On prospects and dangers arising from borders of analytical mastery in exploring scalar mesons within QCD

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1 Light flavored $q'\bar{q}$ nonets with $J^{PC}{}_{n} = 1^{+-}, 1^{++}, 2^{++}$ a *spectroscopic* entry point

Let me begin with the spectroscopic representation of meson resonance nonets ascribed to a valence $q'\bar{q}$; $q = u, d, s$ composition with $L_{q'\bar{q}} = 1$, excluding scalars , as entry point to the discussion of the latter .

The word 'spectroscopy' *must* **be used with clear caution in all situations where the ratio of width to mass of a** *hadronic* **resonance is outside the range of percents.**

I take as mass, e.g. in figure 1 – faute de mieux – the best value as given by the PDG and neglect the width .

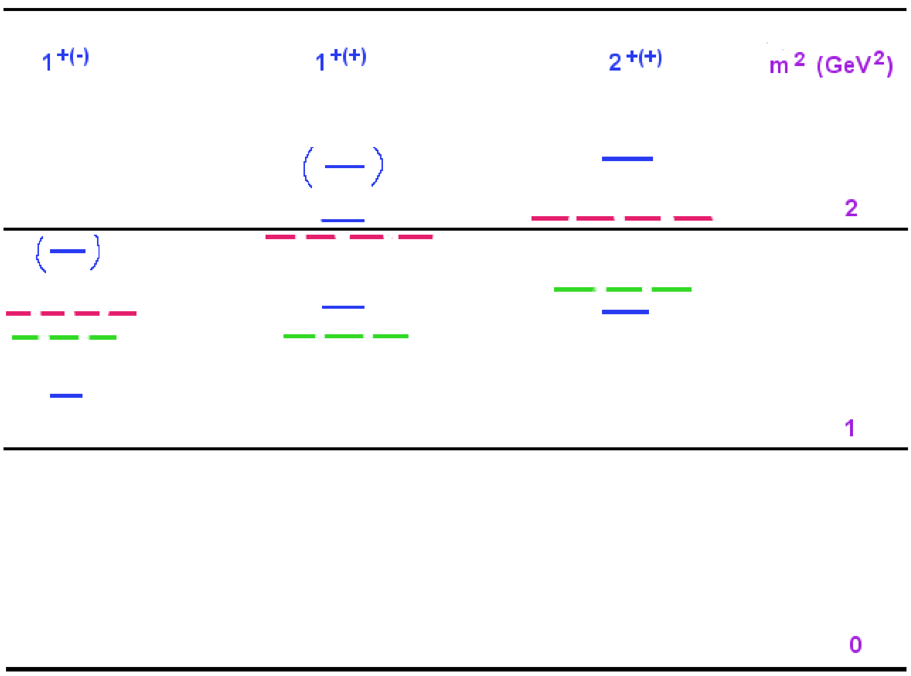

Figure 1:

$J^{PC}{}_{n} = 1^{+-}, 1^{++}, 2^{++}$ – **nonets** *adapted* **from ref. [1]** (→) .

Comments to figure 1 :

1 $J^{PC}{}_{n} = 1^{+-}$ **: This nonet contains only 8 resonances within the 'Summary Tables' of the PDG [1] . I have included a ninth state in parentheses** $h_1(1380)$ **, omitting a tenth candidate** $-\ h_1(1595)$ **, denoting entries by** $X\ (\ M\ \pm\ \Delta M\ ,\ \Gamma\ \pm\ \Delta\Gamma\ [\ \textbf{MeV}\ ]\ )$ **, forming the set**

$$\begin{array}{ll} h_1(\ 1170\ \pm\ 40\ ,\ 360\ \pm\ 40\ ) & I^G = 0^- \\ b_1(\ 1229.5\ \pm\ 3.2\ ,\ 142\ \pm\ 9\ ) & I^G = 1^+ \\ K_1(\ 1272\ \pm\ 7\ ,\ 90\ \pm\ 20\ ) & I = \frac{1}{2} \\ \left(\ h_1(\ 1386\ \pm\ 19\ ,\ 91\ \pm\ 30\ )\ \right) & I^G = 0^- \end{array} \tag{1}$$

The original findings on the state $h_1(1380)$ **were the result of interference analysis by the LASS collaboration [2] , [3] in the reaction**

$$K^- p \ \rightarrow \ K_s \ K^{\pm} \ \pi^{\mp} \ \Lambda \tag{2}$$

The interference involves two – hypothetical – resonances

$h_1(1380)\ ;\ 1^{+-}$ **discussed presently and**

$f_1(1510)\ ;\ 1^{++}$ **, added in parentheses to the** *selected* 1^{++} **– nonet in figure 1 , to be discussed below ,**

and is deduced by comparing the invariant mass distributions of $K_{1^-}(892)\ \overline{K}\ \leftrightarrow\ \overline{K}_{1^-}(892)\ K$

where $K^{+(-)}\ \pi^{-(+)}$ **serve to tag the resonant subsystems** $K_{1^-}(892)\ \left(\ \overline{K}_{1^-}(892)\ \right)$ **.** →

The interference analysis of the LASS collaboration is refined *at least apparently* **by Crystal Barrel at CP-Lear (CERN) [4] , [5] studying final state interactions in the reactions**

$$\begin{array}{ll} a) \ \ p\,\overline{p} \ \rightarrow \ K_l \ K_s \ \pi^0 \ \pi^0 & [4] \\ b) \ \ p\,\overline{p} \ \rightarrow \ K_l \ K^{\pm} \ \pi^{\mp} \ \pi^0 & [5] \end{array} \tag{3}$$

In reaction a) (eq. 3) – due to the definite C - properties of the final state particles – the quantum numbers of $h_1(1380)\ ;\ 1^{+-}$ **can be isolated in the** $K_l\ K_s\ \pi^0$ **three body combination and a signal is observed , albeit with marginal significance, corresponding to mass and width**

$$m \ = \ 1440 \ \pm \ 60 \ \ ; \ \ \Gamma \ = \ 170 \ \pm \ 80 \ \textbf{MeV} \tag{4}$$

This is not in contradiction with the results of LASS in eq. 1.

