Variations concerning QCD

The origins of " the story of the $60\,^{\circ}$ " in $K \ \rightarrow \ \pi\,\pi$ decays

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2008 $\longrightarrow$

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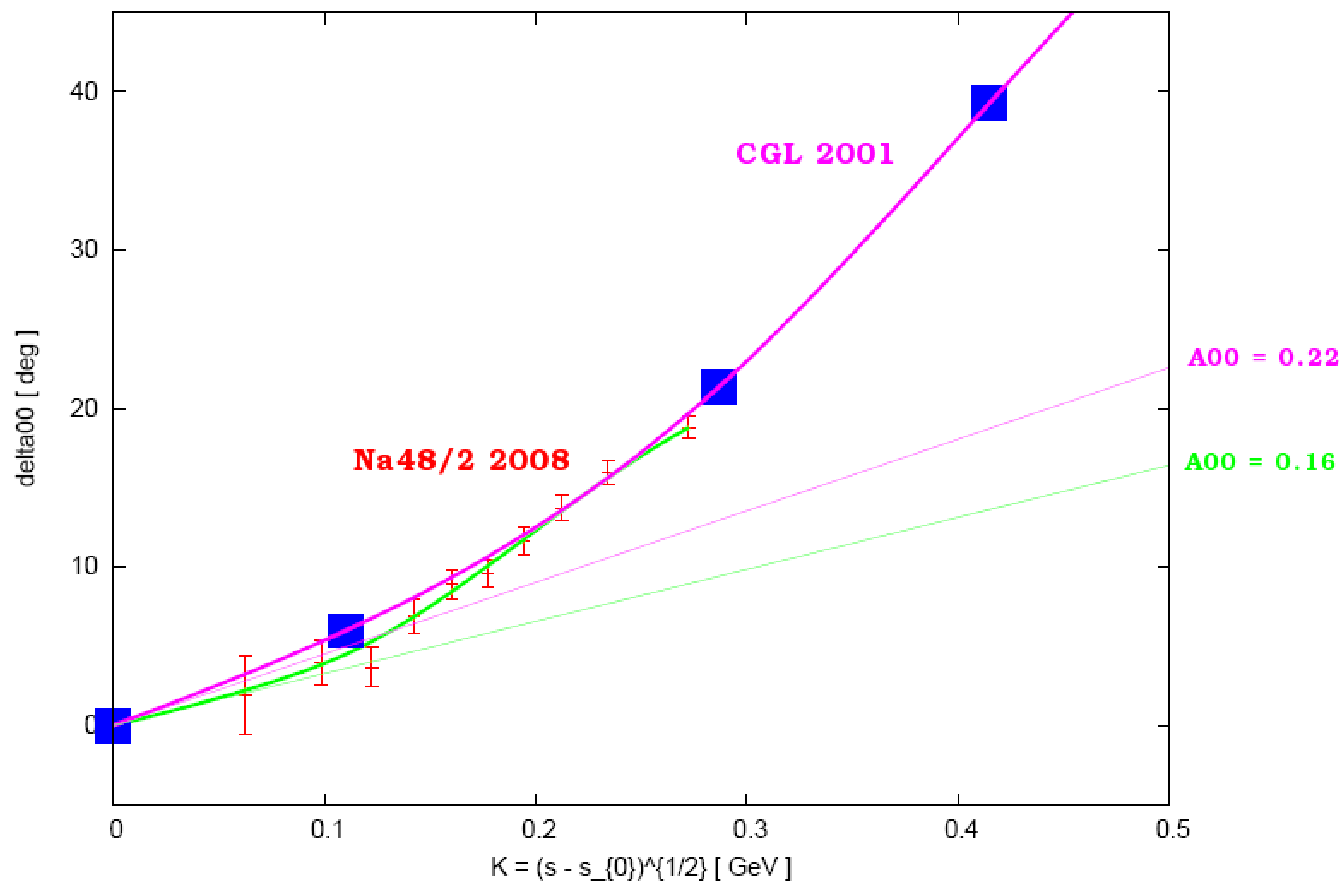
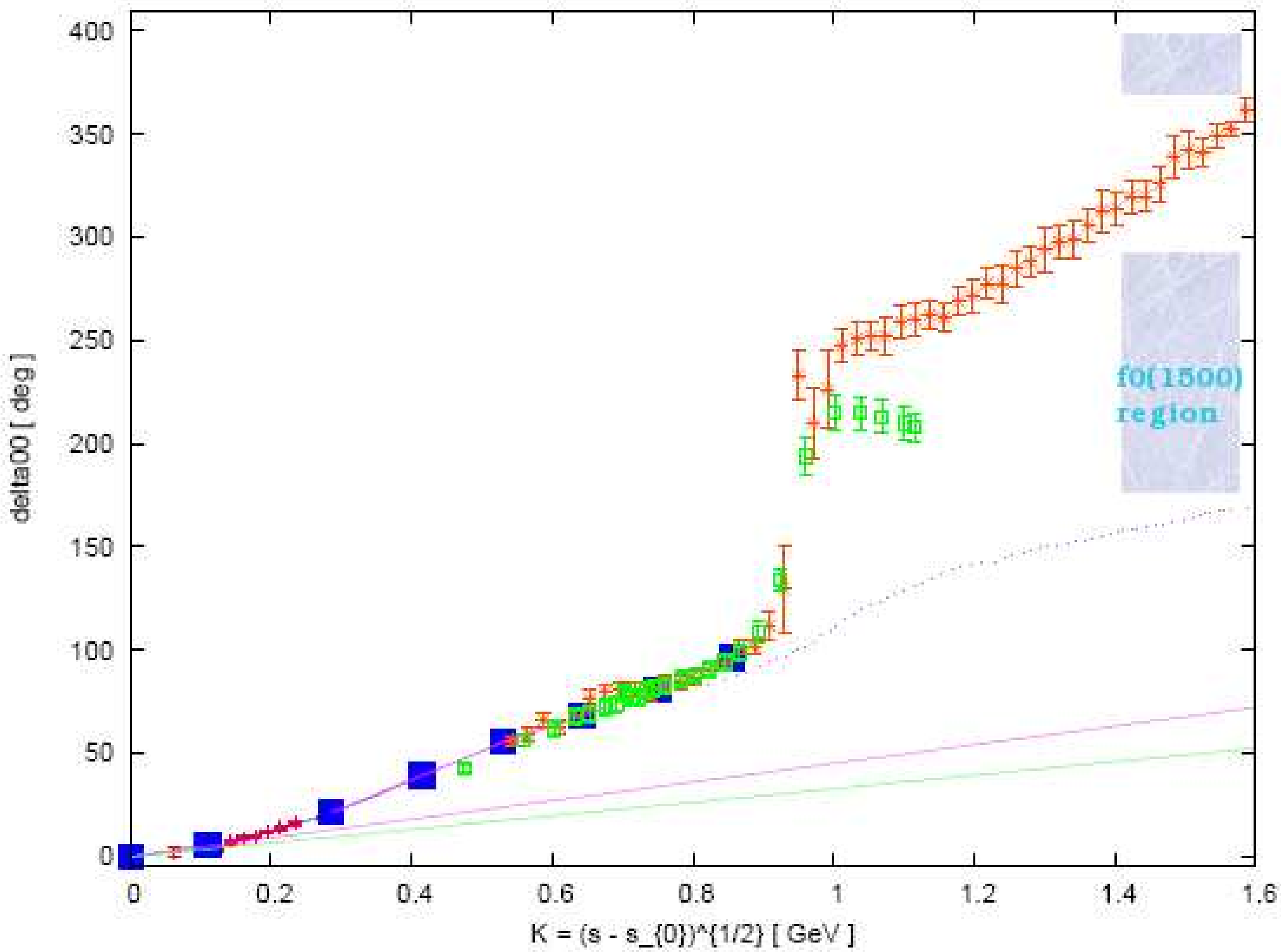
