The taming of the red dragon

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Spontaneous topics in theoretical physics
Bern, May 5, 2006

There is the broad object seen in $\pi\pi$ scattering, often called "background", which extends from about 400 MeV up to about 1700 MeV. This object we consider as a single broad resonance[2] *which we identify as the lightest glueball with quantum numbers* $J^{PC} = 0^{++}\dots$

[2] *we refer to it as red dragon*

P. Minkowski and W. Ochs, Eur. Phys. J. C9 (1999) 283

Low energy pion physics

- Plays a crucial role whenever the strong interaction is involved at low energies

  Example: Standard model prediction for muon magnetic moment

- Main experiments on $\pi\pi$ scattering were done in the seventies. What's new ?
  - Significant theoretical progress, based on ChPT + dispersion theory
  - New precision data:

$K \to \pi\pi\ell\nu$	E865	Brookhaven
pionic atoms	DIRAC	CERN
$K \to 3\pi$	NA48/2	CERN

  - Lattice results on M_π, F_π, a_0^2, $\langle r^2 \rangle_s$

Analyticity and crossing

- $\pi\pi$ scattering is special: crossed channels are identical

$\Rightarrow$ Re $T(s,t)$ can be represented as a twice subtracted dispersion integral over Im $T(s,t)$ in physical region

S.M. Roy 1971

- The 2 subtraction constants can be identified with the S-wave scattering lengths:

$$a_0^0\;,\;\;a_0^2\;\begin{matrix}\leftarrow \text{isospin}\\ \leftarrow \text{angular momentum}\end{matrix}$$

- Representation leads to dispersion relations for the individual partial waves: *Roy equations*

Roy equations

- Pioneering work on the physics of the Roy equations: Basdevant, Froggatt & Petersen 1974
- Dispersion integrals converge rapidly (2 subtractions)

$\Rightarrow$ Crude phenomenological information on Im $T(s,t)$ for energies above 800 MeV suffices

$\Rightarrow$ Given a_0^0, a_0^2, the scattering amplitude can be calculated to within small uncertainties

Ananthanarayan, Colangelo, Gasser & L. 2001
Descotes, Fuchs, Girlanda & Stern 2002

$\Rightarrow$ a_0^0, a_0^2 are the essential parameters at low energy

- Main problem in early work: a_0^0, a_0^2 poorly known
Experimental information near threshold is meagre

Low energy theorems

- Chiral perturbation theory provides the missing piece: theoretical prediction for a_0^0, a_0^2

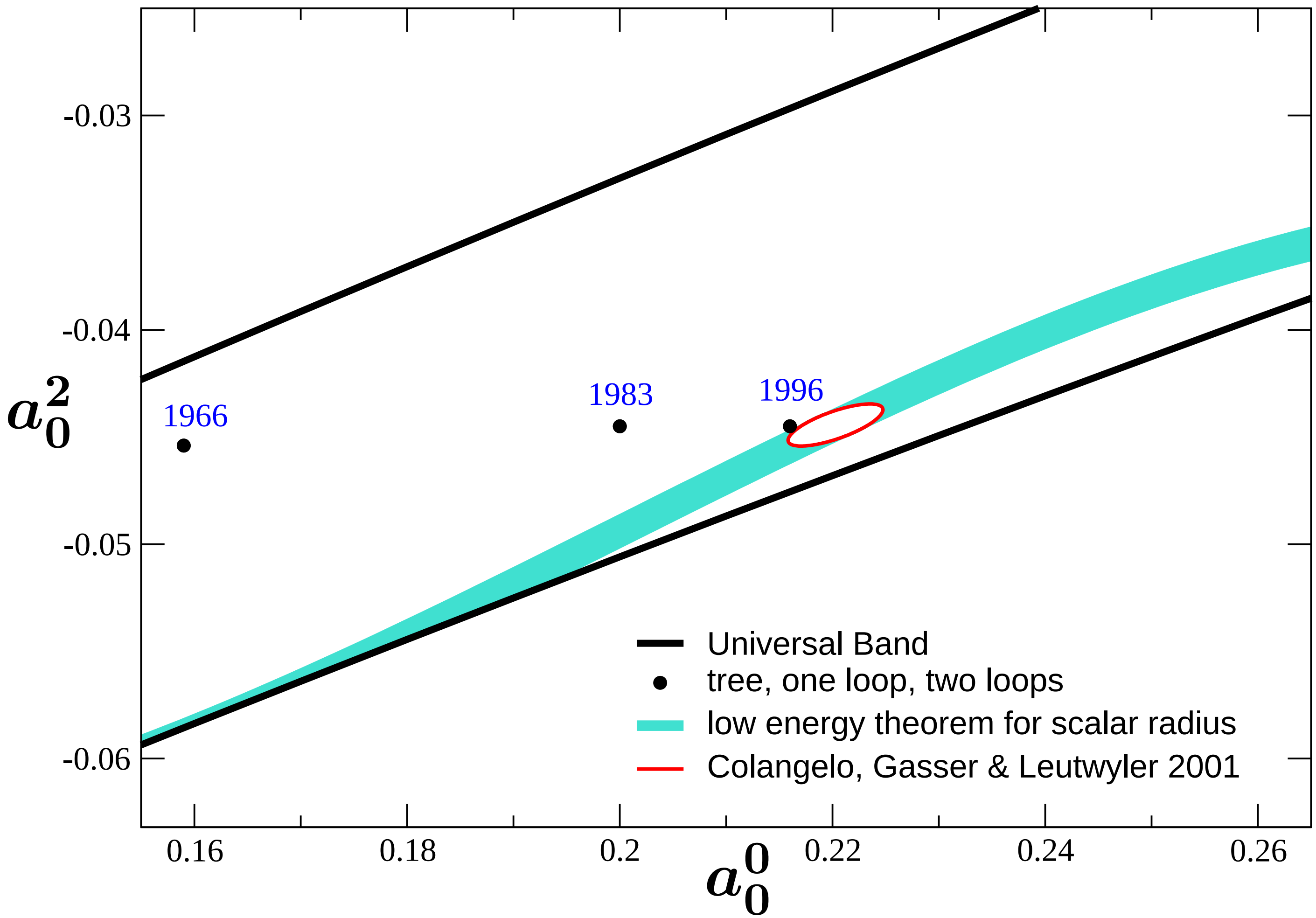
  Weinberg 1966, Gasser & L. 1983, Bijnens, Colangelo, Ecker, Gasser & Sainio 1996