2 $J^{PC\,n} = 1^{++}$ **:**

This nonet contains 9 resonances within the 'Summary Tables' of the PDG [1] . A tenth state is included in parentheses $f_1(1510)$ **, denoting entries like in eq. 1 , forming the set**

$$\begin{array}{lll} a_1 (1230 \pm 40 \, , \, 250 - 600) & I^G = 1^- & \\ f_1 (1281.8 \pm 0.6 \, , \, 24.2 \pm 1.1) & I^G = 0^+ & \\ K_1 (1403 \pm 7 \, , \, 174 \pm 13) & I = \frac{1}{2} & \\ f_1 (1426.3 \pm 0.9 \, , \, 54.9 \pm 2.6) & I^G = 0^+ & \\ (f_1 (1518 \pm 5 \, , \, 73 \pm 25)) & I^G = 0^+ & \end{array} \tag{5}$$

It must be emphasized that in particular the two resonances $h_1(1170)$ **(eq. 1) and** $a_1(1320)$ **(eq. 5) are rather wide**

$$\begin{array}{ll} h_1 (1170 \pm 40 \, , \, 360 \pm 40) & I^G = 0^- \\ a_1 (1230 \pm 40 \, , \, 250 - 600) & I^G = 1^- \end{array} \tag{6}$$

As a consequence the correct 'spectroscopic' mass ([2]) may not correspond – at least within a half-width – to the best value obtained by the PDG,as chosen throughout in figure 1 .

Also I note the different status of the $L = 1 \, , \, q' \, \overline{q}$ **candidate-states in todays tables relative to e.g. 1996 .**

The non-observation of the tenth state $(f_1 (1518))$ **in eq. 5 in virtual** $\gamma \, \gamma -$ **fusion by the TPC/Two Gamma Collaboration [6] does not** *necessarily* **eliminate it from consideration .**

3 $J^{PC\,n} = 2^{++}$ **:**

This nonet is the clearest , on the grounds of the large total angular momentum and ensuing centrifugal barrier protection especially in two body decays . It is formed by the set

$$\begin{array}{ll} f_2 (1275.1 \pm 1.2 \, , \, 185 \; {}^{+2.9}_{-2.4}) & I^G = 0^+ \\ a_2 (1318.3 \pm 1.4 \, , \, 104.7 \pm 1.9) & I^G = 1^- \\ K_2^{(\pm)} (1425.6 \pm 1.5 \, , \, 98.5 \pm 2.7) & I = \frac{1}{2} \\ f_2' (1525 \pm 5 \, , \, 73 \; {}^{+6}_{-5}) & I^G = 0^+ \end{array} \tag{7}$$

We note *withthout further mention some* **additional** f_2 **resonances, omitted from the review tables by the PDG**

$$f_2 (1430) \, , \, f_2 (1565) \, , \, f_2 (1640)$$

$\longrightarrow$

It is not easy (for me) to qualify the insight gained from this attempt of *spectroscopic* $L = 1$, $J > 0$, $q^{\,\prime} \, \overline{q}$ – nonet assignment . The experimental challenges however become quite clear – not to speak of the theoretical ones .

1a Adding scalar candidates *with low enough mass* in order to (over–) complete the $L = 1$, $q^{\,\prime} \overline{q}$ nonets

We extend the *spectroscopic* view in figure 1 to include all scalar resonances below $M \sim 1.8$ GeV in figure 2 and compare with the spectrum of charmonium in figure 3

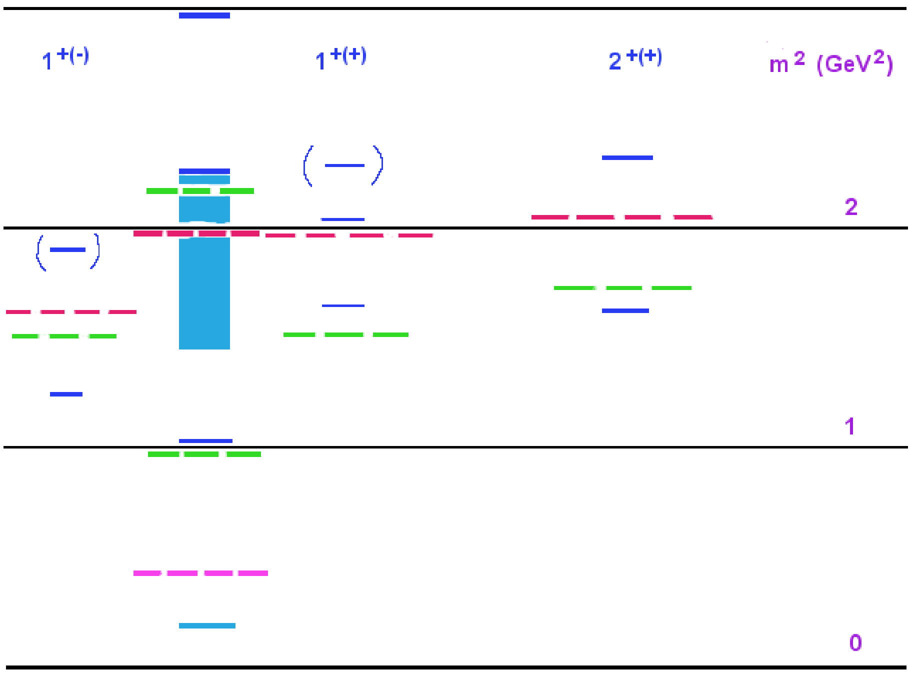

Figure 2:

$J^{\,PC}{}_{n} = 0^{\,++} , 1^{\,+-} , 1^{\,++} , 2^{\,++}$ – nonets *adapted* from refs. [1] , [7] , [8] (→) .

