C}{\leftarrow} x) = P \exp - \int_x^y \Gamma^{(1)} \right\}_\sigma^\varrho$$

The analog of the orientation gauge transformation in eq. (6) corresponds for U to the local coordinate transformation on the fibre F , beyond *or outside* **B**

(31)

$$f \longrightarrow a(z).f \longrightarrow$$

$$U'(y \overset{C}{\leftarrow} x) = U(y)\, U'(y \overset{C}{\leftarrow} x)\, U^{-1}(x)$$

$$U(z) = Ad(a(z))$$

$$T'(y' \overset{C}{\leftarrow} x') = M(y)\, T(y \overset{C}{\leftarrow} x)\, (M(x))^{-1}$$

$$M(z) = \partial_z z'$$

→

Finally – in this section – we compare the gauge transformations on local 1-forms (eq. 13)

(32)

$$\mathcal{W}^{\prime\,(1)} = U\,\mathcal{W}^{(1)}\,U^{-1} + U\,d\,U^{-1}$$

$$U(z) = Ad(a(z))$$

$$\Gamma^{\prime\,(1)} = M\,\Gamma^{(1)}\,M^{-1} + M\,d\,M^{-1}$$

$$M(z) = \partial_z\,z^{\prime}$$

and the curvature 2-form (eqs. 18 - 19)

(33)

$$\mathcal{F}^{(2)} = d\,\mathcal{W}^{(1)} + \left(\mathcal{W}^{(1)}\right)^2$$

$$R^{(2)} = d\,\Gamma^{(1)} + \left(\Gamma^{(1)}\right)^2$$

as well as the covariant transformation rules (eq. 20)

(34)

$$\mathcal{F}^{\prime\,(2)} = U\,\mathcal{F}^{(2)}\,U^{-1}$$

$$R^{\prime\,(2)} = M\,R^{(2)}\,M^{-1}$$

The following questions, undecidable in the context of *classical local fields*, shall prepare the transition to Heraklitean space-time :

1) Can the charge like local gauge structure be obtained as a reduction of the global (E -) extended coordinate transformation gauging ?

2) Is such an extension towards general orientation gauging – specifically for $d > 4$ – at the origin of the apparent similarities, or are these fortuitous ? $\leftrightarrow$ this question gave rise to the original quest for unification .

We first turn towards 'gauging a Lie-transformation group acting on a distinct fibre' i.e. continuing with section 3b .

Now we treat as quantized only the $\mathcal{W}$ gauge fields appropriate for this minimal step (eq. 21) .

(35) $$E \left(B ; F , d_B + d_F \right) \sim B \times F$$

[a]

$\longrightarrow$

[a] **It is worth noting , that Albert Einstein escaped the following quantum complications in a logically consistent way ,** *assuming* **that ultimate field theories shall not be quantized .**

Using the *unperturbed* i.e. unquantized Killing fields defined in eq. 23 repeated below

(36)
$$h^{\,k}_{\,\varrho}\,(\,f\,)\;=\;\partial_{\,b\,\varrho}\;\Omega^{\,k}\,(\,b\,;\,f\,)\,\Big|_{\,b\,=\,0}$$

The $\mathcal{W}$ non - deformed metric in E is of the *classical* form

$$g_{\,A\,B}\;=\;\begin{pmatrix}\eta_{\,\mu\nu} & 0\\ 0 & G_{\,kl}\end{pmatrix}\quad;\quad\mu,\nu\;=\;0,1,2,3\quad,\quad k,l\;=\;1,\cdots,d_{\,F}$$

$$G^{\,kl}\;=\;\sum_{\varrho=1}^{dim\,G}\,h^{\,k}_{\,\varrho}\,(\,f\,)\,h^{\,k}_{\,\varrho}\,(\,f\,)\quad,\qquad{}^{kl}\;\longrightarrow\;{}_{kl}\;\leftrightarrow\;G\;\longrightarrow\;G^{\,-1}$$

(37)
$$\eta_{\,\mu\nu}\;=\;\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\quad;\quad c\;=\;1$$

→

Coordinates and their infinitesimal differentials (in E) shall be denoted by the pairs

(38) $$z^A = (x^\mu, f^k) \;\rightarrow\; dz^A = (dx^\mu, df^k)$$

$\pi\alpha\nu\tau\alpha\ \varrho\varepsilon\iota$ **comes in** $\mu\iota\kappa\varrho\alpha\ \beta\eta\mu\alpha\tau\alpha$ $\rightarrow$

quanta **come in** **small steps**

$$df^k \rightarrow d\underline{\Phi}^k = df^k + dx^\mu \underline{W}_\mu^\varrho(x)\, h_\varrho^k$$

$$d\underline{s}^2 = d_4 s^2 + d\underline{\Phi}^k G_{kl}\, d\underline{\Phi}^l = dz^A\, \underline{g(E)}_{AB}\, dz^B$$

