Leptonic g-2: Standard Model vs measurements

Massimo Passera

Università e INFN Padova

Symposium für Peter Minkowski 'Emeritierung'
Universität Bern - 5 Mai 2006

The present experimental values:

$$a_e = 1159652188.3(4.2) \times 10^{-12}$$

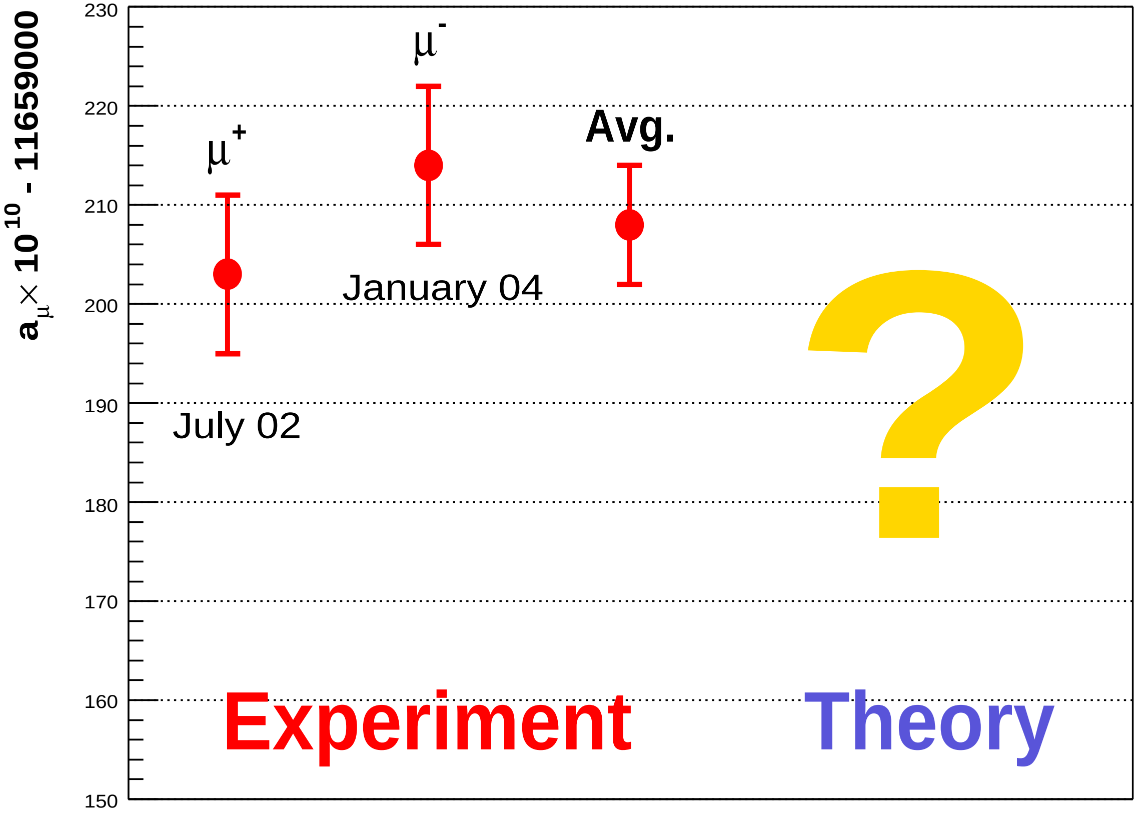
3.6 parts per billion !! CODATA '98 based on Van Dyck et al. 1987

$$a_\mu = 116592080\ (63) \times 10^{-11}$$

0.5 parts per million !! E821 – Final Report: hep-ex/0602035

$$-0.052 < a_\tau < 0.013$$

(95% CL) DELPHI at LEP2 - EPJ35 (2004) 159



$$a_\mu^{EXP} = (116592080 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$$

The anomalous magnetic moment: theory

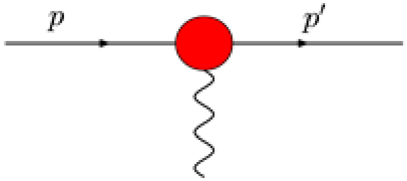
- The Dirac theory predicts for a lepton l=e,μ,τ:

$$\vec{\mu}_l = g_l \left(\frac{e}{2m_l c}\right) \vec{s} \qquad g_l = 2$$

- QFT predicts deviations from the Dirac value:

$$g_l = 2\,(1 + a_l)$$

- Study the photon-lepton vertex:



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \ldots\right]u(p)$$

$$\boxed{F_1(0) = 1 \qquad F_2(0) = a_l}$$

The electron g-2

The electron g-2 and (the best determination of) alpha

a_e^{SM} = $(1/2)(\alpha/\pi)$ - $0.328\,478\,444\,002\,90(60)\,(\alpha/\pi)^2$

Schwinger 1948 Sommerfield, Petermann '57, Suura, Wichmann '57, Elend '66, MP '05

+ $1.181\,234\,016\,827\,(19)\,(\alpha/\pi)^3$

Kinoshita, Barbieri, Laporta, Remiddi, … , Li, Samuel, MP '05

- $1.7283\,(35)\,(\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio July '05

+ $0.0\,(3.8)\,(\alpha/\pi)^5$ In progress (12672 mass-indep. diagrams!)

Mohr & Taylor '05 (CODATA 2002); Kinoshita & Nio, in progress.