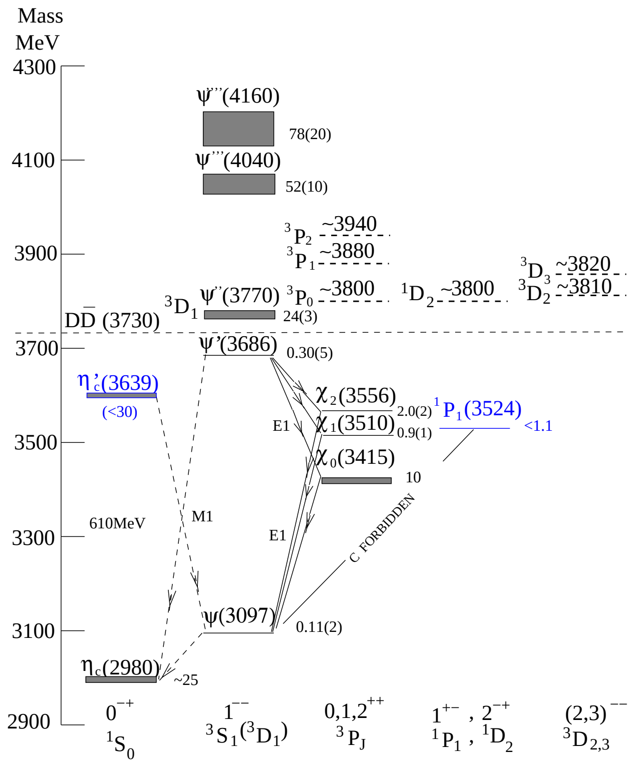

FIGURE 1. Spectra of the states of Charmonium.

Figure 3:

$c\bar{c}$ **spectroscopy , Cleo collaboration [9]** (→) .

1 **To figure 3 : charmonium spectroscopy At the the onset of non-relativistic motion the states** $n = 1$, $L = 1$, $c\,\bar{c}$ **are clearly observed . This should not lead us to abandon exploring the relativistic motion of light flavor states .**

2 **To figure 2 : The set of states displayed in two tables (eqs. 8 and 9) , comprises , using the pole coordinates obtained by Caprini, Colengelo and Leutwyler [7] and name** σ **as well as those obtained by Descotes-Genon and B. Moussallam [8] and name** κ **(see the discussion by Meadows in ref. [10]) , while following the PDG otherwise, except the mass value for** $f_0\ (\ 980\ )\ \rightarrow\ m\ =\ 1010$ **MeV from Hyams et al. [11] , Wolfgang Ochs's thesis [12] and Grayer et al. [13] , see also Protopopescu et al. [14] .** →

2 To figure 2 , continued

name	mass [MeV]	width [MeV]	I^G	#
σ	$441 \; {}^{+16}_{-8}$	$544 \; {}^{+18}_{-25}$	0^+	1
κ	658 ± 13	557 ± 24	$\frac{1}{2}$	4
$a_0(980)$	984.7 ± 1.2	$50 - 100$	1^-	3
$f_0(980)$	1010 ± 20	$40 - 100$	0^+	1
				9

(8)

name	mass [MeV]	width [MeV]	I^G	#
$f_0(1370)$	1200 – 1500	300 – 500	0^+	1
$K_0(1430)$	1414 ± 6	290 ± 21	$\frac{1}{2}$	4
$a_0(1450)$	1474 ± 19	265 ± 13	1^-	3
$f_0(1500)$	1505 ± 6	109 ± 7	0^+	1
$f_0(1710)$	1724 ± 7	137 ± 8	0^+	1
				10 (19)

(9)

This entity of $19 = 9 + 1 + 9$ **scalar resonances is – as far as I can judge – rooted in many ingenious ideas** *yet , on the border of analytic mastery* **.**

2a The $\pi \, \pi -$ threshold resonance $\sigma \, (\, 441 \, , \, 544 \,)$ and $f_0 \, (\, 980 \,)$

In order to document the spirit reigning in 1973 let me quote from the paper of Protopopescu et al. (1973) [14] :

"... We always found one pole $(\, S^* \,)$ **on the second Riemann sheet at** $997 \pm 6 - i \, (\, 27 \pm 8 \,)$ **which can be interpreted as a** $K \, \overline{K}$ **bound state . We also found another pole** $(\, \varepsilon \,)$ **on the second Riemann sheet at** $600 \pm 110 - i \, (\, 320 \pm 70 \,)$ **but ... "** →