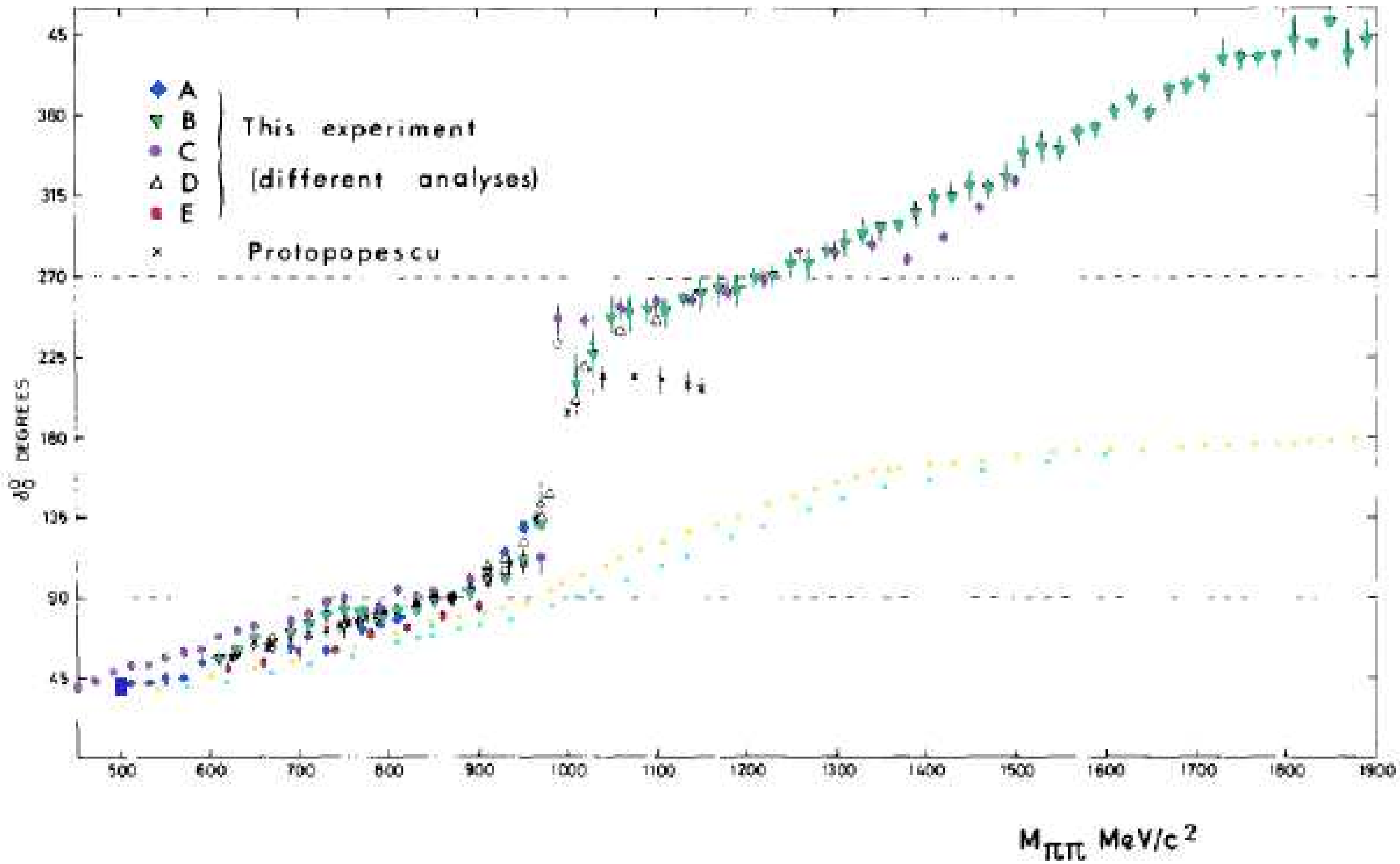
1 Some recent analyses of the $I = 0$ s-wave phase shift $\delta_0^{0)}$ extrapolated to $e = 0$, $m_u = m_d$