+ $1.671\,(19) \times 10^{-12}$ Hadronic

Mohr & Taylor '05 (CODATA 2002), Davier & Hoecker '98, Krause '97, Knecht '03

+ $0.0297\,(5) \times 10^{-12}$ Electroweak

Mohr & Taylor '05 (CODATA 2002)

Comparing $a_e^{SM}(\alpha)$ with $a_e^{exp} = 1159652188.3(4.2) \times 10^{-12}$ CODATA '98 based on Van Dyck et al. 1987

one gets:

$\alpha^{-1} = 137.035\,998\,83$ (50) [3.6 ppb] Kinoshita & Nio '05

versus

$\alpha^{-1} = 137.036\,000\,10$ (110) [7.7 ppb] Wicht et al. 2002

$\alpha^{-1} = 137.035\,999\,11$ (46) [3.3 ppb] CODATA '02 = PDG '04

Check of QED at 4 loop level !

The muon g-2

The QED Contribution to a_μ

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+\ 0.765857410\ (27)\ (\alpha/\pi)^2$

Sommerfield, Petermann, Suura, Wichmann, Elend, MP '04

$+\ 24.05050964\ (43)\ (\alpha/\pi)^3$

Barbieri, Laporta, Remiddi, … , Czarnecki, Skrzypek, MP '04

$+\ 130.992\ (8)\ (\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio July '05

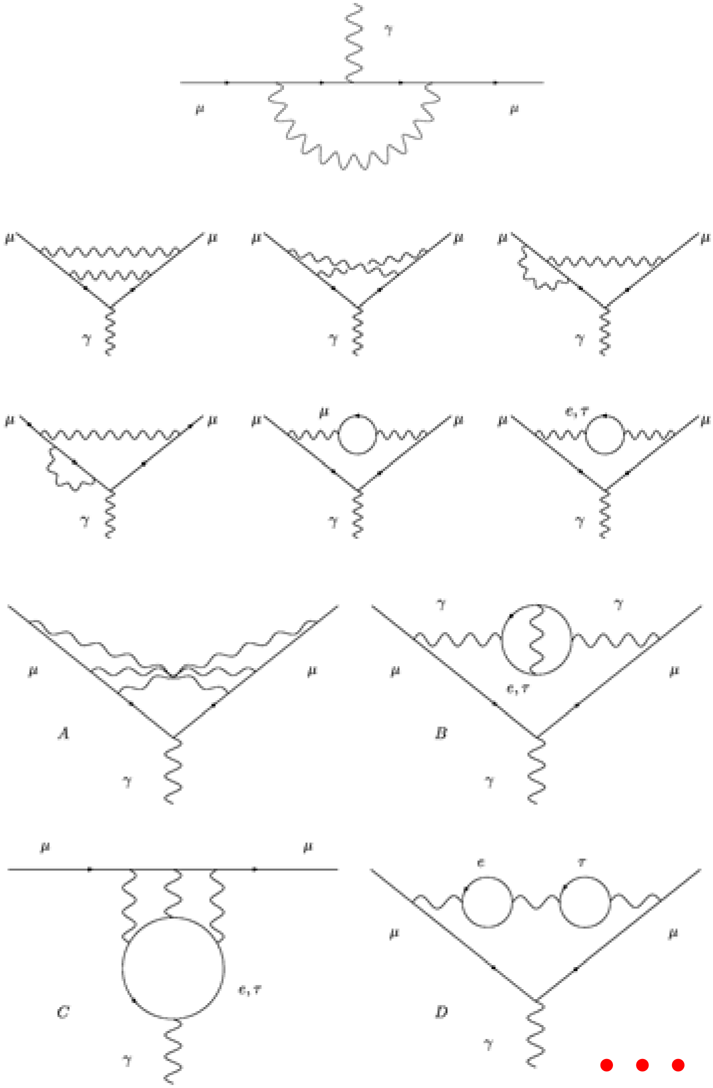
$+\ 663\ (20)\ (\alpha/\pi)^5$ In progress

Kinoshita et al. '90, Yelkhovsky, Milstein, Kataev, Starshenko, Broadhurst, Karshenboim, Laporta, Ellis et al.,…, Kataev '05, Kinoshita & Nio, March '06.

Adding up, I get:

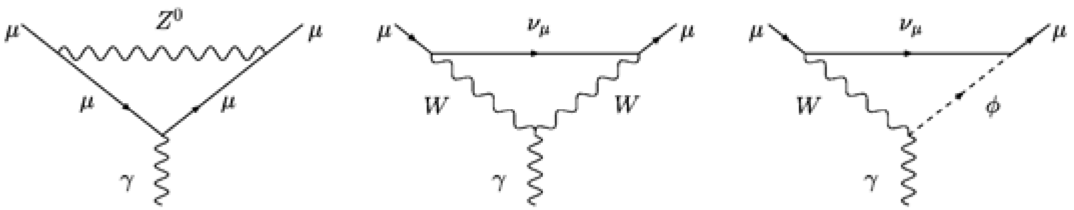
$a_\mu^{QED} = 116584718.6\ (0.1)\ (0.4) \times 10^{-11}$ using

$\alpha = 1/137.03599911\ (46)\ [3.3\ \text{ppb}]$ PDG'04



The Electroweak contribution to a_μ

- ## One-Loop Term:



$$a_\mu^{\mathrm{EW}}(1\ \mathrm{loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa Lee, Sanda.