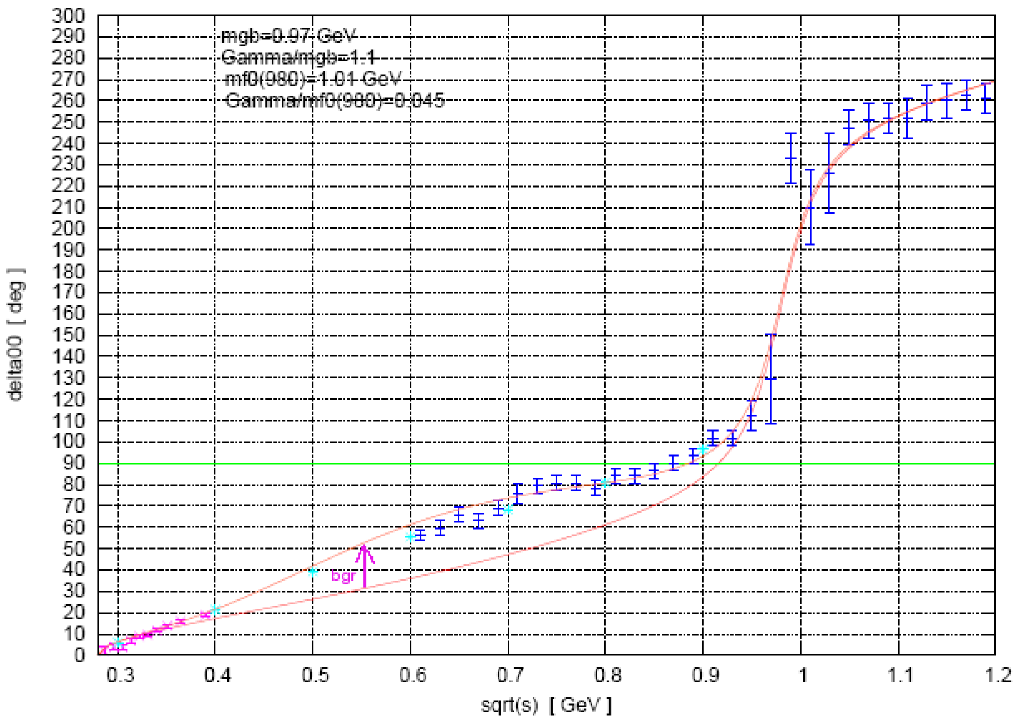

Figure 4:

$\delta_{J=0}^{I=0}$ (π π) **elastic scattering , featuring a form of background parametrization . For the latter modifications are needed for** $\Re\ \sqrt{s} >$ 1.5 **GeV but immaterial for the purpose of illustration here** $(\rightarrow)$ **.**

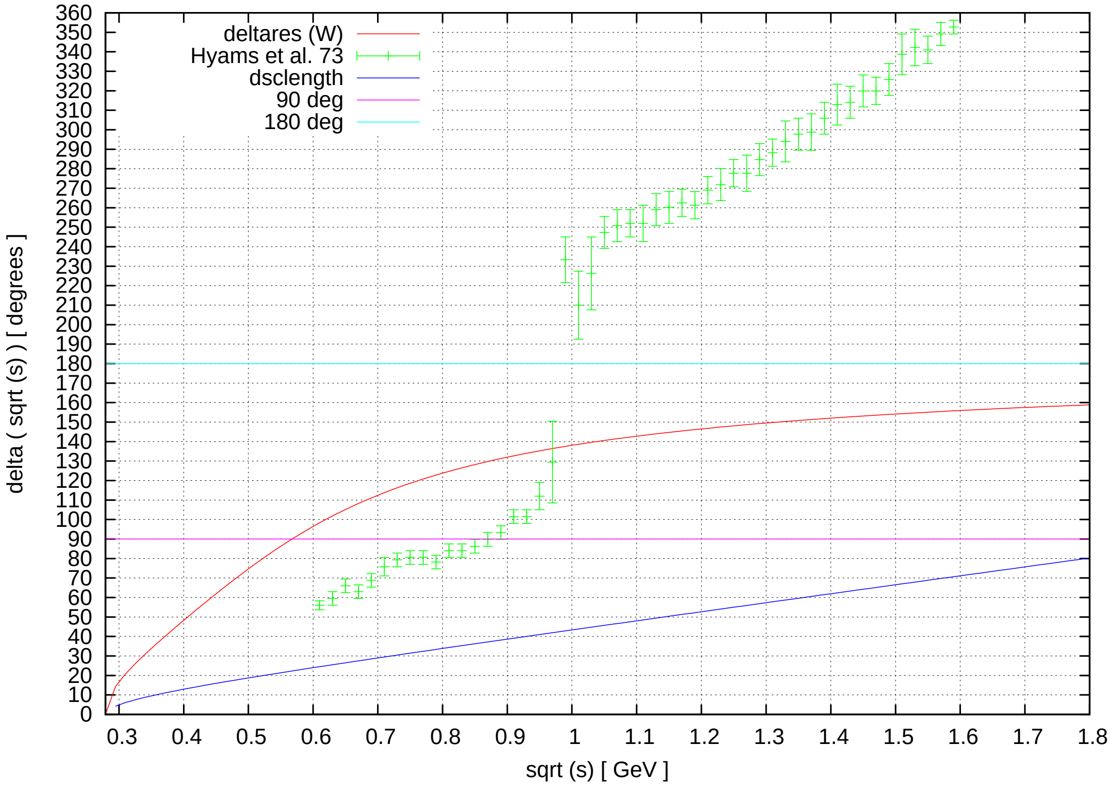

Figure 5:

$\delta_{res} (\sqrt{s})$ **for** σ **from ref. [15]** $\rightarrow$ **.**

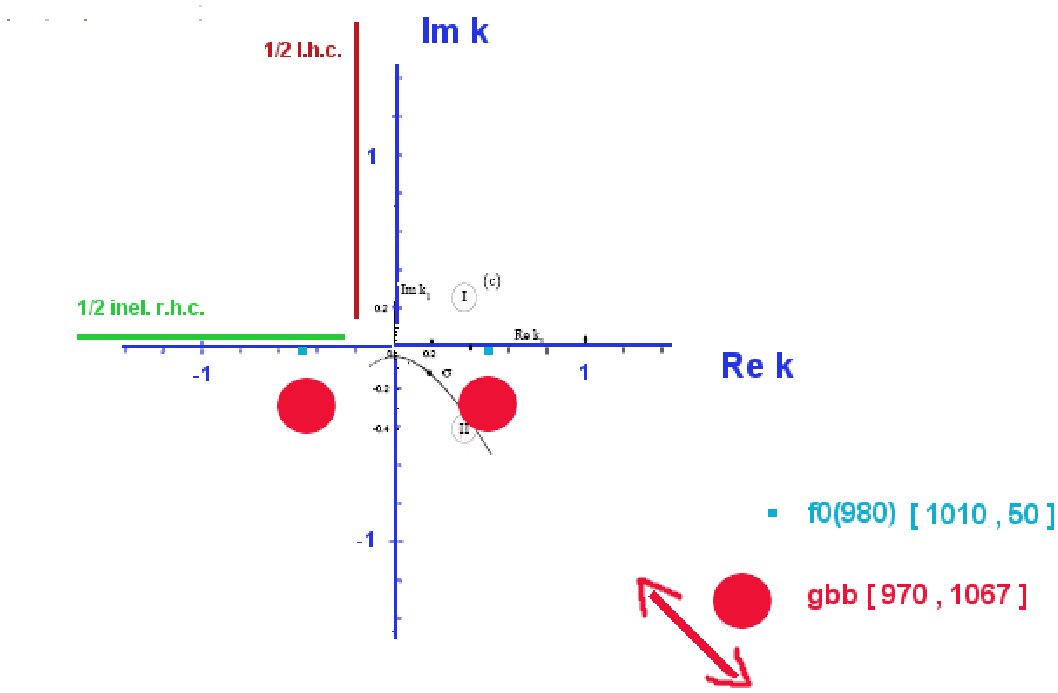

Figure 6: **Poles in the complex $k = \frac{1}{2} K$ plane . σ from Locher and Markushin [16] $\rightarrow$.**

Extended captions to figures 6 - 4 in this order .

a) to figure 6 :

The figure is *extended* **from the original one by Locher and Markushin [16] . For comparison the pole positions – in the $k = \frac{1}{2} K$ plane – of $f_0 (980)$ and gbb (0^{++}) following our analysis with Wolfgang Ochs [17] (presented at the same workshop) are shown . The latter corresponds** *here* **to**

$$[\Re M , 2 \Im M] = [970 , 1067] \text{ MeV} \tag{10}$$

The right hand inelastic cuts from 4π , 6π beginning at $k \sim \pm 0.24$, ± 0.39 GeV as well as the left hand cuts beginning at $k \sim \pm i (0.14)$ GeV – as shown in part in the figure – are very close to the σ pole position as derived

in ref. [16] . The pole position and trajectory (in black in figure 6) , according to Locher and Markushin , correspond to unitarized chiral perturbation theory with the larger pole masses – larger referring to the real part of $\mathcal{M}_{pole}$ **– resulting from an** *increase* **of** f_π **i.e. from a** *decrease* **of the interaction strength as parametrized by** f_π^{-2} **.**

Despite the admittedly large uncertainty in the derived gbb-pole-position the two poles do not *appear* **identifiable – to me.**

b) to figure 5 :

The data points with statistical errors *only* **are the phase shifts** $\delta_{J=0}^{I=0}(\sqrt{s})$ **from Hyams et al. [11] . The blue curve – below the data points – is the scattering length approximation**

$$\delta_0^0 \sim a_0^0 \, k \; ; \; a_0^0 = 0.22 \, m_\pi^+ \tag{11}$$

The red curve well above the onset of the data points at $\sqrt{s} = 0.6$ **GeV represents the** *minimal meromorphic parametrisation* **of a given resonance contribution to the** $I = 0 \, ; \, \pi\pi$ **s-wave amplitude , restricted to s-sheets I and II – to be discussed below, derived by Caprini, Colangelo and Leutwyler** $(M_R , \Gamma_R) = (0.4416 , 0.5438)$ **GeV (ref. [7])**

$$\tau_{m.m.r.} = e^{2 i \delta_{m.m.r.}} = \left[\begin{array}{l} \dfrac{(\mathcal{K}_R + K)(\mathcal{K}_R^* - K)}{(\mathcal{K}_R^* + K)(\mathcal{K}_R - K)} \\ = \dfrac{|\mathcal{K}_R|^2 + i \gamma_R K - K^2}{|\mathcal{K}_R|^2 - i \gamma_R K - K^2} \end{array} \right]$$

$$\begin{array}{l} \mathcal{K}_R = \left(\kappa - \frac{1}{2} i \gamma \right)_R \; ; \; \mathcal{M}_R = \left(M - \frac{1}{2} i \Gamma \right)_R \\ (\mathcal{K}_R)^2 = (\mathcal{M}_R)^2 - 4 m_\pi^2 \; ; \; K^2 = s - 4 m_\pi^2 \end{array} \tag{12}$$

→

c) to figure 4 :

The data points in dark blue are the same as in figure 5 : $\delta\ ^{I=0}_{J=0}\ (\sqrt{s}\ )$ **from Hyams et al. [11] in the interval** $0.6 GeV \leq \sqrt{s} \leq 1.3$ **GeV .**

Those in purple are from Mme Bloch-Devaux for the Na48/2 collaboration [17] , extracted from high statistics measurement of the reactions $K^{\pm} \rightarrow \pi^{+}\pi^{-} e^{\pm} \overset{(-)}{\nu}_{e}$ **, corrected for leading isospin breaking terms as calculated in refs. [18] , [19] and [20] .**

The light blue points represent the solution to the Roy equations by Caprini, Colangelo and Leutwyler in ref. [7] .

