To be submitted to
Physics Letters B

MPI-PAE/PTh 37/78
August 1978

The mass of $\eta'(958)$, CP and P conservation in QCD, manifestations of a Josephson effect

Peter Minkowski[+)]

Max-Planck-Institut für Physik und Astrophysik, München (Fed. Rep. of Germany)

Abstract:

The ground state of QCD reveals among others a spontaneous phase parameter in connection with gauge field configurations with nontrivial topological charge. It is shown to share the basic associated features with the Josephson effects pertaining to electromagnetic gauges in a superconductor.

True and trial groundstates are shown to be distinguishable, the latter showing P(CP) violating effects but not the former.

+) On leave from Inst. for Theoretical Physics, University of Bern, Bern, Switzerland

The masses and mixing pattern of the pseudoscalar meson nonet

$$\{(\pi^{\pm},\pi^{o});(K^{\pm},K^{o},\bar{K}^{o});\eta;\eta'\}$$

as opposed e.g. to the associated vector meson nonet $\{(\rho^{\pm},\rho^{o});\omega;$ $(K^{*\pm},K^{*o},\bar{K}^{*o});\varphi\}$ are thought to reflect the approximate realization of spontaneously broken $SU3_{fl}\times SU3_{fl}$ chiral symmetry[1]. In the symmetry limit the eight axial vector currents

$$\sum_{c}\bar{q}_{s}^{c}\left(\frac{\lambda^{a}}{2}\right)_{st}\gamma_{\mu}\gamma_{5}q_{t}^{c}\;;\quad \mathrm{Tr}\,\lambda^{a}=0 \qquad a=1,\dots,8 \tag{1}$$

(s,t) = (up, down, strange)

c = (red, yellow, blue)

connect the groundstate to eight massless pseudoscalar Nambu-Goldstone-bosons

$$\{(\pi^{\pm},\pi^{o});(K^{\pm},K^{o},\bar{K}^{o});\eta\}_{(o)}$$

The nature of color exchange forces between a color octet of vector gluons and color triplet quarks[2] was conjectured to provide a mechanism to split the η'-meson from the eight Goldstone modes such that

$$(m_{\eta'})_{(o)}\simeq 880\ \mathrm{MeV}\neq 0 \tag{2}$$

(o): referring to the chiral limit

Difficulties connected with the theoretical understanding of this mechanism have become known as the U1-problem[3].

In Quantumchromodynamics (QCD)[2][4] the chiral limit is realised by the vanishing of the quark mass term $\mathcal{L}_m$ in the total Lagrangian

$$\mathcal{L}=\mathcal{L}_o+\mathcal{L}_m+\mathcal{L}_\vartheta+(\mathcal{L}_{gauge\,fixing}+\mathcal{L}_{ghost})$$

$$\mathcal{L}_o=\frac{1}{2g^2}\mathrm{Tr}\left(B_{\mu\nu}B^{\mu\nu}\right)+\bar{q}_s^{c}\,i\gamma^{\mu}\left[\frac{1}{2}\overleftrightarrow{\partial}_{\mu}+B_{\mu}^{cc'}\right]q_s^{c'}$$

$$-\mathcal{L}_m=\bar{q}_s^{c}\left[m_{st}\frac{1+\gamma_5}{2}+\bar{m}_{ts}\frac{1-\gamma_5}{2}\right]q_t^{c} \tag{3}$$

$$\mathcal{L}_\vartheta = -\vartheta \frac{1}{16\pi^2} Tr\left(B_{\mu\nu}\tilde{B}^{\mu\nu}\right); \quad \vartheta : \text{real arbitrary}$$

$$B_\mu = i B_\mu^a \frac{\lambda^a}{2}; \quad B_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu + [B_\nu, B_\mu]$$

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = i f_{abc}\frac{\lambda^c}{2}; \quad Tr\,\lambda^a\lambda^b = 2\delta^{ab}$$

$$a, b, c = 1, \ldots, 8$$

$$\tilde{B}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} B^{\alpha\beta}$$

In eq. 3 m_{st} are arbitrary complex parameters conveyed by spontaneous breakdown of weak gauges.