δ_0^0 as a function of twice the c.m. momentum in $\pi\pi$ scattering for $m_u = m_d$ and $e \to 0$



δ_0^0 **as a function of twice the c.m. momentum in** $\pi\pi$ **scattering for** $m_u = m_d$ **and** $e \to 0$



δ^0_0 as a function of twice the c.m. momentum in $\pi\,\pi$ scattering ($\longrightarrow$) .

2 The (3-) complex of decays $\left\{ \begin{array}{lll} K^{+} \rightarrow \pi^{+} \pi^{0} & (\gamma) \\ K^{0} \rightarrow \pi^{+} \pi^{-} & (\gamma) \\ K^{0} \rightarrow \pi^{0} \pi^{0}{}_{B} & (\gamma) \end{array} \right\} \rightarrow$

Ignoring – first – electroweak corrections , as well as short distance QCD corrections , the three decays are induced by the $\Delta S = -1$ - part of the nonleptonic weak Hamiltonian density

$$\mathcal{H}_{-} = 2\sqrt{2}\, G_{F}\, j_{\mu}^{(\Delta S = 0)}\, j^{\mu\,(\Delta S = -1)}$$

(1) $$j_{\mu}^{(\Delta S = 0)} \sim V_{ud}\, \overline{u}\, \gamma_{\mu}\, P_{L}\, d \quad , \quad j_{\mu}^{(\Delta S = -1)} \sim V_{us}^{*}\, \overline{s}\, \gamma_{\mu}\, P_{L}\, u$$

$$P_{L} = \frac{1}{2}\,(1 - \gamma_{5\,R}) \;\; ; \;\; \gamma_{5\,R} = \frac{1}{i}\, \gamma_{0}\, \gamma_{1}\, \gamma_{2}\, \gamma_{3} = i\, \gamma^{0}\, \gamma^{1}\, \gamma^{2}\, \gamma^{3}$$

In eq. 1 only the simplest approximate form of the hadronic weak currents is retained with

(2) $$V_{ud}\, V_{us}^{*} \sim sin\, \vartheta_{C}\, \cos\, \vartheta_{C} \quad \textbf{real}$$

In the following CP-violation yields only small corrections and will be ignored . →

The current-current product form implies the u-d-isospin assignments , now also ignoring isospin breaking effects

(3)

$$\mathcal{H}_- = \mathcal{H}_-^{\Delta I = 1/2} + \mathcal{H}_-^{\Delta I = 3/2}$$

$$\mathcal{H}_- \leftrightarrow u \otimes \overline{u} \otimes d \longrightarrow \Delta I_3 \mathcal{H}_- = \frac{1}{2}$$

The s-wave projected two pion states in the title of this section transform under u-d isospin in the following way *adopting Condon-Shortley phase conventions*

(4)

$$- \left| \pi^+ \pi^0 \right\rangle \qquad \left\{ \begin{array}{l} I = 2 \\ I_3 = 1 \end{array} \right\}$$

$$- \left| \pi^+ \pi^- \right\rangle = \sqrt{\tfrac{2}{3}} \left\{ \begin{array}{l} I = 0 \\ I_3 = 0 \end{array} \right\} + \sqrt{\tfrac{1}{3}} \left\{ \begin{array}{l} I = 2 \\ I_3 = 0 \end{array} \right\}$$

$$\left| \pi^0 \pi^0 \right\rangle_B \qquad - \sqrt{\tfrac{1}{3}} \left\{ \begin{array}{l} I = 0 \\ I_3 = 0 \end{array} \right\} + \sqrt{\tfrac{2}{3}} \left\{ \begin{array}{l} I = 2 \\ I_3 = 0 \end{array} \right\}$$

→

Thus isospin invariance implies the reduction of three amplitudes to 2 , whereby final state interactins induce interference in the neutral two pion final states

$$T^{+0} = \left(\begin{array}{c|cc} I = 2 & I = \frac{3}{2} & I = \frac{1}{2} \\ I_3 = 1 & I_3 = \frac{1}{2} & I_3 = \frac{1}{2} \end{array} \right) \mathcal{A}^{\left(\frac{3}{2}\right)}$$

(5)
$$T^{+-} = \left\{ \begin{array}{l} \left(\begin{array}{c|cc} I = 0 & I = \frac{1}{2} & I = \frac{1}{2} \\ I_3 = 0 & I_3 = \frac{1}{2} & I_3 = -\frac{1}{2} \end{array} \right) \sqrt{\frac{2}{3}}\, \mathcal{A}^{\left(\frac{1}{2}\right)} \\ + \left(\begin{array}{c|cc} I = 2 & I = \frac{3}{2} & I = \frac{1}{2} \\ I_3 = 0 & I_3 = \frac{1}{2} & I_3 = -\frac{1}{2} \end{array} \right) \sqrt{\frac{1}{3}}\, \mathcal{A}^{\left(\frac{3}{2}\right)} \end{array} \right\}$$

$$T^{00} = \left\{ \begin{array}{l} \left(\begin{array}{c|cc} I = 0 & I = \frac{1}{2} & I = \frac{1}{2} \\ I_3 = 0 & I_3 = \frac{1}{2} & I_3 = -\frac{1}{2} \end{array} \right) \sqrt{\frac{1}{3}}\, \mathcal{A}^{\left(\frac{1}{2}\right)} \\ - \left(\begin{array}{c|cc} I = 2 & I = \frac{3}{2} & I = \frac{1}{2} \\ I_3 = 0 & I_3 = \frac{1}{2} & I_3 = -\frac{1}{2} \end{array} \right) \sqrt{\frac{2}{3}}\, \mathcal{A}^{\left(\frac{3}{2}\right)} \end{array} \right\}$$