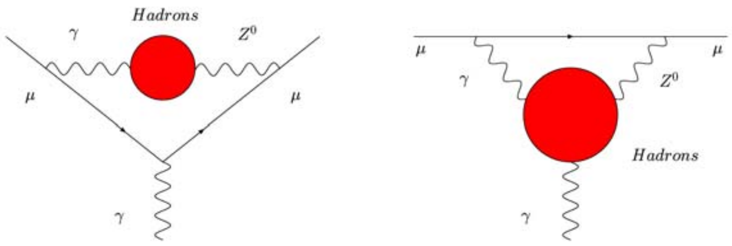
- ## One-Loop plus Higher-Order Terms:

a_μ^{EW} = 154 (2) (1) × 10^{-11}

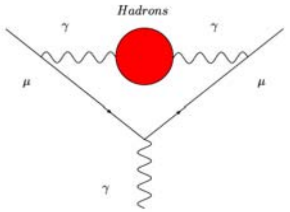
Kukhto et al. '92, Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano & Vainshtein '02; Degrassi & Giudice '98; Heinemeyer, Stockinger & Weiglein '04 Gribouk & Czarnecki '05.

Higgs mass, M_top error, three-loop nonleading logs

Hadronic loop uncertainties:



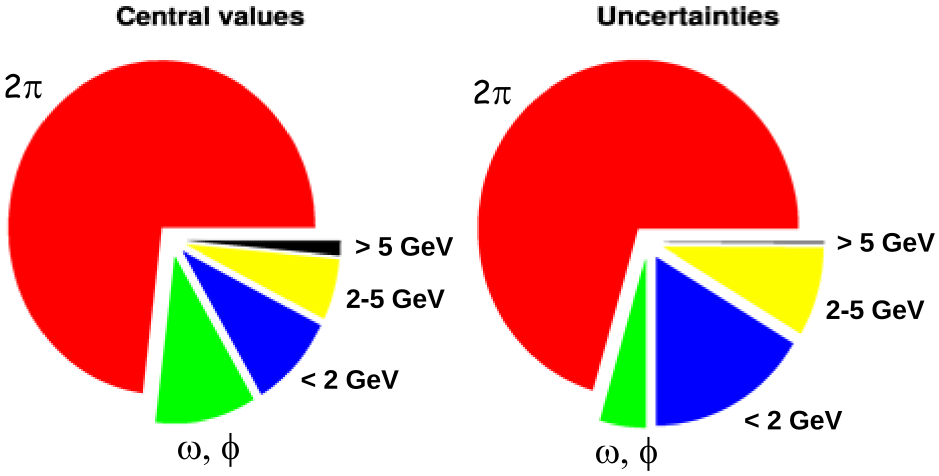
a_μ: hadronic contributions



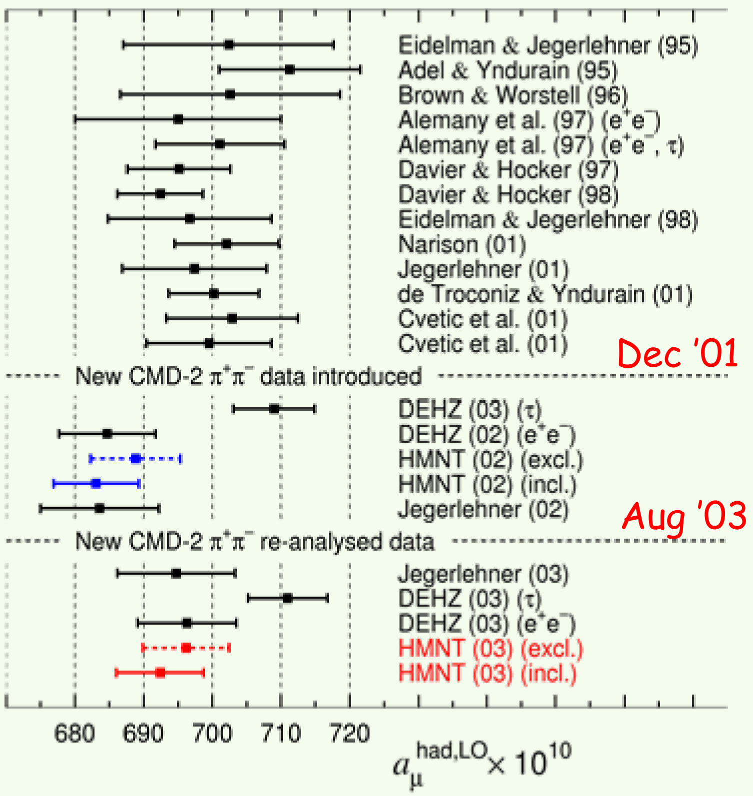
$$a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s)\, R(s)$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

Bouchiat & Michel 1961, Gourdin & de Rafael 1969



S. Eidelman, INFN Roadmap Meeting, LNF, Jan 2006



Hagiwara et al., PRD 69 (2004) 093003

a_μ: hadronic contributions - ii

Data from e^+e^- (CMD2 after August 2003)

a_μ^{HLO} = 6934 (53)$_{exp}$ (35)$_{rad}$ × 10^{-11} A. Hoecker@ICHEP04, hep-ph/0410081

= 6948 (86) × 10^{-11} F. Jegerlehner, Nucl. Phys. Proc. Suppl. 126 (2004) 325

= 6924 (59)$_{exp}$ (24)$_{rad}$ × 10^{-11} K. Hagivara et al., PRD69 (2004) 093003

= 6944 (48)$_{exp}$ (10)$_{rad}$ × 10^{-11} de Troconiz, Yndurain, PRD71 (2005) 073008