The dynamics of the flavor singlet ($\eta'_{(o)}$) $J^P = 0^-$ channel is connected to the axial current anomaly[5)]

$$\partial_\mu\left(\bar{q}_s^c \gamma^\mu\gamma_5 q_s^c\right) = \partial_\mu J_5^\mu = (2n_{fl})\frac{1}{16\pi^2} Tr\left(B_{\mu\nu}\tilde{B}^{\mu\nu}\right) + \bar{q}\left[i(m-m^+) + (m+m^+)i\gamma_5\right] q \tag{4}$$

The expression not vanishing for m → 0 on the right-hand side of eq. 4

$$-\frac{1}{16\pi^2} Tr\, B_{\mu\nu}\tilde{B}^{\mu\nu} = \frac{1}{\vartheta}\mathcal{L}_\vartheta$$

is a total divergence of a gauge variant current

$$-\frac{1}{16\pi^2} Tr\left(B_{\mu\nu}\tilde{B}^{\mu\nu}\right) = \partial_\mu \Delta^\mu(B_\alpha)$$
$$\Delta_\mu = \frac{1}{16\pi^2}\varepsilon_{\mu\alpha\beta\gamma}\, Tr\begin{pmatrix} B^\alpha B^{\beta\gamma} + \\ + \frac{2}{3} B^\alpha B^\beta B^\gamma \end{pmatrix} \tag{5}$$

The conserved current

$$\tilde{J}_\mu^5 = J_\mu^5 + (2n_{fl})\Delta_\mu(B) \tag{6}$$

gives rise to a conserved charge (in the limit of m → 0)

$$Q_5 = \int d^3x\, \tilde{f}_0^{\,5} \tag{7}$$

This charge is not gauge invariant[6)7)]. For fixed time t the mapping

$$\begin{array}{ccccc} S_3 & \longrightarrow & R_3 & \xrightarrow{(n)} & SU3_c \\ \underset{\sim}{e} = e_0 + i\vec{e}\vec{\sigma} & \longrightarrow & \vec{x}(\underset{\sim}{e}) & \xrightarrow{(n)=0,\pm 1,\pm 2,\ldots} & \Omega^{(n)}(t,\vec{x}) \cong (\underset{\sim}{e})^n \\ e_0^2 + \vec{e}^{\,2} = 1 & & & & \end{array} \tag{8}$$

into a subgroup of the gauge group $SU3^c$ defines associated topological classes of gauge transformations with n = 0,±1,±2,... . Any representative of class n has the property

$$\begin{aligned} Q_5\left(B_\alpha^{\,\Omega^{(n)}}\right) &= Q_5\left(B_\alpha\right) + 2n_{fl}\binom{n}{4n} \\ B_\alpha^{\,\Omega^{(n)}} &= \Omega^{(n)} B_\alpha \left(\Omega^{(n)}\right)^{-1} + \Omega^{(n)}\partial_\alpha\left(\Omega^{(n)}\right)^{-1} \end{aligned} \tag{9}$$

n: independent of time.

The two inequivalent embeddings of $SU2 \rightarrow SU3^c$ yield $2n_{fl}\cdot n$, $8n_{fl}\cdot n$ in eq. 9 respectively.

In close analogy with QED in two dimensions (Schwinger model)[8)9)] there exist formally unitary transformations T_n associated with $\Omega^{(n)}$, mandatory for the local representation of quark and gluon fields, with the property

$$\begin{aligned} T_n &= (T)^n \;, \qquad T^{-1} Q_5 T = Q_5 + 2n_{fl} \\ T &= \exp\left[-i(2n_{fl})\cdot S_5\right] \qquad \left[Q_5\,,\,S_5\right] = i(2n_{fl})\mathbb{1} \end{aligned} \tag{8}$$

The conjugate pair (Q_5, S_5) can be represented on a circle as

$$Q_5 \longleftrightarrow i\frac{\partial}{\partial\Theta}(2n_{fl}) \;, \qquad S_5 \longleftrightarrow \Theta \tag{9}$$

The formal construction of the groundstate functional proceeds in close analogy to the representation in eq. (9)[6)]:

$$|\omega_n\rangle \longleftrightarrow e^{-in\Theta} \qquad n=0,\pm 1,\pm 2,\cdots$$

$$Q_5\,|\omega_n\rangle = (2n_{fl})\cdot n\,|\omega_n\rangle$$

$$T|\omega_n\rangle = |\omega_{n+1}\rangle$$

$$|\Omega_{\Theta_V}\rangle = \sum_{n=-\infty}^{+\infty} e^{-in\Theta_V}|\omega_n\rangle \longleftrightarrow \delta(\Theta-\Theta_V) \tag{10}$$

$$\langle\Omega_{\Theta_V'}|\Omega_{\Theta_V}\rangle = (2\pi)\,\delta(\Theta_V'-\Theta_V)$$

We will argue that it is indeed necessary to minimize the action integral with respect to the trial states $|\Omega_{\Theta_V}\rangle$, varying Θ_V to determine the minimum, if the energy of the groundstate corresponds to a true, isolated minimum of energy[10].