$\rightarrow$

In eq. 4 the suffix B indicates the inclusion of a Bose factor $\frac{1}{\sqrt{2}}$ in the definition of the $2\,\pi^{\,0}$ final state.
In eq. 5 the quantities in brackets denote Clebsch-Gordan coefficients

(6)
$$\left(\begin{array}{rcl|rclrcl} I & = & 2 & I & = & \frac{3}{2} & I & = & \frac{1}{2} \\ I_3 & = & 1 & I_3 & = & \frac{1}{2} & I_3 & = & \frac{1}{2} \end{array}\right) = \frac{\sqrt{3}}{2}$$
$$\left(\begin{array}{rcl|rclrcl} I & = & 0 & I & = & \frac{1}{2} & I & = & \frac{1}{2} \\ I_3 & = & 0 & I_3 & = & \frac{1}{2} & I_3 & = & -\frac{1}{2} \end{array}\right) = \frac{1}{\sqrt{2}}$$
$$\left(\begin{array}{rcl|rclrcl} I & = & 2 & I & = & \frac{3}{2} & I & = & \frac{1}{2} \\ I_3 & = & 0 & I_3 & = & \frac{1}{2} & I_3 & = & -\frac{1}{2} \end{array}\right) = \frac{1}{\sqrt{2}}$$

Inserting the C-G-coefficients into eq. 5 the latter becomes $\longrightarrow$

(7)

$$T^{+0} = \frac{\sqrt{3}}{2}\,\mathcal{A}^{\left(\frac{3}{2}\right)}$$

$$T^{+-} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{2}{3}}\,\mathcal{A}^{\left(\frac{1}{2}\right)} + \sqrt{\frac{1}{3}}\,\mathcal{A}^{\left(\frac{3}{2}\right)}\right)$$

$$T^{00} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{1}{3}}\,\mathcal{A}^{\left(\frac{1}{2}\right)} - \sqrt{\frac{2}{3}}\,\mathcal{A}^{\left(\frac{3}{2}\right)}\right)$$

The amplitudes in eqs. 5 and 7 refer to the initial states $K^{+} \leftrightarrow T^{+0}$ and $K^{0} \leftrightarrow T^{+-}, T^{00}$ respectively .

We rescale T^{+-}, T^{00} by a factor $\sqrt{2}$ with respect to the initial state $K_s \sim K_1$

(8)

$$K_s \sim \frac{1}{\sqrt{2}}\left(\left|\, K^{0}\right\rangle - \left|\,\overline{K}^{0}\right\rangle\right) = K_1$$

neglecting CP violation and obtain

(9)

$$T^{+0} \sim \frac{\sqrt{3}}{2}\,\mathcal{A}^{\left(\frac{3}{2}\right)}$$

$$T_s^{+-} \sim \sqrt{\frac{2}{3}}\,\mathcal{A}^{\left(\frac{1}{2}\right)} + \sqrt{\frac{1}{3}}\,\mathcal{A}^{\left(\frac{3}{2}\right)}$$

$$T_s^{00} \sim \sqrt{\frac{1}{3}}\,\mathcal{A}^{\left(\frac{1}{2}\right)} - \sqrt{\frac{2}{3}}\,\mathcal{A}^{\left(\frac{3}{2}\right)}$$

(Approximate) CP invariance implies that the final state phases in the amplitudes $\mathcal{A}^{\left(\frac{1}{2}\right)}$, $\mathcal{A}^{\left(\frac{3}{2}\right)}$ apply also to the $K_s \sim K_1 \longrightarrow 2\pi$ decay modes , as indicated in eq. 9 .

