ON THE EQUILIBRIUM OF 4 LEGS TABLES

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CERN

MOSTLY SQUARE TABLES

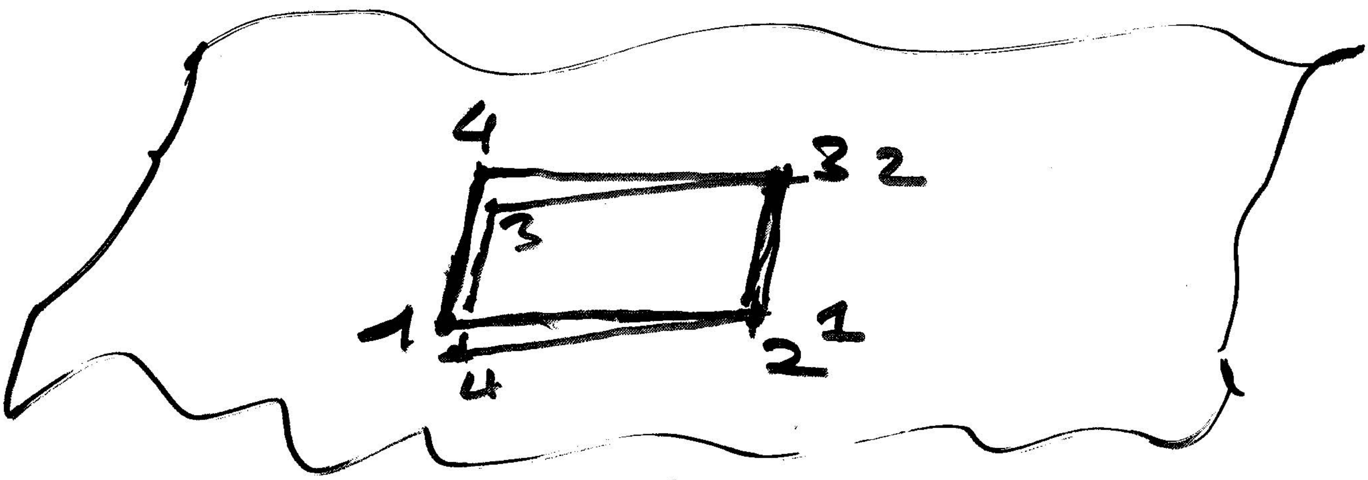
PROBLEM:

YOU SIT, DRINKING COFFEE AT THE CAFETERIA TERRACE MOST OF THE TIME THE TABLE TOUCHES THE GROUND ON 3 LEGS. THE 4th LEG IS IN THE AIR. ANY SMALL PERTURBATION (NOT INFINITESIMALLY SMALL!) WILL MAKE THE TABLE OSCILLATE AND YOU SPILL YOUR COFFEE ON THE SAUCER

HOW TO AVOID THAT?

AN <u>APPROXIMATE</u> REASONING
INDICATES THAT BY
"ROTATING" THE TABLE YOU
WILL FIND AN EQUILIBRIUM
POSITION.

CONDITIONS: - PERFECT TABLE
(4 FEET IN THE SAME PLANE
INFINITELY THIN)
- CONTINUOUS GROUND
(LATER LIPSCHITZ CONDITION!)
(NOT REALLY SATISFIED AT THE
CERN TERRACE!)