---

(39) $$\underline{g(E)}_{AB} = \begin{pmatrix} \underline{g}_{\mu\nu} & \underline{W}_\mu^\varrho(x)\, h_{l\varrho}(f) \\ \underline{W}_\nu^\varrho(x)\, h_{k\varrho}(f) & G_{kl}(f) \end{pmatrix}$$

$$\underline{g}_{\mu\nu} = \eta_{\mu\nu} + \underline{W}_\mu^\varrho(x)\,\underline{W}_\mu^\tau(x)\, F_{\varrho\tau}(f) \quad ; \quad d_4 s^2 = dx^\mu \eta_{\mu\nu}\, dx^\nu$$

$d = 1 + 3$ **gravity switched off**

$$F_{\varrho\tau} = h_\varrho^k G_{kl} h_\tau^l \quad ; \quad h_{m\varrho} = G_{ml} h_\varrho^l$$

$\rightarrow$

In looking at the form of the full metric in eq. 39 (again)

(40) $$\underline{g\,(\,E\,)}_{\;A\;B} \;=\; \left(\begin{array}{cc} \underline{g}_{\;\mu\nu} & \underline{W}^{\;\varrho}_{\;\mu}\,(\,x\,)\,h_{\;l\;\varrho}\,(\,f\,) \\ \underline{W}^{\;\varrho}_{\;\nu}\,(\,x\,)\,h_{\;k\;\varrho}\,(\,f\,) & G_{\;kl}\,(\,f\,) \end{array}\right)$$

It takes some ingenuity to realize that it is for a semi-simple group equivalent *and helpful* **to replace the fibre associated gauge fields by Lie algebra valued ones , remembering eq. 29 for the definition of the adjoint Lie - algebra representation , and here we reduce the gauge group to one simple factor**

(41) $$\begin{array}{l} \underline{W}^{\;\varrho}_{\;\mu} \;\rightarrow\; \left(\underline{\mathcal{W}}_{\;\mu}\right)^{\;\sigma}_{\;\;\;\tau} \;=\; \underline{W}^{\;\varrho}_{\;\mu}\;(\,ad_{\;\varrho}\,)^{\;\sigma}_{\;\;\tau} \\ h_{\;l\;\varrho} \;\rightarrow\; (\,h_{\;l}\,)^{\;\sigma}_{\;\tau} \;=\; h_{\;l\;\varrho}\;(\,ad^{\;\varrho}\,)^{\;\sigma}_{\;\;\tau} \\ (\,ad_{\;\varrho}\,)^{\;\sigma}_{\;\;\tau} \;=\; f_{\;\sigma\;\varrho\;\tau} \quad ; \quad (\,ad^{\;\varrho}\,) \;=\; const\,(\,ad_{\;\varrho}\,) \\ -\,tr\;ad_{\;\varrho}\;ad_{\;\varrho^{\,\prime}} \;=\; f_{\;\varrho\sigma\tau}\;f_{\;\varrho^{\,\prime}\sigma\tau} \;=\; C_{\;2}\,(\,G_{\;simple}\,)\,\delta_{\;\varrho\;\varrho^{\,\prime}} \end{array}$$

adjusting the normalizations of Killing fields , and the dual adjoint matrices in the way indicated in eq. 41.

$\rightarrow$

The next step consists – within charge like fibre gauging – to compute the Riemann tensor and scalar for the full metric on E and its (reduced) integral over the fibre (modulo irrelevant constants)

(42)

$$\underline{S} = \int d^4 x \, \underline{\mathcal{L}}_4 = \int d^4 x \, d^{d_F} f \, \sqrt{\left| \underline{g(E)} \right|} \, \underline{R(E)}$$

$$\left(\underline{\mathcal{L}}_4 = \frac{1}{4} \mathcal{N} \, tr \left(\underline{\mathcal{W}}_{\mu\nu} \underline{\mathcal{W}}^{\mu\nu} \right) \rightarrow - \frac{1}{4} g^{-2} \underline{W}^{\varrho}_{\mu\nu} \underline{W}^{\mu\nu \, \varrho} \right) (x)$$

$$\underline{\mathcal{W}}_{\mu\nu} = \partial_\nu \underline{\mathcal{W}}_\mu - \partial_\mu \underline{\mathcal{W}}_\nu + \left[\underline{\mathcal{W}}_\nu , \underline{\mathcal{W}}_\mu \right] = \underline{W}^{\varrho}_{\mu\nu} \left(ad_{\varrho} \right)$$

$$\underline{W}^{\varrho}_{\mu\nu} = \partial_\nu \underline{W}^{\varrho}_{\mu} - \partial_\mu \underline{W}^{\varrho}_{\nu} + f_{\varrho\sigma\tau} \underline{W}^{\sigma}_{\nu} \underline{W}^{\tau}_{\mu}$$