- Most accurate results for a_0^0, a_0^2 are obtained by matching the chiral and dispersive representations near the center of the Mandelstam triangle

  Colangelo, Gasser & L. 2001

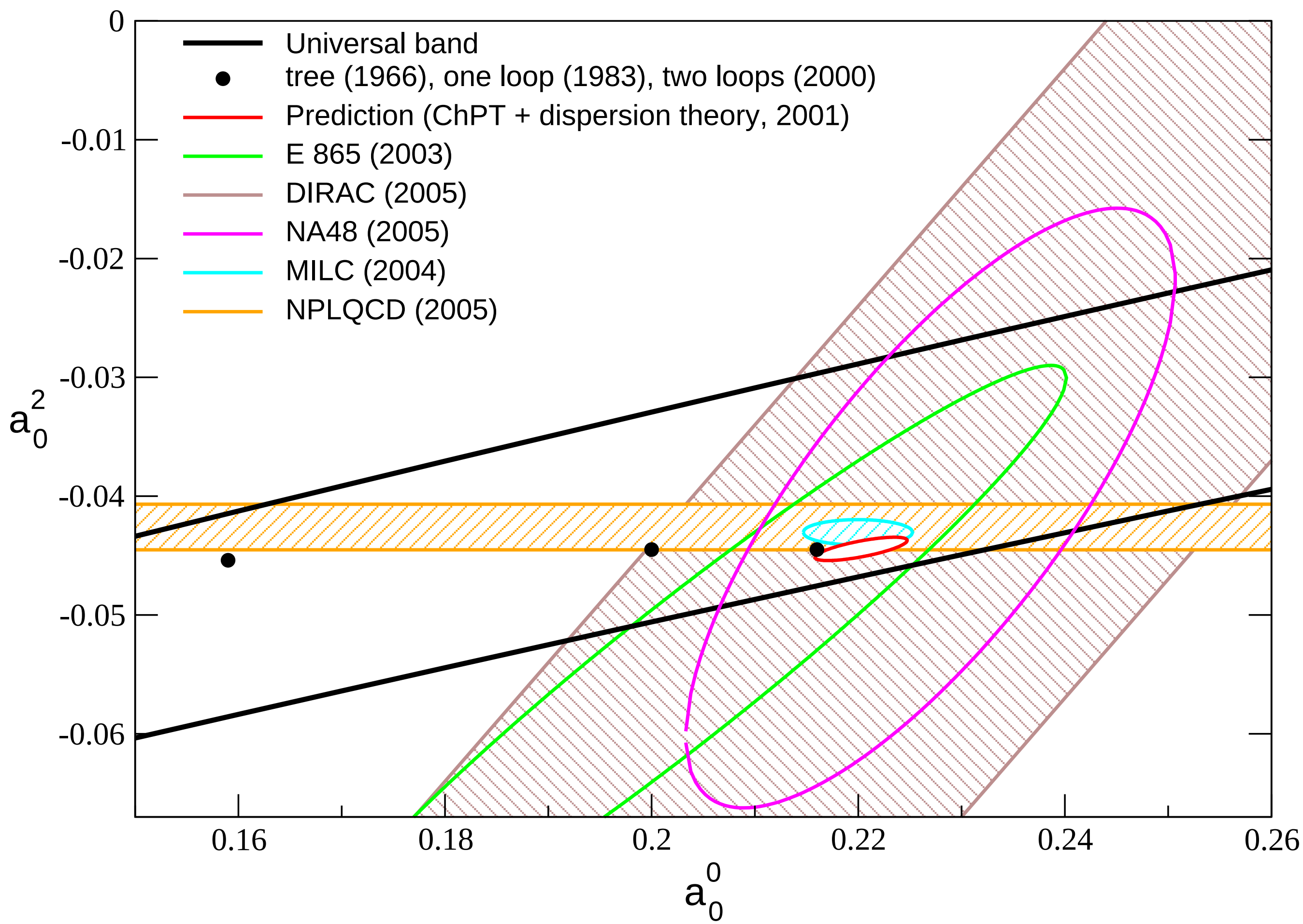
- In combination with the low energy theorems for a_0^0, a_0^2, the dispersion relations for the partial waves fix the $\pi\pi$ scattering amplitude to an incredible degree of accuracy

Predictions for the S-wave $\pi\pi$ scattering lengths



Sizeable corrections in a_0^0, while a_0^2 nearly stays put

Tests of the predictions for a_0^0, a_0^2: experiment & lattice



Theory is ahead of experiment . . .

The red dragon

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001

- Does QCD have a resonance near threshold ?
- Why care ?
  - Concerns the nonperturbative domain of QCD
  - Quark and gluon degrees of freedom useless there

  $\Rightarrow$ Understanding very poor, pattern of energy levels ?
  - Lowest resonance: σ ? ρ ?
- Resonance $\leftrightarrow$ pole on second sheet
  - Poles are universal
  - Pole position is unambiguous, even if width is large
  - Where is the pole closest to the origin ?