—— : INITIAL POSITION
1, 2, 3 ON THE GROUND 4 <u>ABOVE</u>

—— : FINAL POSITION
1 2 3 ON THE GROUND 4 <u>BELOW</u>

IF THERE IS A <u>CONTINUOUS</u> MOTION
FROM 1 2 3 4 TO 1 2 3 4
$\exists$ INTERMEDIATE POSITION WHERE
1 2 3 4 ARE ALL ON THE GROUND

PROBLEMS: - MAKE SURE THAT SUCH A CONTINUOUS MOTION EXISTS

- MAKE SURE THAT NOT ONLY FEET ON THE GROUND, BUT, ALSO LEGS ABOVE THE GROUND

HISTORY: I FOUND THIS IN 1995
PRESENTED WITH RIGOUROUS PROOF IN 1998 AT IHES
WROTE AN ARTICLE ACCEPTED BY ARCHIVES

arXiv:org: math-ph 17 oct 2005

THEN AN ARTICLE BY A GROUP OF MATHEMATICIANS (PROBABLY AUSTRALIAN)
APPEARED IN

arXiv.org: math.HO/0511490 19 Nov 2005

THIS CONTAINS MANY REFERENCES TO PREVIOUS WORK AND PRESENTS A PROOF VALID FOR RECTANGULAR TABLES

AMONG PREVIOUS PROOF

1) APPROXIMATE PROOFS LIKE THE ONE I PRESENTED
(GARDNER, KRAFT)

2) PROOF OF GLOBAL EQUIBRIUM
(DYSON, ON A DEFORMED SPHERE)

SKETCH OF THE PROOF(S)

- CHOOSE A HORIZONTAL PLANE (ARBITRARY, BUT CAN BE OPTIMIZED)
- IMPOSE A CONDITION ON THE ALTITUDE, Z, OF THE GROUND

$$\boxed{|Z(x,y) - Z(x',y')| < |\vec{x} - \vec{y}| \tan \theta_M}$$

- TRY TO DEFINE A CONTINUOUS PATH FROM 1234 TO 1234

METHOD # 1 1233' FORMS A TETRAHEDRON (IF 1233 in a PLANE, NO PROBLEM!)