- Radiative Corrections (Luminosity, ISR, Vacuum Polarization, **FSR**) are a very delicate issue! All under control?
- CMD2's new (1998) $\pi^+\pi^-$ data presented at EPS 2005 and at Novosibirsk 2006 agree well with their earlier (1995) ones.
- SND's $\pi^+\pi^-$ data released in June 2005 have been recently reanalyzed (RC fixed, σ decreased – see Achasov's talk at Novosibirsk 2006). There is now good agreement with the $\pi^+\pi^-$ data of CMD2.

a_μ: hadronic contributions - iii

- Radiative Return (KLOE & BABAR): The collider operates at fixed energy but s_π can vary continuously. This is an important independent method!
- Some discrepancies between KLOE's and CMD2's results, although their contributions to a_μ^{HLO} are similar.
- SND's JETP101 (2005) 1053 data were significantly higher than KLOE's ones above the ρ peak, but they now decreased.
- Comparison in the range $s_\pi \in [0.37, 0.93]$ GeV^2:

$a_\mu^{\pi\pi} = (3786 \pm 27_{stat} \pm 23_{sys+th}) \times 10^{-11}$	CMD2 (95)	PLB578 (2004) 285
$a_\mu^{\pi\pi} = (3770 \pm 22_{stat} \pm 15_{sys+th}) \times 10^{-11}$	CMD2 (95+98)	Eidelman 2006 prelim.
$a_\mu^{\pi\pi} = (3756 \pm 8_{stat} \pm 48_{sys+th}) \times 10^{-11}$	KLOE	Venanzoni@ICHEP'04
$a_\mu^{\pi\pi} = (3767 \pm 13_{stat} \pm 49_{sys+th}) \times 10^{-11}$	SND new	Eidelman 2006 prelim.
$a_\mu^{\pi\pi} = (3856 \pm 14_{stat} \pm 50_{sys+th}) \times 10^{-11}$	SND old	JETP 101 (2005)1053

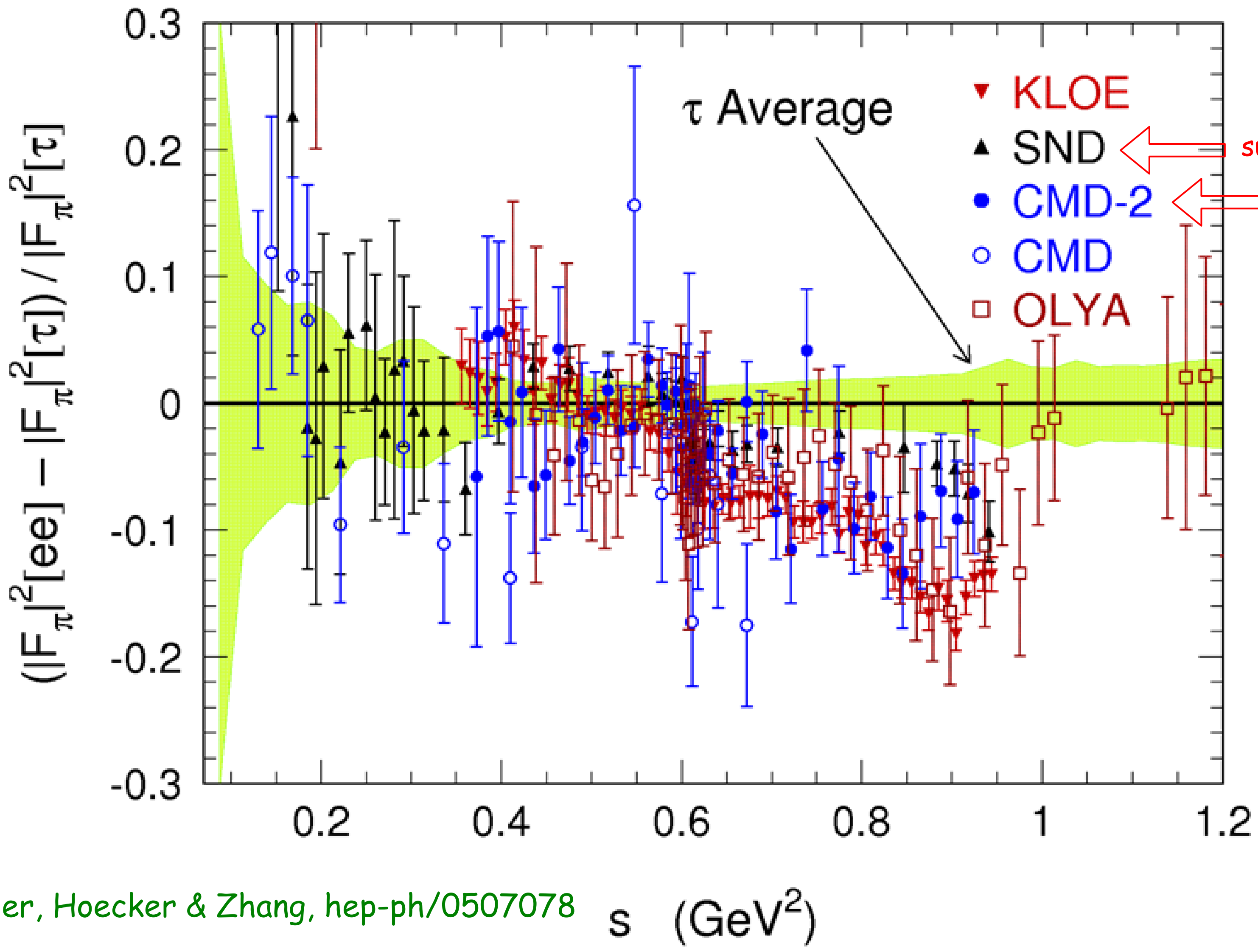
Tau Data (ALEPH, CLEO, OPAL and BELLE)