$f_0(600)$ or σ

$I^G(J^{PC}) = 0^+(0^{++})$

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$f_0(600)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\,\mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
(400–1200)−*i*(300–500) OUR ESTIMATE				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$(541 \pm 39) - i(252 \pm 42)$	[1] ABLIKIM	04A	BES2	$J/\psi \to \omega\pi^+\pi^-$
$(528 \pm 32) - i(207 \pm 23)$	[2] GALLEGOS	04	RVUE	Compilation
$(440 \pm 8) - i(212 \pm 15)$	[3] PELAEZ	04A	RVUE	$\pi\pi \to \pi\pi$
$(533 \pm 25) - i(247 \pm 25)$	[4] BUGG	03	RVUE	
$532 - i272$	BLACK	01	RVUE	$\pi^0\pi^0 \to \pi^0\pi^0$
$(470 \pm 30) - i(295 \pm 20)$	[5] COLANGELO	01	RVUE	$\pi\pi \to \pi\pi$
$(535^{+48}_{-36}) - i(155^{+76}_{-53})$	[6] ISHIDA	01		$\Upsilon(3S) \to \Upsilon\pi\pi$
$610 \pm 14 - i620 \pm 26$	[7] SUROVTSEV	01	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(558^{+34}_{-27}) - i(196^{+32}_{-41})$	ISHIDA	00B		$p\overline{p} \to \pi^0\pi^0\pi^0$
$445 - i235$	HANNAH	99	RVUE	π scalar form factor
$(523 \pm 12) - i(259 \pm 7)$	KAMINSKI	99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\sigma\sigma$
$442 - i\,227$	OLLER	99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$469 - i203$	OLLER	99B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$445 - i221$	OLLER	99C	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$
$(1530^{+\ 90}_{-250}) - i(560 \pm 40)$	ANISOVICH	98B	RVUE	Compilation
$420 - i\,212$	LOCHER	98	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(602 \pm 26) - i(196 \pm 27)$	[8] ISHIDA	97		$\pi\pi \to \pi\pi$
$(537 \pm 20) - i(250 \pm 17)$	[9] KAMINSKI	97B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, 4π
$470 - i250$	[10,11] TORNQVIST	96	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $K\pi$, $\eta\pi$
$\sim (1100 - i300)$	AMSLER	95B	CBAR	$\overline{p}p \to 3\pi^0$
$400 - i500$	[11,12] AMSLER	95D	CBAR	$\overline{p}p \to 3\pi^0$
$1100 - i137$	[11,13] AMSLER	95D	CBAR	$\overline{p}p \to 3\pi^0$
$387 - i305$	[11,14] JANSSEN	95	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$525 - i269$	[15] ACHASOV	94	RVUE	$\pi\pi \to \pi\pi$
$(506 \pm 10) - i(247 \pm 3)$	KAMINSKI	94	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$370 - i356$	[16] ZOU	94B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$408 - i342$	[11,16] ZOU	93	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$870 - i370$	[11,17] AU	87	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$470 - i208$	[18] BEVEREN	86	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$, ...
$(750 \pm 50) - i(450 \pm 50)$	[19] ESTABROOKS	79	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(660 \pm 100) - i(320 \pm 70)$	PROTOPOP...	73	HBC	$\pi\pi \to \pi\pi$, $K\overline{K}$
$650 - i370$	[20] BASDEVANT	72	RVUE	$\pi\pi \to \pi\pi$

Model independent determination of the pole

- All of the results quoted by the PDG are obtained by
  (a) parametrizing the data for real values of s
  (b) continuing this parametrization analytically in s

$\Rightarrow$ Result is sensitive to the parametrization used

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- We found a model independent method:
  1. Poles on second sheet are zeros on first sheet
  2. The Roy equations are valid for complex values of s, in a limited region of the first sheet

  $\Rightarrow$ Exact representation of the partial waves in terms of observable quantities, valid for complex values of s

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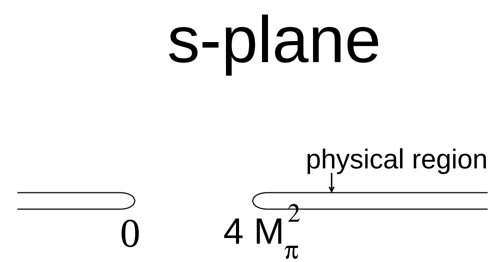
  $\Rightarrow$ Exact representation of the partial waves in terms of observable quantities, valid for complex values of s

  3. Can evaluate this representation to good precision and determine the zeros numerically

Pole on second sheet $\longleftrightarrow$ zero on first sheet

- $S_0^0(s) = \eta_0^0(s)\exp 2i\delta_0^0(s)$

  $S_0^0(s)$ is analytic in the cut plane



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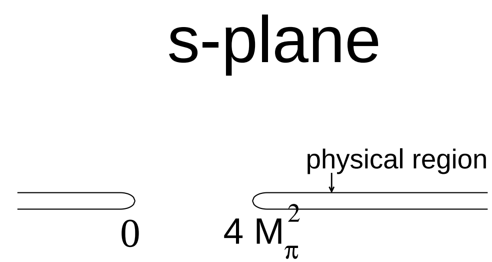
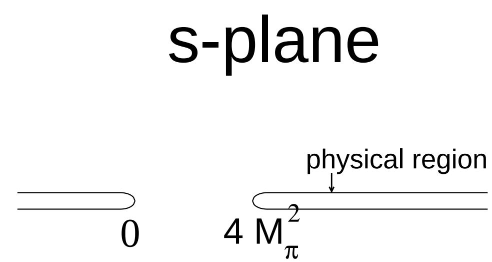


- For $0 < s < 4M_\pi^2$, $S_0^0(s)$ is real

$\Rightarrow$ $S_0^0(s^\star) = S_0^0(s)^\star$

x in elastic interval: $S_0^0(x \pm i\epsilon) = \exp \pm 2i\delta_0^0(x)$

Pole on second sheet ⟷ zero on first sheet



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- Second sheet is reached by continuation across the elastic interval of the right hand cut