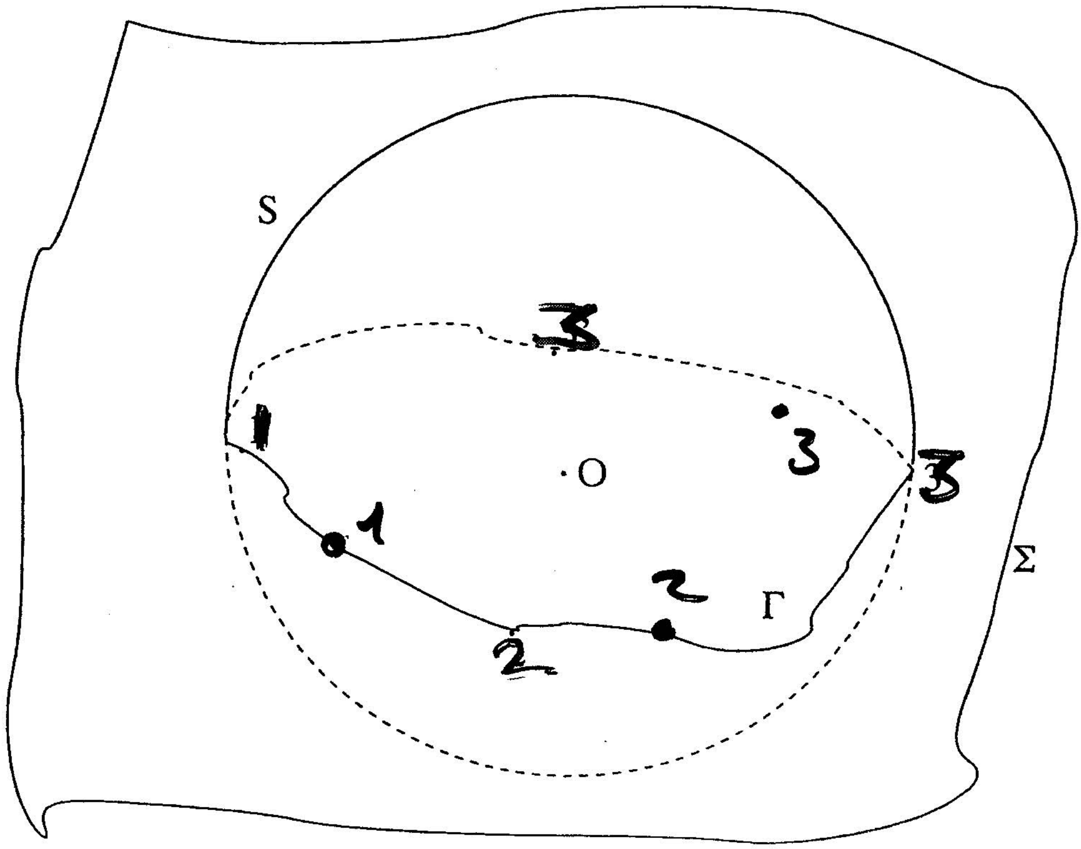
DRAW A SPHERE S WITH 1233 ON IT

Γ, INTERSECTION OF THE SPHERE WITH THE SURFACE Σ WILL BE THE PATH ALONG WHICH WE MOVE 1 and 2.

WE IMPOSE THAT O, CENTER OF THE SPHERE BE INSIDE THE TETRAHEDRON. THIS IS TRUE IF

$$\boxed{\theta_M < 35^\circ \ldots}$$

WE MOVE 1 AND IMPOSE THAT 2 EXISTS, IS UNIQUE, AND THAT THE LEG IN 2 IS ABOVE THE GROUND THIS GIVES $\boxed{\theta_M < 14^\circ \ldots}$


S
3
1
O
3
3
1
2
Γ
Σ
2

ONCE 12 is fixed condition
$\Theta_M < 35^\circ$ IS ENOUGH TO
GARANTEE THAT 3 (ON THE
GROUND, BUT NOT NECESSARILY
ON Γ) EXISTS IS UNIQUE
AND THAT ALL 3 LEGS ARE
ABOVE THE GROUND.

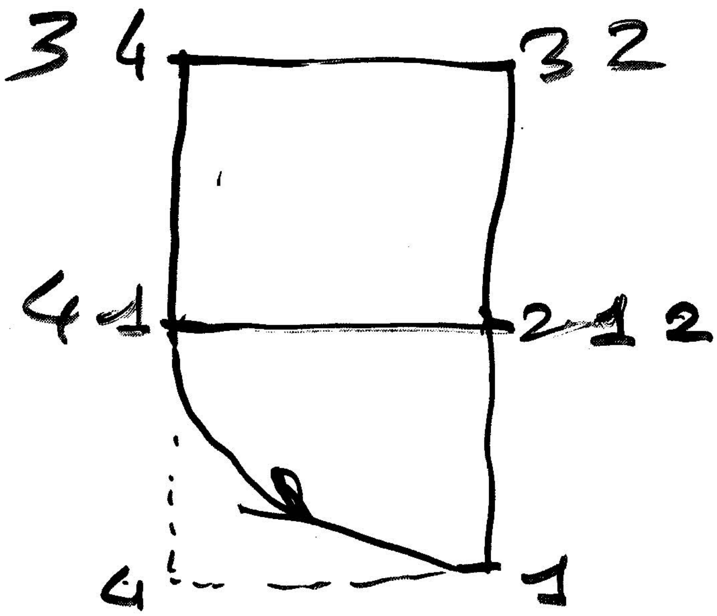
HENCE, IF $\Theta_M < 14^\circ$ A
CONTINUOUS MOTION EXISTS,
WITH 3 LEGS ABOVE THE GROUND.
AT ONE POINT, THE 4th LEG
CROSSES THE GROUND AND
WE ~~HAVE~~ AN EQUILIBRIUM
POSITION

NOTICE THAT WE NEED
INFORMATION ON THE SURFACE
IN A REGION OF LINEAR SIZE
LESS THAN 3 TIMES THE
DISTANCE BETWEEN 2 FEET.