- The tau data of ALEPH and CLEO are significantly higher than CMD2 e+e- ones above ~ 0.85 GeV. KLOE confirms this discrepancy with the tau data (see plot in a moment).
- In the same region, SND [no longer] agrees with ALEPH.
- The recent preliminary tau results of BELLE seem to be in better agreement with e+e- data (see plot in a moment).
- Latest value (Davier, Eidelman, Hoecker & Zhang, EPJC31 (2003) 503):

$$\boxed{a_\mu^{HLO} = 7110\ (58) \times 10^{-11}}$$

- Inconsistencies in the e^+e^- or tau data? Are all possible isospin-breaking effects properly taken into account?? (Marciano & Sirlin 1988; Cirigliano, Ecker, Neufeld 2001-02, ...)

a_μ: hadronic contributions - v

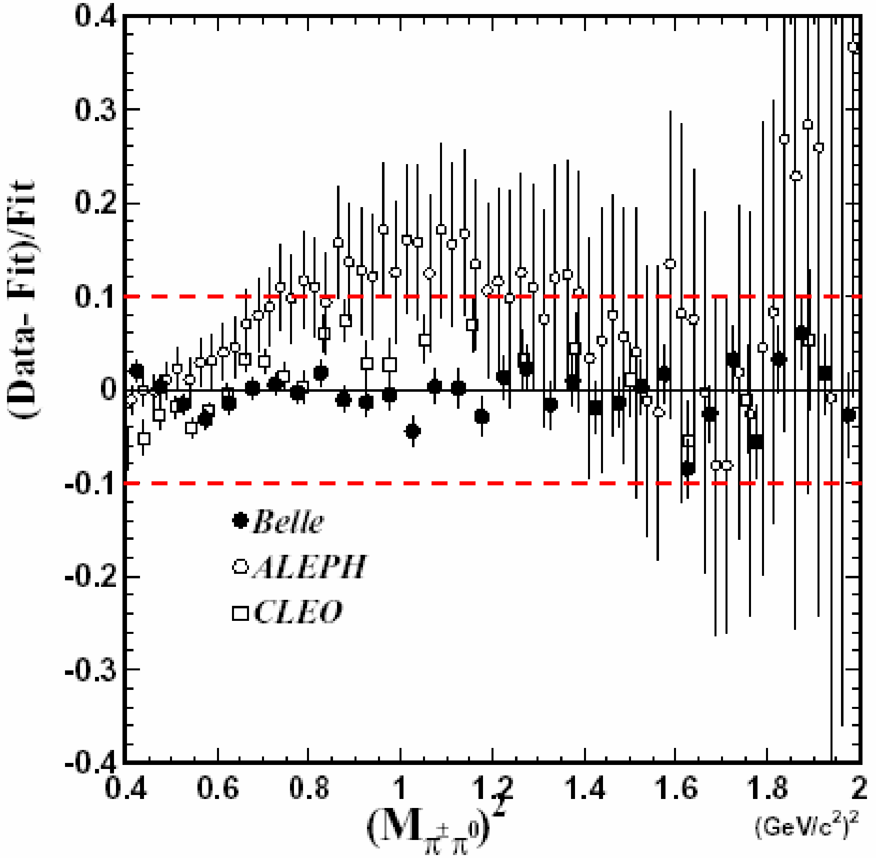


superseded

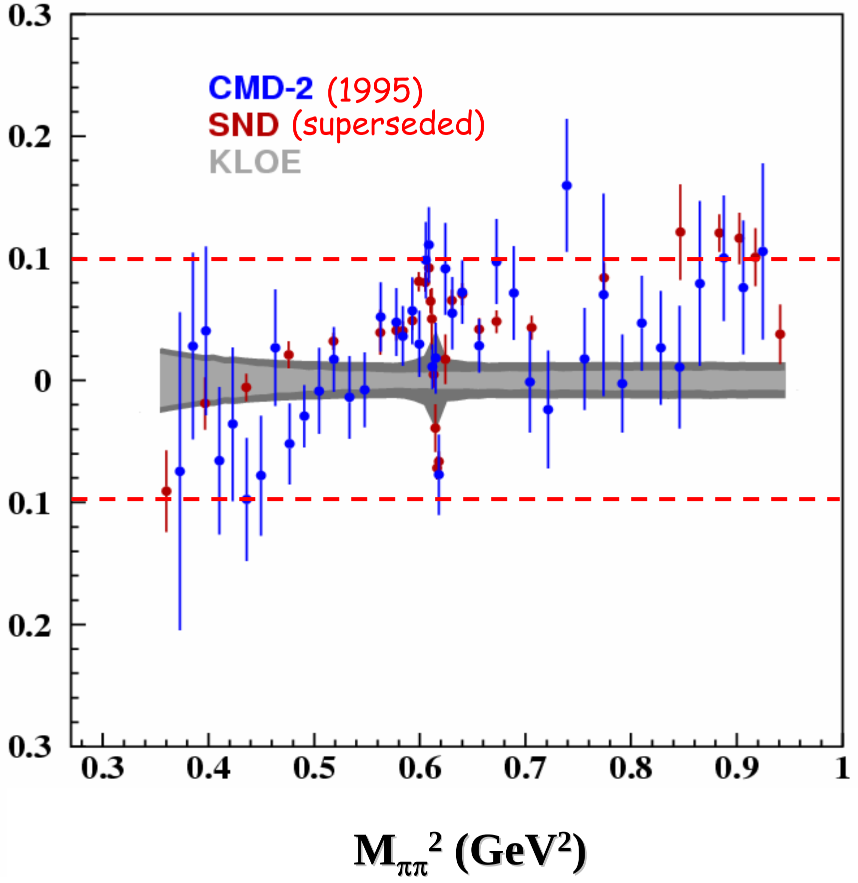
1995

Davier, Hoecker & Zhang, hep-ph/0507078

a_μ: hadronic contributions - vi