The lower red curve describes the logarithmic convolution of two resonance contributions :

1) $f_{0}(980)\ ;\ [\ M_{f_0}\ ,\ \Gamma_{f_0}] = [\ 1010\ ,\ \ 50\ ]$ **MeV**

2) $gbb\ ;\ [\ M_{gbb}\ ,\ \Gamma_{gbb}] = [\ 970\ ,\ 1067\ ]$ **MeV**

$$\delta_{1+2} = \sum_{j=1}^{2} \textbf{arctang} \frac{\gamma_{R_j} K}{\left| \mathcal{K}_{R_j} \right|^{2} - K^{2}} \tag{13}$$

The resonance parameters $\left[\ M_{R_j}\ ,\ \Gamma_{R_j}\right]$ **in eq. 13 are chosen to** *approximately* **follow the shape of the phase shift in the interval** $0.92 < \sqrt{s} < 1.1$ **GeV .**

This then defines a *background* **relative to the specified parametrisation which is added linearly to the phase shift resulting in an additional attraction yielding the upper red curve in figure 4 .**

This concludes the extended figure captions to figures 4 - 6.
One clarification concerns the nature of the *background* **as defined for figure 4 and the logarithmic convolution(s) in eq. 13 :**
the $I = 0\ ,\ J = 0\ ;\ \pi\pi$ **amplitude is – dropping all quantum number labels (and for** $e = 0\ ,\ m_u = m_d\ ,\ \cdots$ **)**

$$\tau = \tau\ (\ K\ ) = \frac{\exp\ (\ 2\ i\ \Phi\ )\ -\ 1}{2\ i}\ ;\ \Phi = \Phi\ (\ K\ )$$
$$\Phi = \tfrac{1}{2}\log\ (\ 1 + 2\ i\ \tau\ ) \longleftrightarrow t = v^{-1}\ \tau = \frac{\sqrt{s}}{K}\ \tau \tag{14}$$

$\rightarrow$

Then the (triple) logarithmic convolution using δ_{1+2} in eq. 13 defines Φ

$$\Phi = \Phi_{bgr} + \delta_{1+2} \; ; \; \Phi_{bgr} \not\equiv 0 \tag{15}$$

The inequality in eq. 15 follows from the inherited analytic structures *and* **their limits (inelastic cuts , 'l.h.' cuts, Lehmann ellipse(s) , $\cdots$) . For real positive K**

$$\begin{aligned} & 1 + 2 \, i \, \tau = \eta \exp 2 \, i \, \delta \; ; \; \eta = e^{-2 \, \Im \, \Phi_{bgr}} \\ & \delta = \Re \, \Phi_{bgr} + \delta_{1+2} \; (\textbf{mod } \pi) \end{aligned} \tag{16}$$

2b The $K \pi -$ threshold resonances $\kappa (658 , 557)$ from Roy-Steiner equations and $K_0 (1430)$

I used the best values for mass and width of κ as obtained by Descotes-Genon and Moussallam in ref. [21]. The analyses of ref. [21] following a paper by Büttiker, Descotes-Genon and Moussallam [22] uses as main experimental input the results of two reports by the LASS collaboration, Aston et al. [23] and Estabrooks et al. [24].
The invariant amplitude denoted T , for elastic $K^{\pm} \pi^{\mp} \rightarrow K^{\pm} \pi^{\mp}$ scattering with the conventional relativistic normalizations of one particle states and scattering angle and momenta referring to the c.m. system is

$$\begin{aligned} & T = (16\pi) \sum_l (2l + 1) P_l (z) (\sqrt{s} / K) \tau_l (K) \\ & d \sigma_{el} / d \Omega_{c.m.} = | T / (8\pi \sqrt{s}) |^2 = | f |^2 \\ & f = \sum_l (2l + 1) P_l (z) (1 / k) \tau_l \; ; \; k = q = \tfrac{1}{2} K \\ & \tau_l = \tfrac{2}{3} \tau_l^{I=\frac{1}{2}} + \tfrac{1}{3} \tau_l^{I=\frac{3}{2}} \end{aligned} \tag{17}$$

$\tau_l \, , \, \tau_l^I \; ; \; I = \frac{1}{2}, \frac{3}{2}$ **are normalized in the 'Argand convention', i.e. lie as compex numbers inside the Argand circle . Always in the $SU2_{u,d} \, , \, e = 0$ limit τ_l^I are on the boundary of that circle** *below inelastic threshold* $(\sqrt{s} \leq m_K + 2 \, m_\pi)$ **. This is shown in figure 6 below** $\rightarrow$