  $S_0^0(x - i\epsilon)^{II} = S_0^0(x + i\epsilon)^I = 1/S_0^0(x - i\epsilon)^I$

  Analyticity $\Rightarrow$ $\boxed{S_0^0(s)^{II} = 1/S_0^0(s)^I}$ valid $\forall s$

  Pole in $S_0^0(s)^{II} \Longleftrightarrow$ zero in $S_0^0(s)^I$

Roy equation for the isoscalar S-wave

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$$t_0^0(s) = a + (s - 4M_\pi^2)\, b + \int_{4M_\pi^2}^{\infty} ds' \left\{ K_0(s,s')\, \mathrm{Im}\, t_0^0(s') + K_1(s,s')\, \mathrm{Im}\, t_1^1(s') + K_2(s,s')\, \mathrm{Im}\, t_0^2(s') \right\}$$

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- The subtraction constants are determined by a_0^0, a_0^2:
  $a = a_0^0\,, \qquad b = (2a_0^0 - 5a_0^2)/(12M_\pi^2)$

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  $$a = a_0^0\,, \qquad b = (2a_0^0 - 5a_0^2)/(12 M_\pi^2)$$
- The kernels are elementary functions, e.g.
  $$K_0(s,s') = \underbrace{\frac{1}{\pi(s'-s)}}_{r.h.cut} + \underbrace{\frac{2\, ln\{(s+s'-4M_\pi^2)/s'\}}{3\pi(s-4M_\pi^2)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'(s'-4M_\pi^2)}}_{l.h.cut}$$

Roy equation for the isoscalar S-wave

$$S_0^0(s)=1+2\,i\rho\,t_0^0(s) \qquad \rho=\sqrt{1-4M_\pi^2/s}$$

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$$+K_1(s,s')\,\mathrm{Im}\,t_1^1(s')+K_2(s,s')\,\mathrm{Im}\,t_0^2(s')\}$$

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- Left hand cut is essential for convergence:
  $K_0(s,s')\sim 1/{s'}^3$ for large s'

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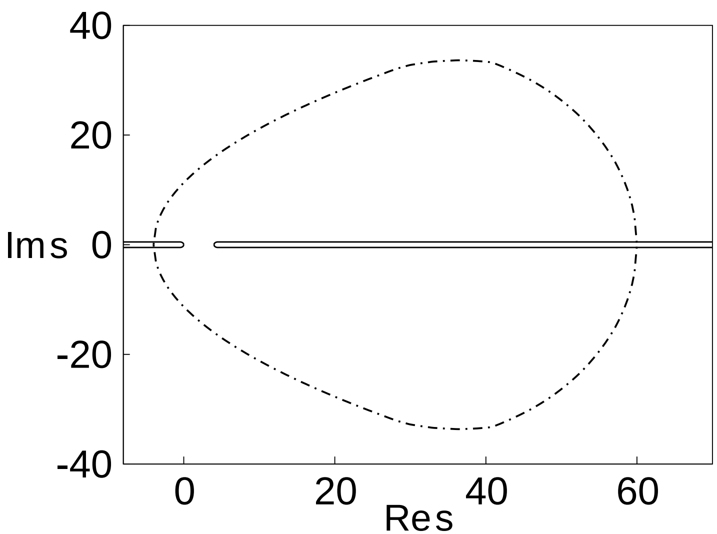

s in units of M_π^2

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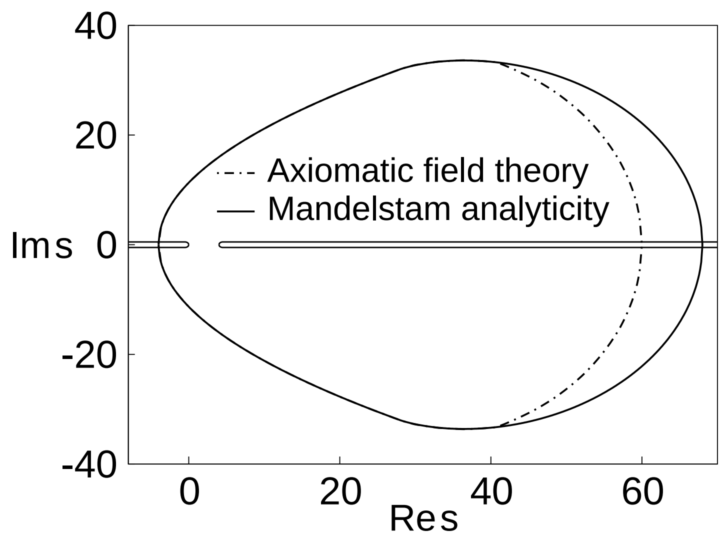

s in units of M_π^2

- Proof is based on first principles, general quantum field theory

A. Martin, Scattering Theory:
Unitarity, Analyticity and Crossing
Lecture Notes in Physics, vol. 3, 1969

G. Mahoux, S. M. Roy and G. Wanders
Nucl. Phys. B 70 (1974) 297

$\Rightarrow$ Exact representation for $S_0^0(s)$ in this region
Do not need to parametrize the amplitude

Evaluation of the pole position

- Insert our solutions of the Roy equations
  For the central solution, $S_0^0(s)$ has two pairs of zeros in the region of validity of the representation:
  $s = (6.2 \pm i\, 12.3)\, M_\pi^2 \quad \sigma$
  $s = (51.4 \pm i\, 1.4)\, M_\pi^2 \quad f_0(980)$