The action functional due to instanton effects has been shown by t'Hooft[12] to correspond to the individual contributions

$$\langle\Omega_{\Theta_V'}|\left(\int\mathcal{D}[B_\mu,\xi_{ghost},q^+,q]\exp i\,S\right)|\Omega_{\Theta_V}\rangle =$$

$$= (2\pi)\,\delta(\Theta_V'-\Theta_V)\left(\exp - W(\Theta_V)\right) \tag{11}$$

$$\exp\left(-W(\Theta_V)\right) = \sum_{\Delta n=-\infty}^{+\infty}\exp(i\,\Delta n\,\Theta_V)\langle\omega_{\Delta n}|\int\mathcal{D}[X]e^{iS}|\omega_0\rangle$$

$$= \sum_{\Delta n}\exp[i(\Theta_V-\bar\vartheta)\Delta n]\,K_{\Delta n}$$

In eq. (11) $\bar\vartheta = \vartheta$ - arg det m, and $K_{\Delta n}$ denote functions depending only on the n_{fl} chiral invariants of the quark mass matrix (if spontaneous $SUn_{fl}\times SUn_{fl}$ symmetry breaking is neglected)

$$K_{\Delta n} = K_{\Delta n}(x_1,\cdots,x_{n_{fl}}) = K_{-\Delta n} \quad \text{(by CPT invariance)}$$

$$x_\nu = Tr\,(m^+m)^\nu\,;\ \nu=1,\cdots,n_{fl} \tag{12}$$

We consider the operators

$$\mathcal{U}_\alpha = \exp(-i\alpha\, Q_5)\quad , \quad \mathcal{U}_5 = \mathcal{U}\left(\alpha = \frac{\pi}{n_{fl}}\right)$$

$$\Omega_{\Theta_V} \longleftrightarrow \sum_n \exp[in(\Theta - \Theta_V)] \tag{13}$$

$$\mathcal{U}_\alpha\, \Omega_{\Theta_V} = \Omega_{\Theta_V + 2n_{fl}\cdot\alpha} \longleftrightarrow \sum_n \exp[in(\Theta - \Theta_V - 2n_{fl}\cdot\alpha)]$$

The gauge invariant discrete symmetry generated by Q_5 corresponds to a phase shift of 2π.

For small values of m with respect to the renormalization group invariant mass characterising the chiral limit

$$\mu^* = \lim_{\substack{\mu \to \infty \\ m \to 0}} \mu \exp - \frac{8\pi^2}{b g^2(\mu)} \tag{14}$$

$$b = 11 - \frac{2}{3} n_{fl}$$

$K_{\Delta n}$ vanishes proportionally to $\left|\mathrm{Det}\left(\frac{m}{\mu^*}\right)\right|^{|\Delta n|}$

if we neglect all effects due to spontaneous symmetry breaking of $SUn_{fl} \times SUn_{fl}$. We shall denote the part of $K_{\Delta n}$ pertaining to the limit of vanishing <u>spontaneous</u> $SUn_{fl} \times SUn_{fl}$ breaking parameters (not to be confused with quark masses) by $k_{\Delta n}$. These parameters are the BCS-like pairing strengths of quark-antiquark pairs in the groundstate[13].