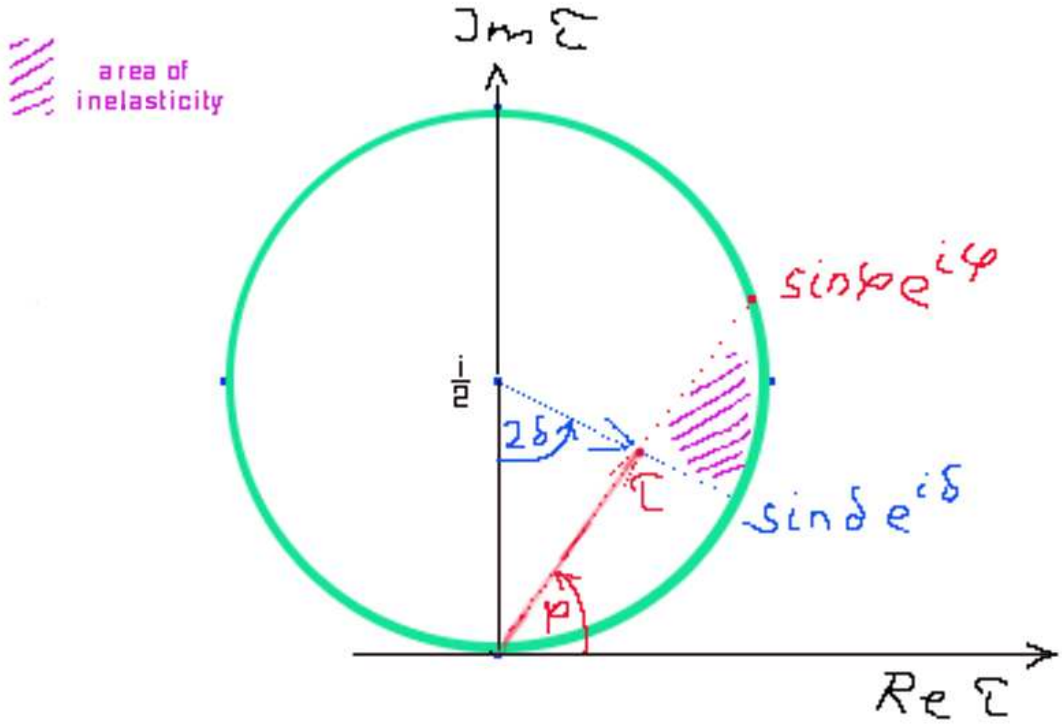

Figure 7: **Argand diagram .**

Now the LASS collaboration in ref. [23] does not use the conventions laid out in eq. 17 and in figure 7 , instead a complex partial wave

$$a_l^I \propto \tau_l^I \tag{18}$$

where the proportionality constant is a matter of *convention* **, the latter remains unclear to me but I hope that those who use it in the Roy-Steiner equations (and beyond) know it .**
Nevertheless the representation in ref. [23] resembles Argand diagrams as shown in figure 8 restricting to s-waves, both isospins mixed first, and projected on $I = \frac{1}{2}$ in figures 9 and 10 below $\rightarrow$

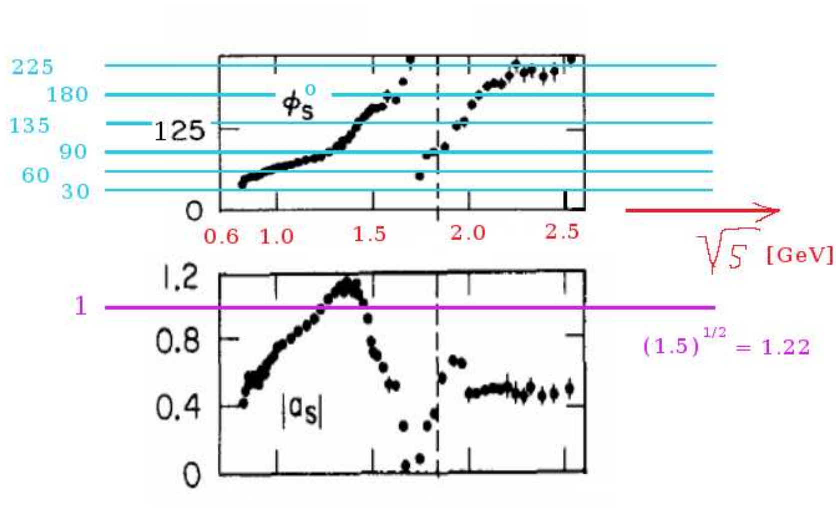

Figure 8: **Argand diagram for $a_s = a_s^{I=\frac{1}{2}} + \frac{1}{2} a_s^{I=\frac{3}{2}}$ from ref. [23] .**

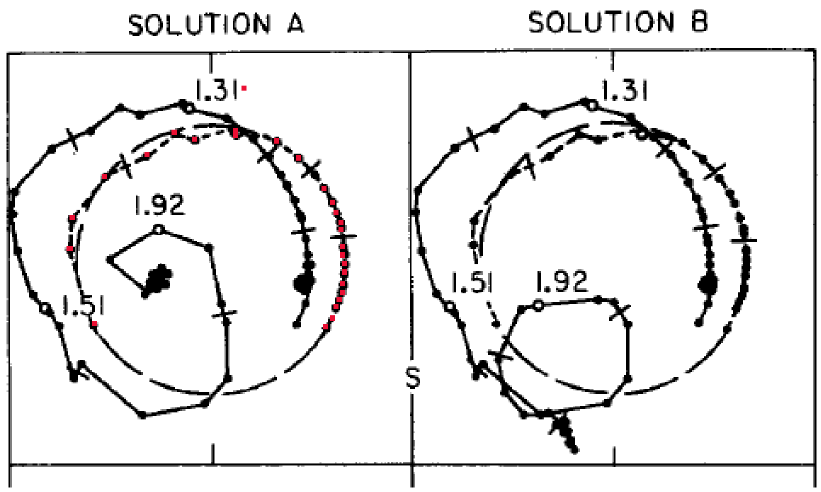

Figure 9: **Argand diagram for $a_s^{I=\frac{1}{2}}$ from ref. [23] $\rightarrow$.**