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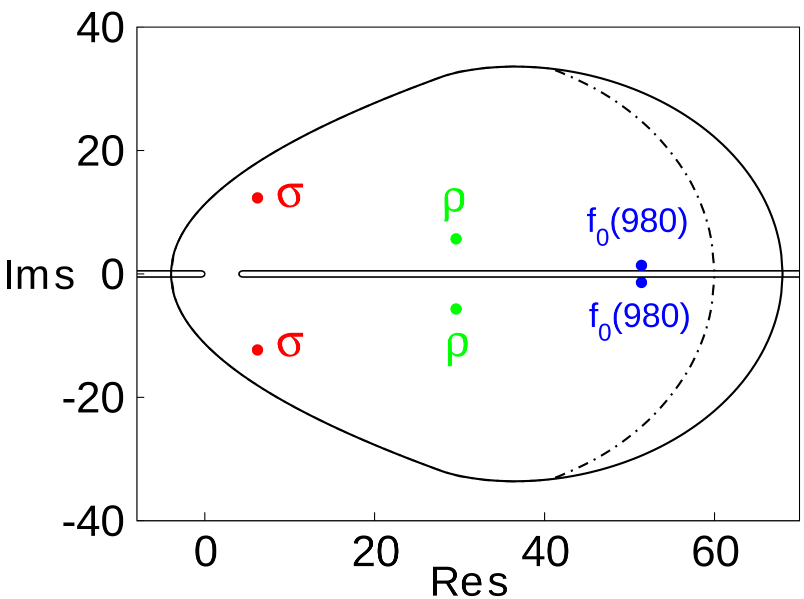

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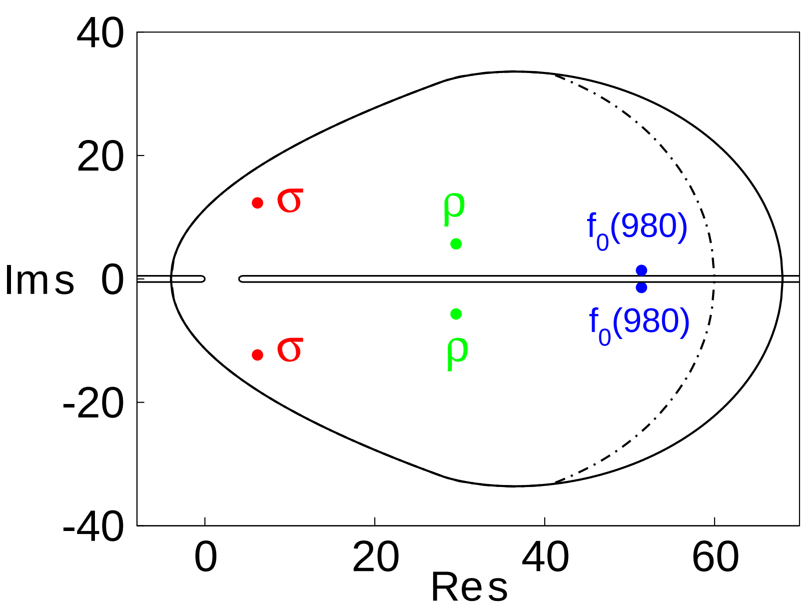

s in units of M_π^2

The eyes of the red dragon

Tail at 1.7 GeV: $s \simeq 150\,M_\pi^2$

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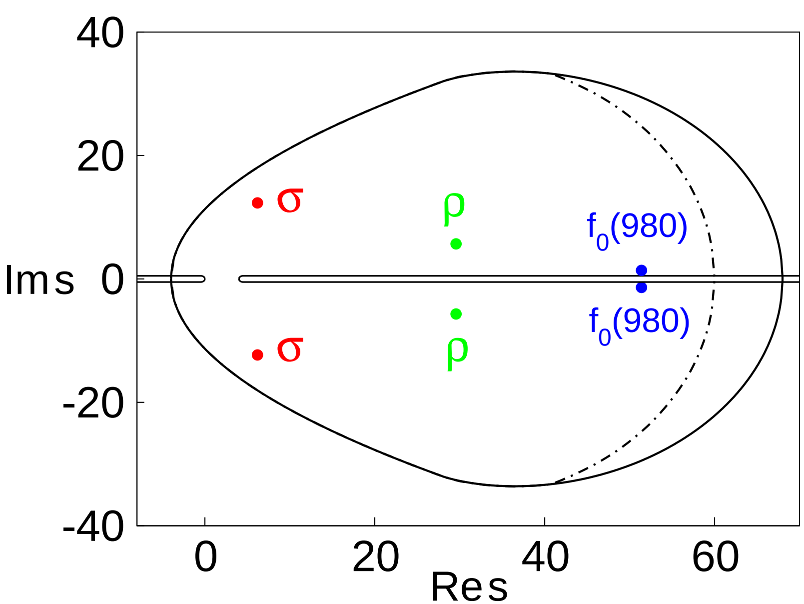

s in units of M_π^2

$\Rightarrow$ 1. Lowest resonance of QCD has vacuum quantum numbers

2. Pole on lower half of sheet II occurs in vicinity of

$$m_\sigma = 441 - i\,272\,\text{MeV}$$
$$= M_\sigma - \tfrac{i}{2}\Gamma_\sigma$$

Error analysis

- Results depend on phenomenological input used when solving the Roy equations, subject to uncertainties
  Can follow error propagation explicitly
- Pole position of σ mainly depends on 3 input variables:

  $a_0^0, a_0^2, \delta_A \equiv \delta_0^0$(800 MeV)

  - Substantial uncertainties in phenomenology of δ_A
  - Use the range: $\delta_A = 82.3^\circ{}^{+10^\circ}_{-4^\circ}$

Error analysis

- Noise from remaining input variables is very small:

  $m_\sigma^0 = (441 \pm 4) - i(272 \pm 6)$ MeV

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- Values of a_0^0, a_0^2, δ_A are crucial:

$$\Delta m_\sigma = (-2.4 + i\,3.8)\,\frac{a_0^0 - 0.22}{0.005} + (0.8 - i\,4.0)\,\frac{a_0^2 + 0.0444}{0.001} + (5.3 + i\,3.3)\,\frac{\delta_A - 82.3}{3.4}$$

numbers in MeV

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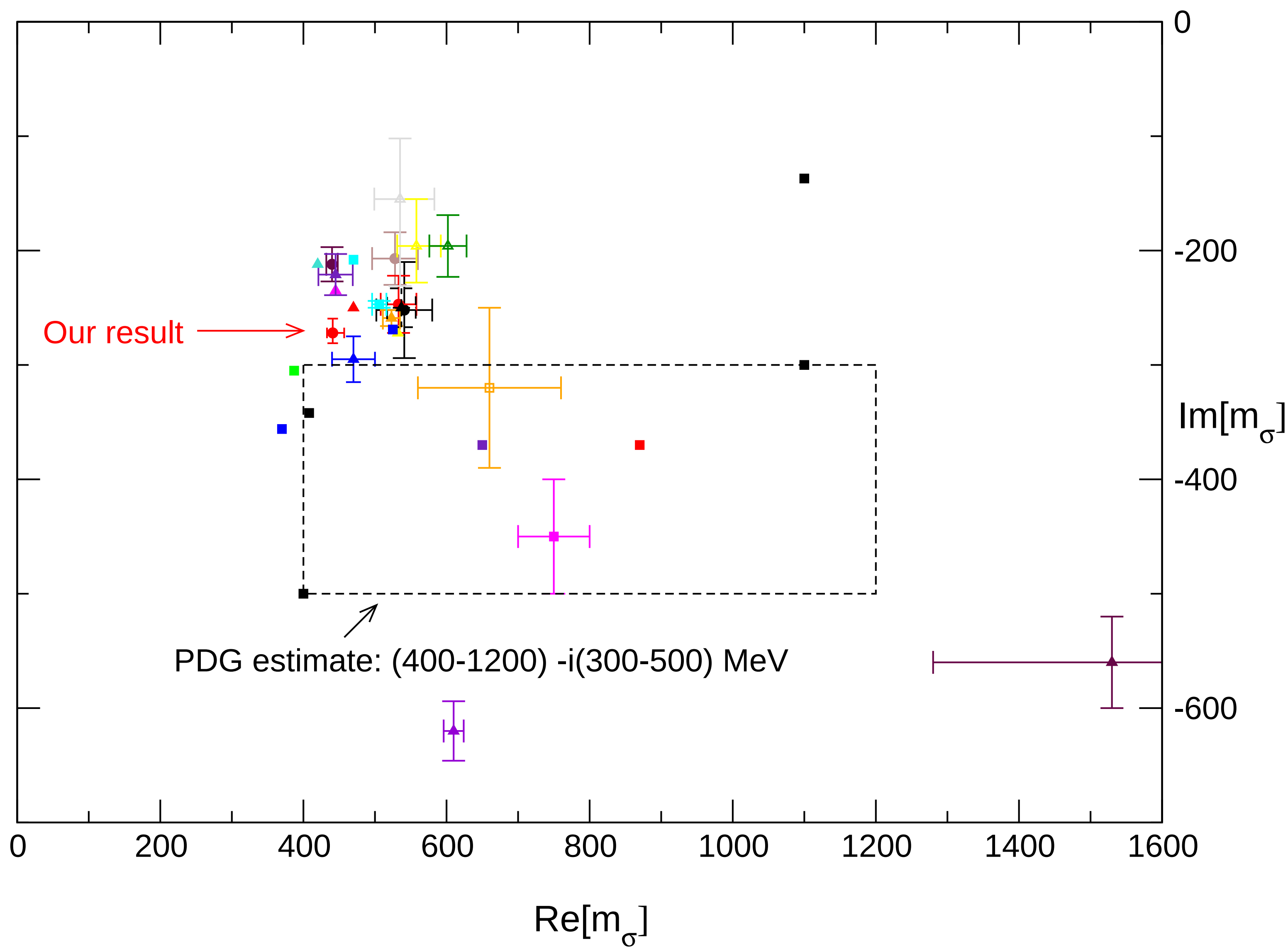
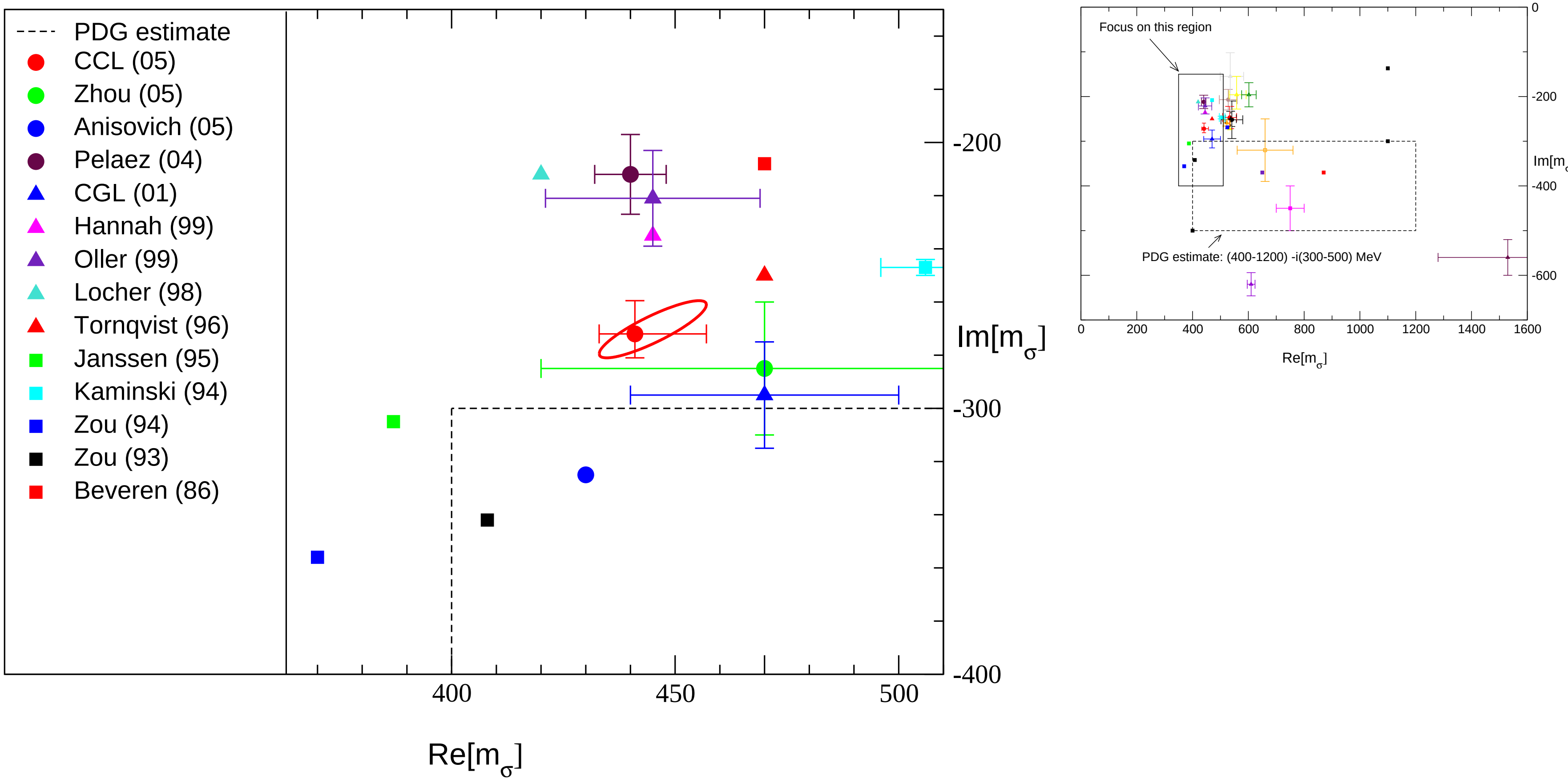
- Final result: insert the predictions for a_0^0, a_0^2, use the phenomenological range for δ_A and add errors up:

$$m_\sigma = 441\,{}^{+16}_{-8} - i\,272\,{}^{+9}_{-13}\ \text{MeV}$$

Comparison with compilation of PDG



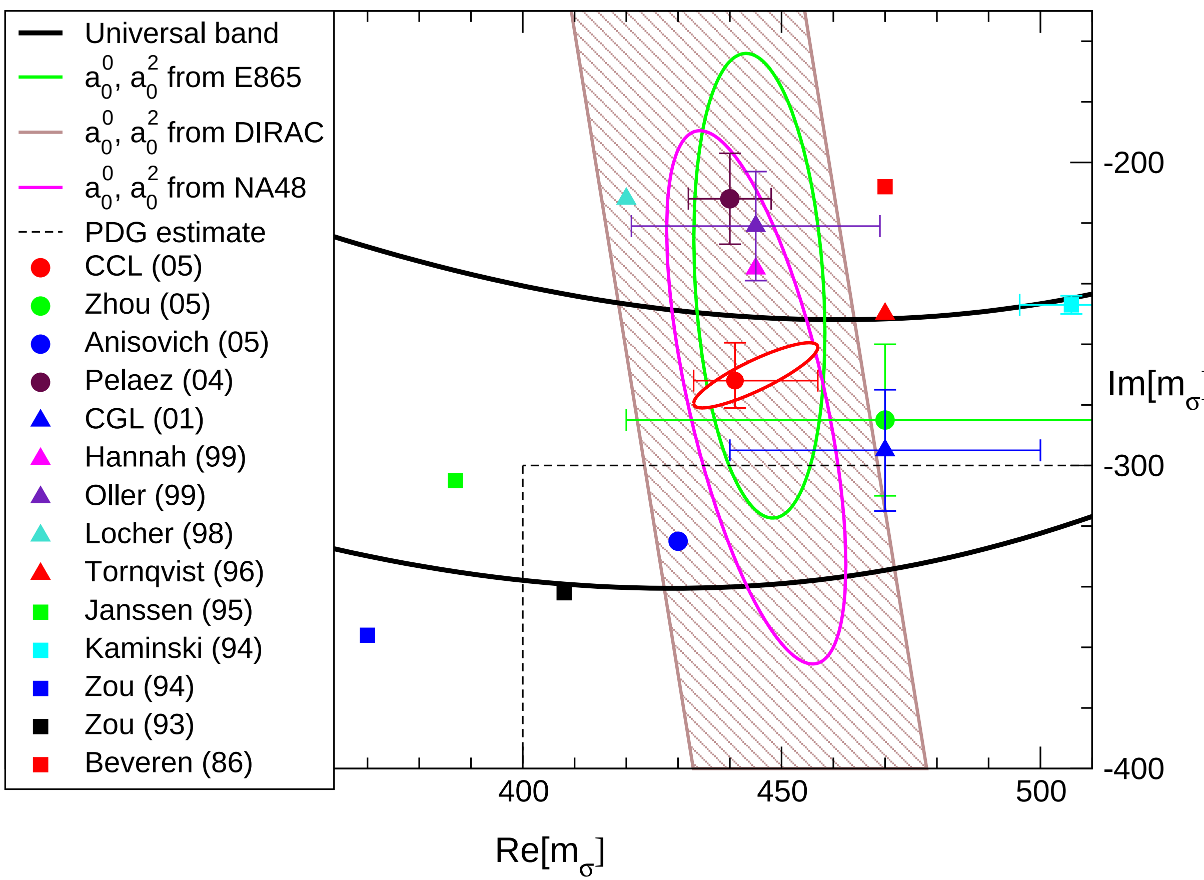
Vicinity of the pole



Results for Re$[m_\sigma]$ and Im$[m_\sigma]$ are strongly correlated

Ignore the theoretical predictions for a_0^0, a_0^2

- Replace the low energy theorems for a_0^0, a_0^2 by the experimental results from E865, DIRAC and NA48
- $a_0^0, a_0^2 \in$ universal band



Why are our errors so incredibly small ?