BELLE hep-ex/0512071



G. Venanzoni, INFN Roadmap WG, LNF, Jan-06

a_μ: higher-order hadronic contributions

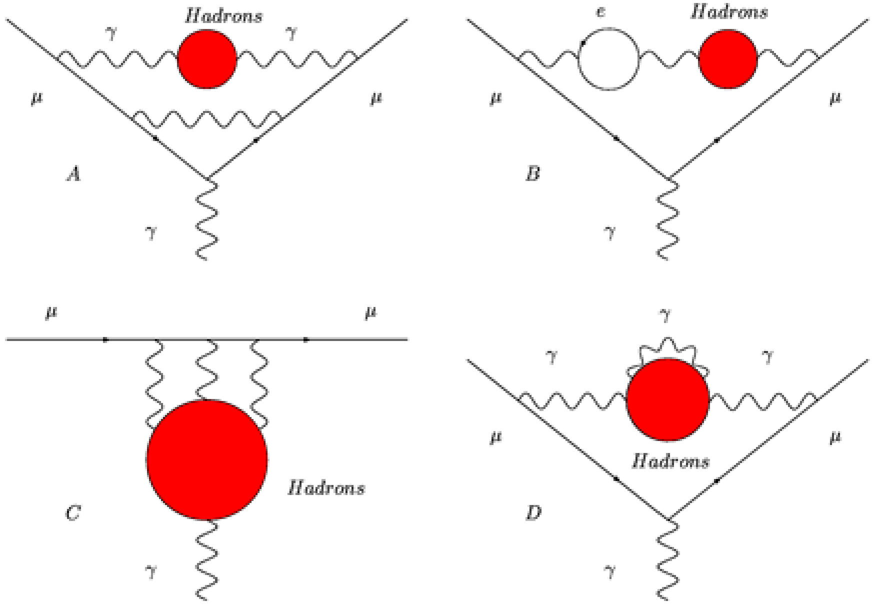
Vacuum Polarization

$O(\alpha^3)$ contribution of diagrams containing hadronic vacuum polarization insertions:

$a_\mu^{HHO}(vp) = -98\ (1) \times 10^{-11}$

Krause '96, Alemany et al. '98, Hagivara et al. '03

Shifts by $\sim -3 \times 10^{-11}$ if tau data are used instead of the e^+e^- ones. Davier & Marciano '04



Light-by-Light

The contribution of the $O(\alpha^3)$ hadronic light-by-light diagram had a troubled life. The latest vales are:

$a_\mu^{HHO}(lbl) = +\ 80\ (40) \times 10^{-11}$ Knecht & Nyffeler 2002

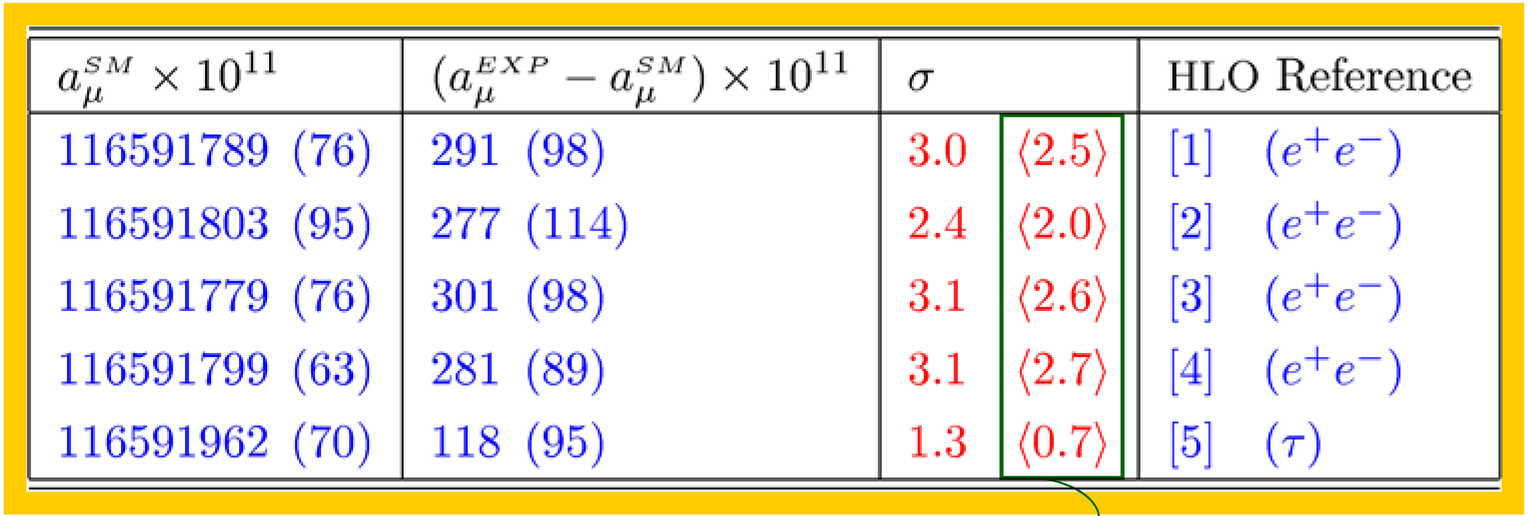
$a_\mu^{HHO}(lbl) = +136\ (25) \times 10^{-11}$ Melnikov & Vainshtein 2003

Hayakawa, Kinoshita 2001; Bijnens, Pallante, Prades 2001; Knecht, Nyffeler 2001, ...