Hence we obtain for the 2π decay width

(10)
$$\begin{array}{lll}
\Gamma \left(K^{+} \rightarrow \pi^{+} \pi^{0} \left(\gamma \right) \right) & \sim & \frac{3}{4} \left| \mathcal{A}^{\left(\frac{3}{2}\right)} \right|^{2} \\
\Gamma \left(K_{s} \rightarrow \pi^{+} \pi^{-} \left(\gamma \right) \right) & \sim & \left[\begin{array}{l} \frac{2}{3} \left| \mathcal{A}^{\left(\frac{1}{2}\right)} \right|^{2} + \frac{1}{3} \left| \mathcal{A}^{\left(\frac{3}{2}\right)} \right|^{2} \\ + \frac{2}{3} \sqrt{2} \left| \mathcal{A}^{\left(\frac{1}{2}\right)} \mathcal{A}^{\left(\frac{3}{2}\right)} \right| \cos \Delta \delta \end{array} \right] \\
\Gamma \left(K_{s} \rightarrow \pi^{0} \pi^{0} \left(\gamma \right) \right) & \sim & \left[\begin{array}{l} \frac{1}{3} \left| \mathcal{A}^{\left(\frac{1}{2}\right)} \right|^{2} + \frac{2}{3} \left| \mathcal{A}^{\left(\frac{3}{2}\right)} \right|^{2} \\ - \frac{2}{3} \sqrt{2} \left| \mathcal{A}^{\left(\frac{1}{2}\right)} \mathcal{A}^{\left(\frac{3}{2}\right)} \right| \cos \Delta \delta \end{array} \right] \\
\Gamma \left(K_{s} \rightarrow 2\pi \left(\gamma \right) \right) & \sim & \left| \mathcal{A}^{\left(\frac{1}{2}\right)} \right|^{2} + \left| \mathcal{A}^{\left(\frac{3}{2}\right)} \right|^{2}
\end{array}$$

The final state interaction phases $\delta^{I}_{J=0}$ are

(11)
$$\textbf{arg} \left(\mathcal{A}^{\left(\frac{1}{2}\right)} \right) = \delta^{0}_{0} \; , \; \textbf{arg} \left(\mathcal{A}^{\left(\frac{3}{2}\right)} \right) = \delta^{2}_{0} \; ; \; \Delta \delta = \left(\delta^{0}_{0} - \delta^{2}_{0} \right) \left[m_K \right]$$

→

We define the three ratios

$$R = \left| \mathcal{A}^{(\frac{1}{2})} / \mathcal{A}^{(\frac{3}{2})} \right|$$

$$g = \frac{Br(K_s \to 2\pi \ (\gamma))}{Br(K^+ \to \pi^+\pi^0(\gamma))} \frac{\tau(K^+)}{\tau(K_s)}$$

$$\sim \frac{4}{3}\left(R^2 + 1\right)$$

(12)

$$f = \frac{Br(K_s \to \pi^+\pi^-(\gamma)) - 2\,Br(K_s \to \pi^0\pi^0 \ (\gamma))}{Br(K_s \to 2\pi \ (\gamma))}$$

$$\sim \left(2\sqrt{2}\,R\cos\Delta\delta - 1\right)\frac{1}{R^2+1}$$

→

In eq. 12 $\tau\left(K^+\right)$, $\tau\left(K_s\right)$ denote the total lifetimes of K^+ , K_s respectively .
We find from recent results reported by the KLOE collaboration [a b]

$$\frac{3}{4} g = 495.39204 \sim R^2 + 1$$

(13)
$$f = \frac{69.2 - 61.38}{99.89} = 0.078286$$

$$\Delta\delta \sim 50.76^\circ \quad ; \quad R \sim 22.235$$

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[a] **F. Ambrosino et al., KLOE Collaboration, 'Measurement of the absolute branching ratio of the $K^+ \rightarrow \pi^+\pi^0(\gamma)$ decay with the KLOE detector', April 2008. 13pp., arXiv:0804.4577 [hep-ex].**

[b] **F. Ambrosino et al., KLOE Collaboration , 'Precise measurement of $\Gamma\left(K_s \rightarrow \pi^+\pi^-(\gamma)\right) / \Gamma\left(K_s \rightarrow \pi^0\pi^0\right)$ with the KLOE detector at DAFNE' , Eur.Phys.J. C48 (2006) 767-780, arXiv:hep-ex/0601025.**