The spontaneous breaking of the U1 symmetry generated by Q_5 can be made explicit considering a variation of e^{-W} with respect to m, and involves the nonvanishing matrix element of the operators[1)12)] $F_{L(R)} = \mathrm{Det}_{s,t}\left(\bar{q}_s^{\,c}\, \frac{1(\pm)\gamma_5}{2}\, q_t^{\,c}\right)$

$$\left\langle \Omega_{\Theta_V'} \middle| \mathrm{Det}_{s,t}\left(\bar{q}_s^{\,c}\, \frac{1(\pm)\gamma_5}{2}\, q_t^{\,c}\right) \middle| \Omega_{\Theta_V} \right\rangle_{(0)} =$$

$$= (2\pi)\, \delta(\Theta_V' - \Theta_V)\, e^{\mp i[\Theta_V - \vartheta]} \lim_{m\to 0}\left(\frac{k_1}{|\mathrm{Det}\, m|}\right) \tag{15}$$

We propose to call this collection of quarks and antiquarks (in the groundstate) flavor cycle.

The action functional when varied with respect to ϑ with fixed Θ_V (or $-\Theta_V$ with fixed ϑ) yields the vacuum expectation value of the operator $B_{\mu\nu}\tilde{B}^{\mu\nu}$:

$$\left(\frac{\partial}{\partial \Theta_V} W\right)_{\vartheta, m} = \frac{1}{32\pi^2}\langle B^a_{\mu\nu}\tilde{B}^{\mu\nu a}\rangle_{\Theta_V} \qquad (16)$$

We now use the Ward identity involving the axial anomaly

$$\partial_\mu J^\mu_5 = -(2n_{fl})\frac{1}{32\pi^2} B^a_{\mu\nu}\tilde{B}^{\mu\nu a} + \bar{q}\left[i(m-m^+)+(m+m^+)i\gamma_5\right]q \qquad (17)$$

and evaluate it with respect to the groundstate. If the latter is an eigenstate of momentum (with vanishing eigenvalues) then

$$\langle \partial_\mu J^\mu_5 \rangle_{\Theta_V} = 0 \qquad (18)$$

Now either the simple pairing strengths

$$\left\langle \sum_c \bar{q}^c_s \frac{1+\gamma_5}{2} q^c_t \right\rangle_{\Theta_V} = \overline{M}_{ts} \qquad (19)$$

are zero, in which case eq. (16), (17) and (18) yield

$$\frac{\partial}{\partial \Theta_V}\left(W_{\Theta_V}\right) = 0 \qquad (20)$$

This condition determines the phase Θ_V dynamically such as to minimize the action. It was pointed out in ref. 13) that neglecting effects due to $M_{ts} \neq 0$ in eq. (19), at least for the light flavors, is incorrect

$$M_{ts} \xrightarrow{(m\to 0)} \delta_{ts} M \quad \text{for } t,s = \text{up, down}$$

$$(m^*_u + m^*_d)\,|M| = \frac{1}{4}F_\pi^2 m_\pi^2 \rightarrow |M|^{1/3} \simeq 200\ \text{MeV} \qquad (21)$$

for $m^*_u + m^*_d = 11$ MeV [7) 14)]

m^*_s denote the nonnegative eigenvalues of the matrix $(mm^+)^{1/2}$.

For $m \rightarrow 0$ there still is tunneling from $|\omega_n\rangle$ to $|\omega_{n+\Delta n}\rangle$ due to single $q\bar{q}$-pairs breaking $SUn_{fl} \times SUn_{fl}$ invariance[15].

Thus in the chiral limit $(m = 0)$ the same argument as before yields

$$\partial_{\Theta_V} W_{\Theta_V}(M) = 0, \qquad (22)$$
$$K_{\Delta n} = K_{\Delta n}(M, m=0) \neq 0, \quad \Delta n = 0, \pm 1, \pm 2, \dots$$

In eq. (22) the dependence of W_{Θ_V} on M comes entirely from the dependence of the trial groundstate $|\Omega_{\Theta_V}(M)\rangle$ on M.

For general externally given (fixed) mass matrix m and nontrivial vacuum expectation values M the action functional is a function of the variables Θ_V, M, m, ϑ

$$e^{-W_{\Theta_V}} = \sum_{\Delta n} e^{i\Delta n(\Theta_V - \bar{\vartheta})} K_{\Delta n}\left[m^+m, m^+M, M^+m, M^+M\right] \qquad (23)$$

In eq. (23) we have specialised to one flavor. The following discussion is not dependent on the number of flavors. One obtains the equations

$$\left(\frac{\partial W}{\partial m^+}\right)_{\Theta_V, \vartheta, M} = M \qquad (24)^{FN1)}$$