This term may become the ultimate limitation of the Standard Model prediction.

a_μ: Standard Model vs. experiment

Adding up all the above contribution we get the following SM predictions for a_μ and comparisons with the measured value:



$a_\mu^{SM} \times 10^{11}$	$(a_\mu^{EXP} - a_\mu^{SM}) \times 10^{11}$	σ		HLO Reference	
116591789 (76)	291 (98)	3.0	$\langle 2.5 \rangle$	[1]	(e^+e^-)
116591803 (95)	277 (114)	2.4	$\langle 2.0 \rangle$	[2]	(e^+e^-)
116591779 (76)	301 (98)	3.1	$\langle 2.6 \rangle$	[3]	(e^+e^-)
116591799 (63)	281 (89)	3.1	$\langle 2.7 \rangle$	[4]	(e^+e^-)
116591962 (70)	118 (95)	1.3	$\langle 0.7 \rangle$	[5]	(τ)

a_μ^{HHO}(lbl) = 80 (40) x 10^{-11} in all table except angle brackets.

$\rightarrow$ a_μ^{HHO}(lbl) = 136 (25) x 10^{-11}

[1] A. Hoecker@ICHEP04, hep-ph/0410081.
[2] F. Jegerlehner, Nucl. Phys. Proc. Suppl. 126 (2004) 325.
[3] Hagivara, Martin, Nomura & Teubner, PRD69 (2004) 093003.
[4] J.F. de Troconiz and F.J. Yndurain, PRD71 (2005) 073008.
[5] Davier, Eidelman, Hoecker and Zhang, EPJC31 (2003) 503.

The g-2 of the tau

The SM prediction of $(g\text{-}2)_\tau$

a_τ^{SM} = $(1/2)(\alpha/\pi)$

Schwinger 1948

\+ $2.057\,457\,(93)\,(\alpha/\pi)^2$ Preliminary

Sommerfield, Petermann '57, Suura, Wichmann '57, Elend '66, MP '06

\+ $57.9315\,(27)\,(\alpha/\pi)^3$ Preliminary

Barbieri, Laporta, Remiddi, … , Li & Samuel '91, MP '06

\+ $?\,(??)\,(\alpha/\pi)^4$

Who? When?

\+ $351.2\,(9.7) \times 10^{-8}$ Hadronic Leading Order

Eidelman and Jegerlehner 1995

\+ $27.6\,(5.8) \times 10^{-8}$ Hadronic Higher Order

Narison 2001

\+ $46.9\,(1.2) \times 10^{-8}$ Electroweak

Narison 2001

Adding up, I get:

$a_\tau^{SM} = 117749\,(11) \times 10^{-8}$ using the value

$\alpha = 1/137.03599911\,(46)$ [3.3 ppb] PDG'04

$(m_\tau/m_\mu)^2 \sim 280$: great opportunity to look for New Physics…

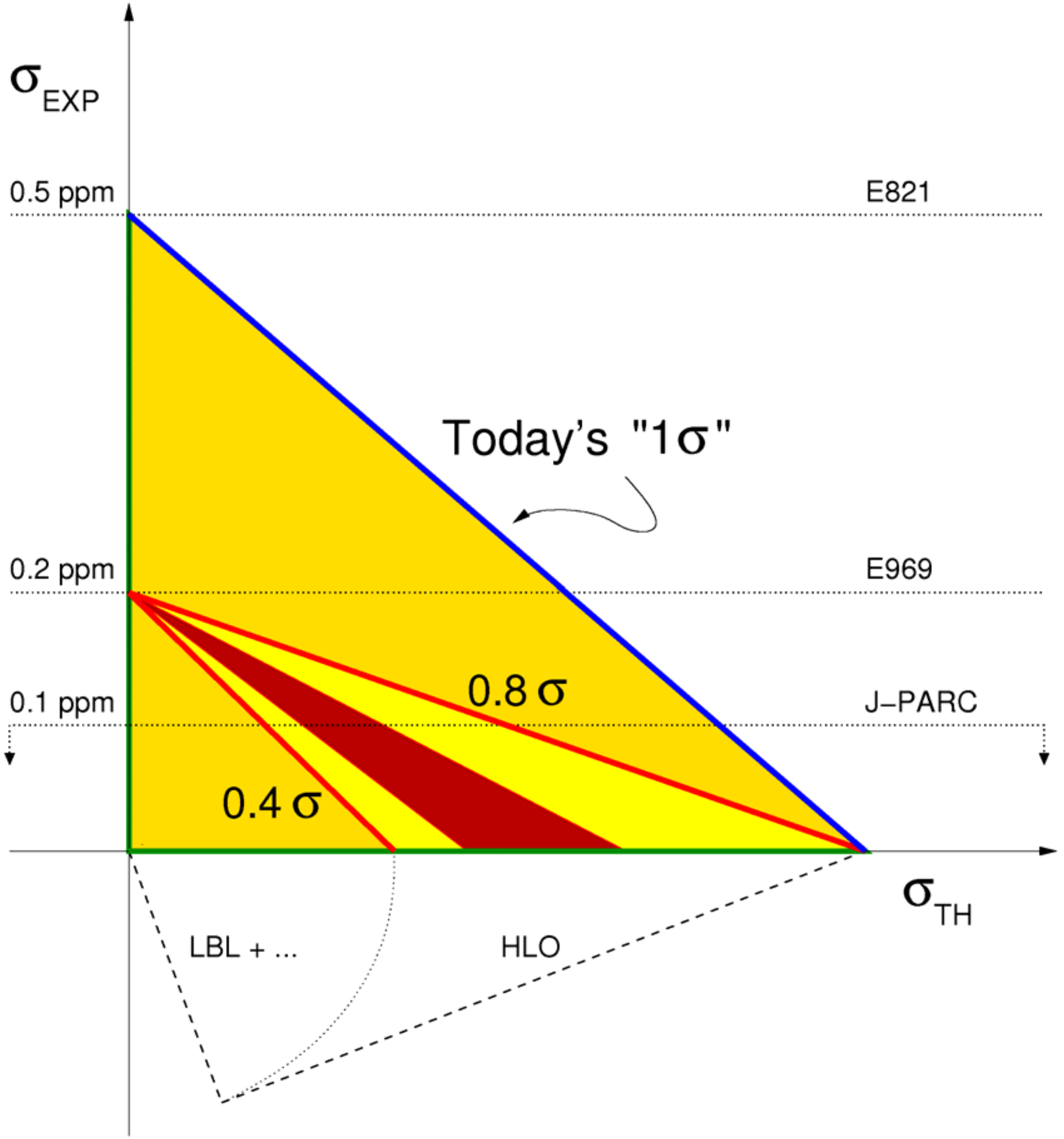
…if only we could measure it!