- The σ occurs at low energies
- At low energies, the subtraction term dominates

$$t_0^0(s) \simeq a_0^0 + (2a_0^0 - 5a_0^2)\frac{(s-4M_\pi^2)}{12M_\pi^2}$$

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$$t_0^0(s) \simeq a_0^0 + (2a_0^0 - 5a_0^2)\frac{(s - 4M_\pi^2)}{12M_\pi^2}$$

Insert low energy theorem for a_0^0, a_0^2

$\Rightarrow$ Roy equation reduces to Weinberg formula

$$t_0^0(s) \simeq \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Dispersion integrals only represent a correction

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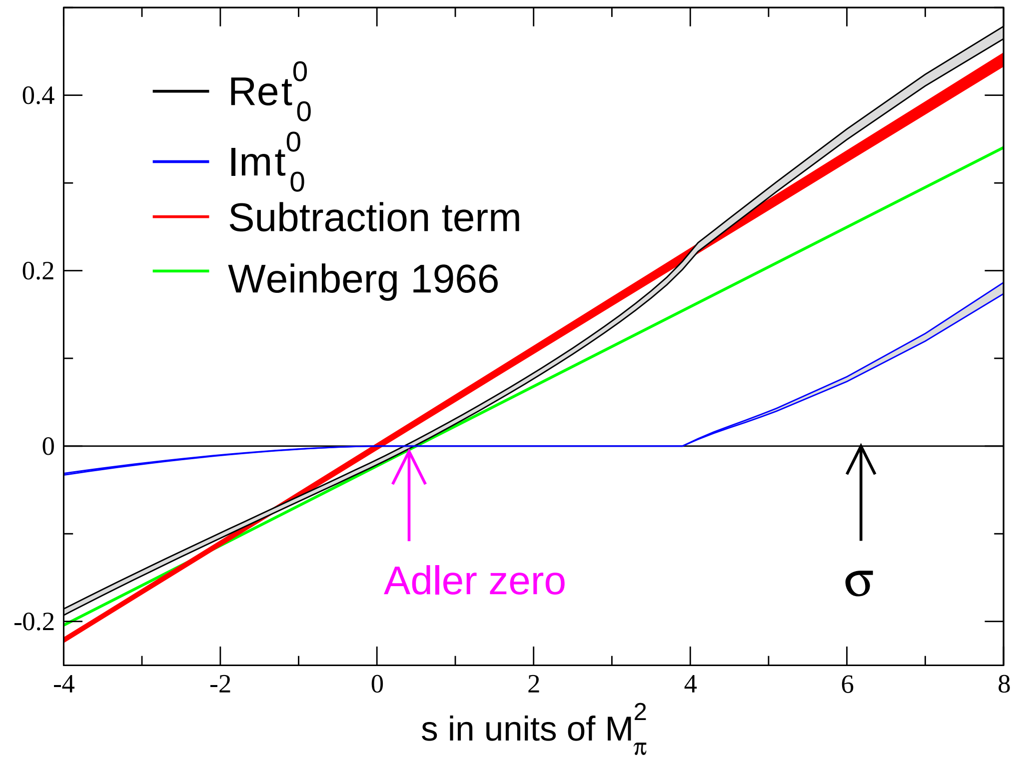

$$s = (0.41 \pm 0.06)\, M_\pi^2 \quad \text{Adler zero}$$

$$s = (6.2 - i\,12.3)\, M_\pi^2 \quad \text{pole from } \sigma$$

Estimate pole position on back of an envelope

- Approximate $t_0^0(s)$ with the Weinberg formula

$$t_0^0(s) = \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Where are the zeros of $S_0^0(s)$ in this approximation ?

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$$1 + 2\,i\,\sqrt{1 - 4M_\pi^2/s}\;t_0^0(s) = 0$$

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Where are the zeros of $S_0^0(s)$ in this approximation ?

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$\Rightarrow$ Cubic equation for s

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- Real zero on sheet II, near $s = 0$ (full amplitude has kinematic singularity: vanishes on sheet II at $s = 0$)

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  - Precision experiments carried out and under way
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  - So far, all tests confirm the theory
- Limitations of our approach:
  - Calculations cannot be done on back of an envelope
  - Method only covers low energies
  - Only a few applications have been worked out:
    $\pi\pi$ scattering, pion form factors, hadronic vacuum polarization in SM prediction for muon $g-2$
    $\gamma\gamma \to \pi^0\pi^0$ M. Pennington, hep-ph/0604212
- Much is yet to be done: $J/\psi \to \omega\pi\pi$, $D \to 3\pi$, ...
  πK, κ, ...

Conclusion

- Model independent method for analytic continuation
  - The lowest resonance of QCD occurs at

    $$M_\sigma = 441\,{}^{+16}_{-8}\ \text{MeV} \qquad \Gamma_\sigma = 544\,{}^{+18}_{-25}\ \text{MeV}$$

    and carries vacuum quantum numbers
  - Crossing symmetry plays an essential role:
    Fixes contributions from left hand cut
    Ensures fast convergence, low energy dominance
  - Pole occurs at low value of s, closer to left hand cut than to singularities from $K\bar{K}$, $f_0(980)$
  - Result for Γ_σ relies on theory for a_0^2
    Experiments concerning a_0^2 would be most welcome

VISIT THE TAMED RED DRAGON

GENTLE ANIMAL

LOOK IN HIS EYES FROM CLOSE

SMELL HIS GOOD BREATH

BRING YOUR PIONS ALONG AND

FEED HIM YOURSELF

The management denies responsibility for incidents involving the dragon's tail