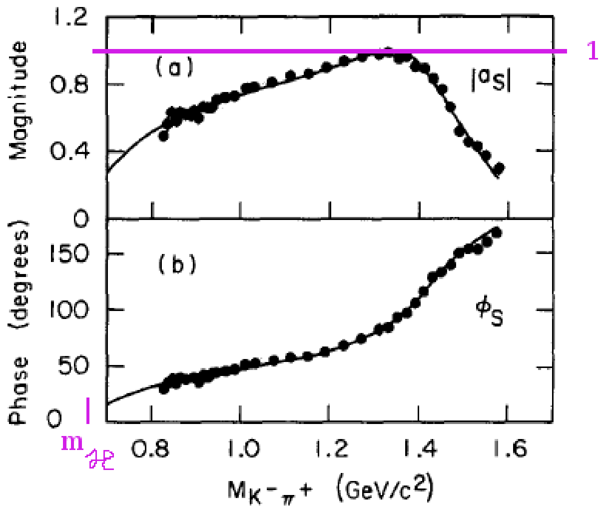

Figure 10: $a_s^{I=\frac{1}{2}}$ **from ref. [23] .**

Just two remarks concerning the work of Büttiker and (Descotes-Genon and Moussallam)

a) threshold resonance

In refs. [21] and [22] the charged pion - and mean kaon masses were used (in MeV)

$$
\begin{array}{l}
m_K = 495.70 \left(\textbf{PDG} : \begin{array}{rcl} m_{K^0} & = & 497.648 \pm 0.022 \\ \Delta_{0\,ch}(m_K) & = & 3.972 \pm 0.027 \end{array} \right) \\
m_\pi = 139.57 \left(\textbf{PDG} : \begin{array}{rcl} m_{\pi^+} & = & 139.57018 \pm 0.00035 \\ \Delta_{ch\,0}(m_\pi) & = & 4.5936 \pm 0.0005 \end{array} \right) \\
m_K + m_\pi = 635.27 \leftrightarrow \Re\mathcal{M}_\kappa = 658 \pm 13 \text{ MeV} \\
\qquad \Delta = 23 \pm 13 \text{ MeV}
\end{array}
\tag{19}
$$

b) complex pole mass square and boundaries deduced from Lehman ellipses

The square of the complex pole mass deduced for κ in ref. [21] in units of the charged pion mass squared is

$$
\left(\Re\mathcal{M}_\kappa - \tfrac{1}{2}\, i\, \Gamma_\kappa \right)^2 / m_\pi^2 = 18.24 - i\, 18.81 \tag{20}
$$

The position of $\left(\Re\mathcal{M}_\kappa - \frac{1}{2}\, i\, \Gamma_\kappa \right)^2 / m_\pi^2$ is indicated in figure 11 below .

$\rightarrow$

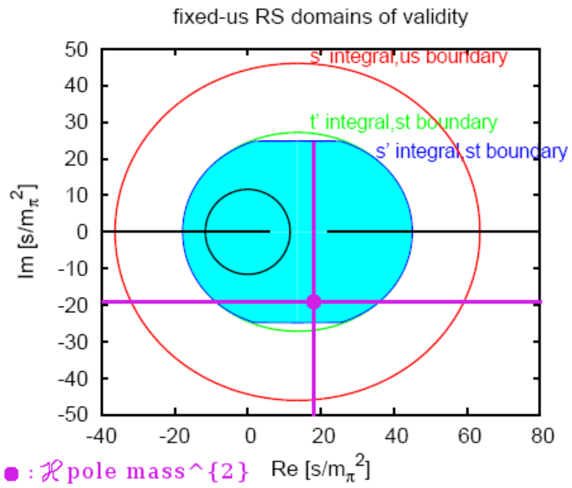

Figure 11: **$\mathcal{M}(\kappa)^2$ from ref. [21] and limits of analytic validity from Lehmann ellipses .**

3 Some conclusions and outlook

1) The fourfold $L = 1$, $\bar{q}\,' q$; $q = u, d, s$ and $n = 1$ nonets

Despite much effort no convincing spectroscopic identification has yet been achieved of these nonets.

This is due to the plethora of scalar resonances , as observed *and* **deduced and a mixing pattern increasingly difficult to unravel for the three light quark flavors .**

2) Analiticity , crossing and dispersion relations for pseudoscalar strong interaction scattering states

The remarkable revived efforts of many present at this meeting – as well as others , unable to attend – use as basic elements analytic methods deeply rooted in fundamentals of mathematics .

Combined with results deduced from experiments , which have to resort to Chew - Low extrapolation – in order to determine the elastic scattering amplitudes searched for – the analytic *extrapolation* **to complex regions of the kinematical parameters leads to borders of analytic mastery .**

This I discussed here – incompletely – in two sections devoted to 5 scalar threshold resonances : σ (441 $^{+16}_{-8}$, 544 $^{+18}_{-25}$) and κ (658 $\pm$ 13 , 557 $\pm$ 24) . The latter when combined with the 4 scalar resonances – (f_0 , a_0) (980) – can indeed be spectroscopically interpreted as an $SU3_{u,d,s}$ – nonet of tetraquark (di – $\overline{\text{di}}$) – composites .

3) QCD

The unbroken local $SU3_c$ – gauge theory was originally understood only as 'on the same footing' as QED and its electroweak extension . The elaboration of secure regions of analytic mastery remains a promising task .

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