We note that the integrations in eq. 42 over x *and* f **– base space and fibre – are performed over** *classical* **variables , which for** x **imples that light-cones are universally 'given' and rigid .**

The direct projection – sidestepping the entire fibre space-time extension – first proposed by Chen-Ning Yang and Robert Mills [6] , has led upon the addition of Yukawa - and sclar field interactions to a very successful description , always leaving $d = 1 + 3$ **space-time rigid** *i.e. untouched* **, of all non-gravitational interactions known to this day. This is referred to as the (neutrino mass extended) Standard Model of elementary particle interactions.** $\longrightarrow$

4 The logical path to Heraklitean space-time

The derived similarities between charge like gauges and gauging of orientation as derived in eqs. 32 - 34 repeated below

$$\mathcal{W}^{\prime\,(1)} = U\,\mathcal{W}^{(1)}\,U^{-1} + U\,d\,U^{-1} \quad \rightarrow \quad \underline{\mathcal{W}^{(1)}}$$

$$U(z) = Ad(a(z))$$

(43)

$$\Gamma^{\prime\,(1)} = M\,\Gamma^{(1)}\,M^{-1} + M\,d\,M^{-1}$$

$$M(z) = \partial_z\, z^{\prime}$$

$$\mathcal{F}^{(2)} = d\,\mathcal{W}^{(1)} + \left(\mathcal{W}^{(1)}\right)^2 \quad \rightarrow \quad \underline{\mathcal{W}}^{(2)}$$

$$R^{(2)} = d\,\Gamma^{(1)} + \left(\Gamma^{(1)}\right)^2$$

(44)

$$\mathcal{F}^{\prime\,(2)} = U\,\mathcal{F}^{(2)}\,U^{-1} \quad \rightarrow \quad \underline{\mathcal{W}}^{(2)}$$

$$R^{\prime\,(2)} = M\,R^{(2)}\,M^{-1}$$

and the entry of quantized fields through ’gauging a Lie tranformation group acting on a distinct fibre’ (section 3b) , lead to the following path of thoughts →

1) If the said similarity in geometric conception of gauging orientation and gauging distinct fibres is *assumed not* **accidental, then the 'projecting away' of the fibre is not legitimate, no matter how small, i.e. invisible to the 'naked eye', the space associated with fibre(s) may be.**

This leads even without gauging orientation to a loss of locality in the complete fibred space E .

2) Also the simultaneous freezing of gravity associated fields in $d = 1 + 3$ **space-time is** *perfectly known* **to be incorrect , despite this forming up to short** *distances* **explored so far an excellent approximation.**

3) The account of multiple obstacles to perfect locality of quantized operators including a metric in a general number of dimensions shall be concentrated in the following :

→

3) continued

consider the differential equation for a geodesic for a quantized metric and metric connection

$$\left(g_{AB} \, , \, (\Gamma_C)^A{}_B \rightarrow \underline{g}_{AB} \, , \, (\underline{\Gamma}_C)^A{}_B \right) (z)$$

(45) $$\ddot{Z}^A + (\underline{\Gamma}_C)^A{}_B (Z) \dot{Z}^C \dot{Z}^B = 0$$

for a path : $Z = Z (\tau)$; τ : path parameter, with $\dot{(\bullet)} = d / d \tau (\bullet)$

cannot have any solution , unless Z is promoted to a quantized field , with classical base space beeing 'adapted'

(46) $$Z \rightarrow \underline{Z} (z)$$

Determining how classical base space shoud be 'adapted' is here immaterial .
The mapping

(47) $$z \rightarrow \underline{Z} (z)$$

does not represent a coordinate transformation , but a 'Heraklitean' one .

....

Hermann Minkowski , 21. September 1908 in Cologne :

”Von Stund an sollen Raum für sich und Zeit für sich völlig zu Schatten herabsinken und nur noch eine Art Union der beiden soll Selbständigkeit bewahren.”

Thank you

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