MY COMPETITORS, USING A
CONTINUOUS MOTION OF A
COMPLETELY DIFFERENT
SPIRIT FIND THAT
$\Theta_M < 35^\circ$ is ENOUGH

FOLLOWING A SUGGESTION OF A. KRIWICKI I TRIED A DIFFERENT MOTION, KEEPING AGAIN 1, 2, 3 ON THE GROUND

I FOUND, SCHEMATICALLY THE FOLLOWING



1) "ROTATE" 12 AROUND 2 UNTIL 1 is in THE VERTICAL PLANE CONTAINING 12 (1234)

2) PULL 12 IN THIS VERTICAL, STAYING ON Σ UNTIL 12 COINCIDES WITH 23 OR 12

FOR THIS WE NEED ONLY $\boxed{\theta_m < 45°}$

IF WE ACCEPT A NON MONOTONOUS MOTION, THE THIRD FOOT FOLLOWS

HOWEVER, IF WE WANT THE 3rd LEG TO BE ABOVE THE GROUND, WE NEED

$$\boxed{\Theta_M < 35^\circ}$$

I.E. THE SAME CONDITION AS MY COMPETITORS

NON SQUARE TABLES

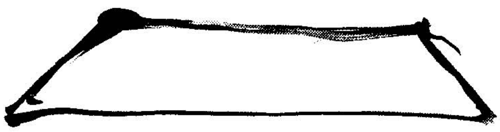
FOLLOWING A QUESTION BY D. DALLMAN

OBVIOUS NECESSARY CONDITION

THE 4 FEET MUST BE **ON A CIRCLE**, BECAUSE THE SURFACE MAY BE A PIECE OF A SPHERE

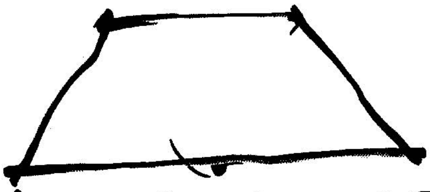
THIS IS THE CASE OF

- RECTANGULAR TABLES (PROVED BY THE OTHER)
- TRAPEZES WITH 2 PARALLEL SIDES AND AXIS OF SYMMETRY



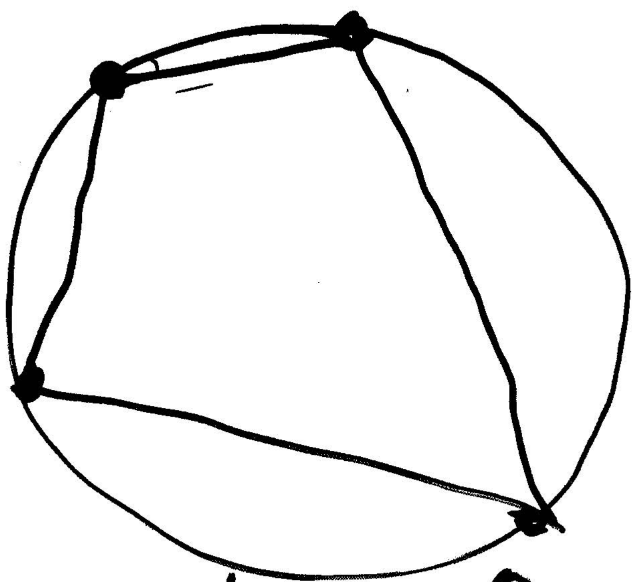
EXAMPLE

HALF REGULAR HEXAGONS



(USED IN CONFERENCE ROOMS)

→ COMPLETELY RANDOM TABES



AT PRESENT, if Θ_m is SMALL WE HAVE THE ALTERNATIVE

– $\exists$ 2 EQUILIBRIUM POSITIONS

– $\exists$ ∞ APPROXIMATE EQUILIBRIUM POSITIONS WITH AN ERROR (DISTANCE TO THE GROUND) OF ORDER $(\Theta_m)^3$