Now eq. (17) takes the form

$$i\left[m\frac{\partial}{\partial m} - m^+\frac{\partial}{\partial m^+}\right]W = (2n_{fl})\frac{\partial}{\partial \Theta_V} W \qquad (25)$$

Since $K_{\Delta n}$ depends nontrivially on m^+M, M^+m, the above equation is by no means a consequence of $U1_5$-covariance. In general the solutions to eq. (24) and (25), of which the previously discussed limits M = 0 and m = 0 are special cases demand the two conditions

$$mM^+ = m^+M \quad ; \quad \frac{\partial}{\partial \Theta_V} W = 0 \qquad (26)$$

It is the conflict of the implementation of both energy and momentum conservation and the candidate symmetry denoted S_5 here which forces the phase Θ_V to cancel the phases $\bar{\vartheta} = \vartheta - \arg\det m$. The associated phase fluctuations forcing a candidate superselection rule to choose a phase bear all similarity to the Josephson effect[16] FN2).

As a consequence of eq. (24), (26) we can perform the following joint change of spontaneous and explicit parameters

$$\Theta_V = \bar{\vartheta} \longrightarrow 0$$

$$M = V_R\, M^* V_L^{-1} \longrightarrow M^* = \begin{pmatrix} M_u^* & & 0 \\ & M_d^* & \\ 0 & & M_s^* \ddots \end{pmatrix} \qquad M_q^* : \text{real} \tag{27}$$

$$m = V_R\, m^* V_L^{-1} \longrightarrow m^* = \begin{pmatrix} m_u^* & & 0 \\ & m_d^* & \\ 0 & & m_s^* \ddots \end{pmatrix} \qquad m_q^* \geq 0$$

$$\vartheta \longrightarrow 0$$

It is only after this change of variables that the discrete symmetries P, CP take the form of simple transformation rules on the quark and gluon fields. They are although not symmetries of the forced groundstates, exact symmetries of the true groundstate when weak effects are neglected.

The dynamically constrained nature of Θ_V was first stated by Peccei and Quinn[17]. As a matter of principle it also follows from their work, that Θ_V is not without constraint relative to the scalar vacuum expectation values $\phi_\alpha = \langle \Omega / \varphi_\alpha / \Omega \rangle$ in the true groundstate.

Finally the Schwinger model[8] does admit the analogy of Θ_V in two-dimensional massless QED[9]. In the sector with even fermion number which contains all the forced groundstates there is no genuine interaction to distinguish among them.

Acknowledgements

It is a pleasure to thank R. Peccei for many controversial and stimulating discussions.

The warm hospitality extended to me by the members of the Max-Planck-Institute and by W. Ochs in particular is gratefully acknowledged.

Footnotes

FN1) The meaning of eq. (24) is the following:

The variation with respect to m should be performed not varying the spontaneous parameters (Θ_V, M) characterising a true or forced groundstate $|\Omega(\Theta_V, M)\rangle$. Even if M happens to be the true value of M for a given m, it will not remain so while m is varied. Thus it is mandatory to compute the action functional for unconstrained values of (Θ_V, M) first:

$$2\pi\,\delta(\Theta_V'-\Theta_V)\,e^{-W} = \langle\Omega(\Theta_V', M)|\int\mathcal{D}X \exp i\,S[X,\vartheta,m]|\Omega(\Theta_V, M)\rangle$$

$$W = W(\Theta_V, M; \vartheta, m)$$

The values pertaining to the true grounstate are subsequently determined through equations of the logical structure of eq. (24) (equilibrium conditions in statistical mechanics).
In this sense the parameters in the Lagrangian (ϑ, m) and the spontaneous parameters characterising the groundstate (Θ_V, M) are conjugate variables. This is best seen when eq. (24) is replaced by the associated Legendre transform

$$T(\Theta_V, M; \vartheta, m) = W - Tr[mM^+ + M^+m]$$

$$\left(\frac{\partial T}{\partial m}\right)_{\Theta_V, \vartheta, M} = 0$$

Symmetries which are broken explicitly and spontaneously e.g. rotations in the Heisenberg ferromagnet in the presence of a fixed external magnetic field, can differentiate between forced and true groundstate.
In the ferromagnet the critical symmetry is a rotation around the axis of the magnetic field. The forced groundstate corresponds to all spins aligned in a direction forming an angle Θ with the magnetic field. The true groundstate has $\Theta = 0$ and the critical symmetry is recovered.
The critical symmetry in our case is CP (or P). The fact that it takes infinite energy to force the above family of trial groundstates is immaterial.

FN2) We conjecture that the η' meson (and similarly the $\eta_c(2830)$) is actually associated with another value of Θ :

$$\Theta_{\eta'} - \Theta_V = \pm(2\pi)/(2\, n_{fl})$$

and that it is this fact which causes the hadronic width of η' to be reduced

$$\Gamma_{\eta' \to \eta\pi\pi} < 1\,\mathrm{MeV}$$

whereas despite the essential mass differnece

$$\langle \Omega \,|\, J_\mu^{(5)} \,|\, \eta' \rangle \simeq \langle \Omega \,|\, (\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) \sqrt{\frac{n_{fl}}{2}} \,|\, \pi^0 \rangle$$

yielding

$$\Gamma_{\eta' \to 2\gamma} \simeq \left(\frac{m_{\eta'}}{m_\pi}\right)^3 \left(\frac{F_{\eta'}^2}{F_\pi^2}\right) \frac{8}{3}\, \Gamma_{\pi^0 \to 2\gamma} \simeq 7.5\,\mathrm{keV}$$

convoluting with the known branching fraction

$\Gamma_{\eta' \to 2\gamma} / \Gamma_{\eta' \to all} = (2.0 \pm 0.3)\ 10^{-2}$ one then obtains

$$\Gamma_{\eta' \to hadrons} \simeq 250\,\mathrm{keV}$$

References

1) M. Gell-Mann, Phys. Rev. 125 (1962), 1067
M. Gell-Mann and M. Lévy, Il Nuovo Cimento 16 (1960), 53
Y. Nambu and D. Lurié, Phys. Rev. 125 (1962), 1429
The connection between spontaneously broken γ_5-invariance, massless Goldstone modes and superconductivity was orginally emphasized by
Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961), 345
124 (1961), 246

2) H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Letters B47 (1972), 365
Y. Nambu, "Preludes in Theoretical Physics", North Holland (1966)
S. Weinberg, Phys. Rev. Letters 31 (1973), 494

4) see e.g. H.D. Politzer, Physics Reports 14C, No. 4 (1974), 129
D.J. Grossand, F. Wilczek, Phys. Rev. D8 (1973) 3633
W. Marciano and H. Pagels, Physics Reports 36C, No. 3 (1978), 137

5) S. Adler, Phys. Rev. 177 (1969), 2426
J.S. Bell and R. Jackiw, Nuovo Cimento 60A (1969), 47

6) R. Jackiw and C. Rebbi, Phys. Rev. Letters 37 (1976), 172
C. Callan, R. Dashen and D. Gross, Phys. Letters 63B (1976) 334

7) P. Minkowski, University of Bern preprint (1977)

8) J. Schwinger, Phys. Rev. 128 (1962), 2425

9) J.H. Lowenstein and J.A. Swieca, Ann. of Physics 68 (1971), 172
J. Kogut and L. Susskind, Phys. Rev. D11 (1975) 3594

10) see e.g.: S. Weinberg, Harvard University prepring HUTP-78/A005 (1978)
C.H. Woo, University of Maryland preprint (1978)

11) A.A. Belavin, A.M. Polyakov, A.S. Schwartz and Yu. S. Tyupkin, Phys. Letters. 59B (1975), 85

12) G. t'Hooft, Phys. Rev. Letters 37 (1976), 8
Phys. Rev. D14 (1976), 432

13) P. Minkowski, Physics Letters 76B (1978), 439

14) for evaluations of the quark masses see e.g.:
H. Leutwyler, Nucl. Phys. B76 (1974) 413
J. Gasser and H. Leutwyler, Nucl. Phys. B94 (1975), 269
S. Weinberg, Harvard University preprint HUTP 77/1064 (1977)

15) The phenomenological necessity of this fact was noted by
R. Crewther, Phys. Letters, 70B (1977), 349

16) see e.g. B.D. Josephson, Weakly coupled superconductors, in
Superconductivity, R.D. Parks Editor,
Dekker Inc., New York 1969, Vol. 1, p. 423

17) R.D. Peccei and H.R.Quinn, Phys. Rev. Letters 38 (1977), 1440
Phys. Rev. D16 (1977), 1791