Conclusions

- Beautiful examples of interplay between theory and experiment. g_e probed at ~ ppt, g_μ at less than ppb! g_τ... theory ahead of exp!
- a_μ: The discrepancies Δ(Exp-SM) range in [2.0, 3.1] σ, according to the values chosen for the hadronic contributions, if e^+e^- data are used (recent CMD2 and SND results are not yet included).
- a_μ: With tau data, Δ(Exp-SM) $\sim$ 1 σ only! The e^+e^- vs tau puzzle is still unsolved. Unaccounted isospin viol. corrections? Problems in the e^+e^- or τ data? Recent news: SND no longer agrees with Aleph; Preliminary Belle's τ data seem to be in better agreement with e^+e^-. More work and data needed from KLOE, Babar, Belle...
- Future: New a_e^{EXP} in June?? a_μ: QED and EW sectors ready for the E969 challenge! The Hadronic sector needs more work and future experimental results: VEPP-2000 (DAFNE-2?). An improvement by a factor of 2 is challenging but possible! The effort is certainly worth the opportunity to unveil (or just constrain) "New Physics" effects!

The End

The future of a_μ?



The Hadronic Contribution to $\alpha(M_Z^2)$

The effective fine-structure constant at the scale s is given by:

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha} \qquad \text{with} \qquad \Delta\alpha = \Delta\alpha_{lep} + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$

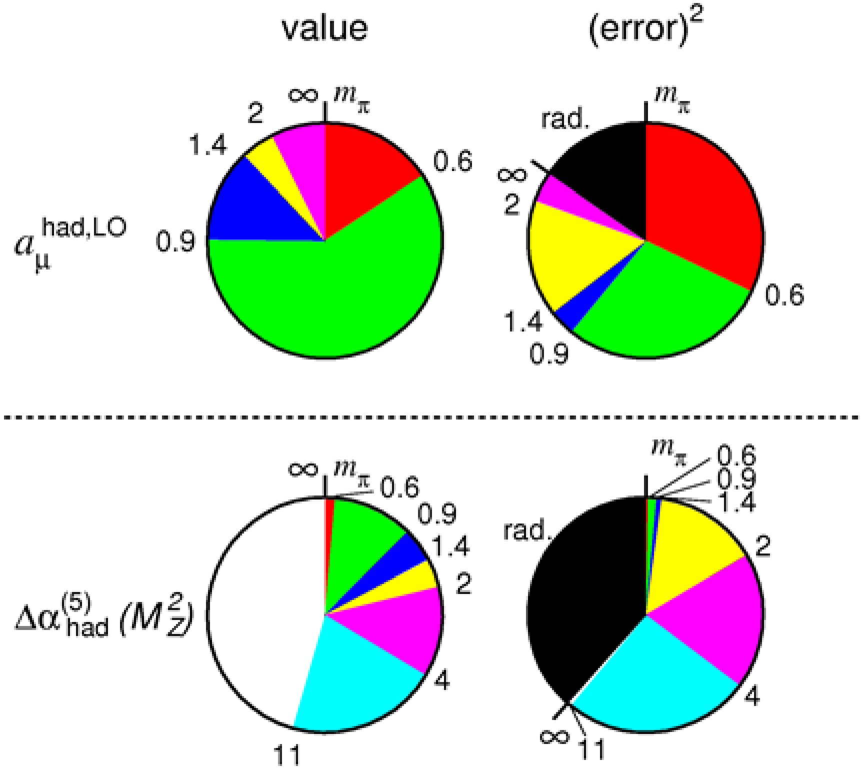
The light quarks part is determined by:

$$\Delta\alpha^{(5)}_{had}(M_z^2) = -\frac{\alpha M_z^2}{3\pi}\int_{s_{thr}}^{\infty} ds \frac{R(s)}{s(s-M_z^2-i\epsilon)}.$$

Progress due to significant improvement of the data (mostly CMD-2 and BES):

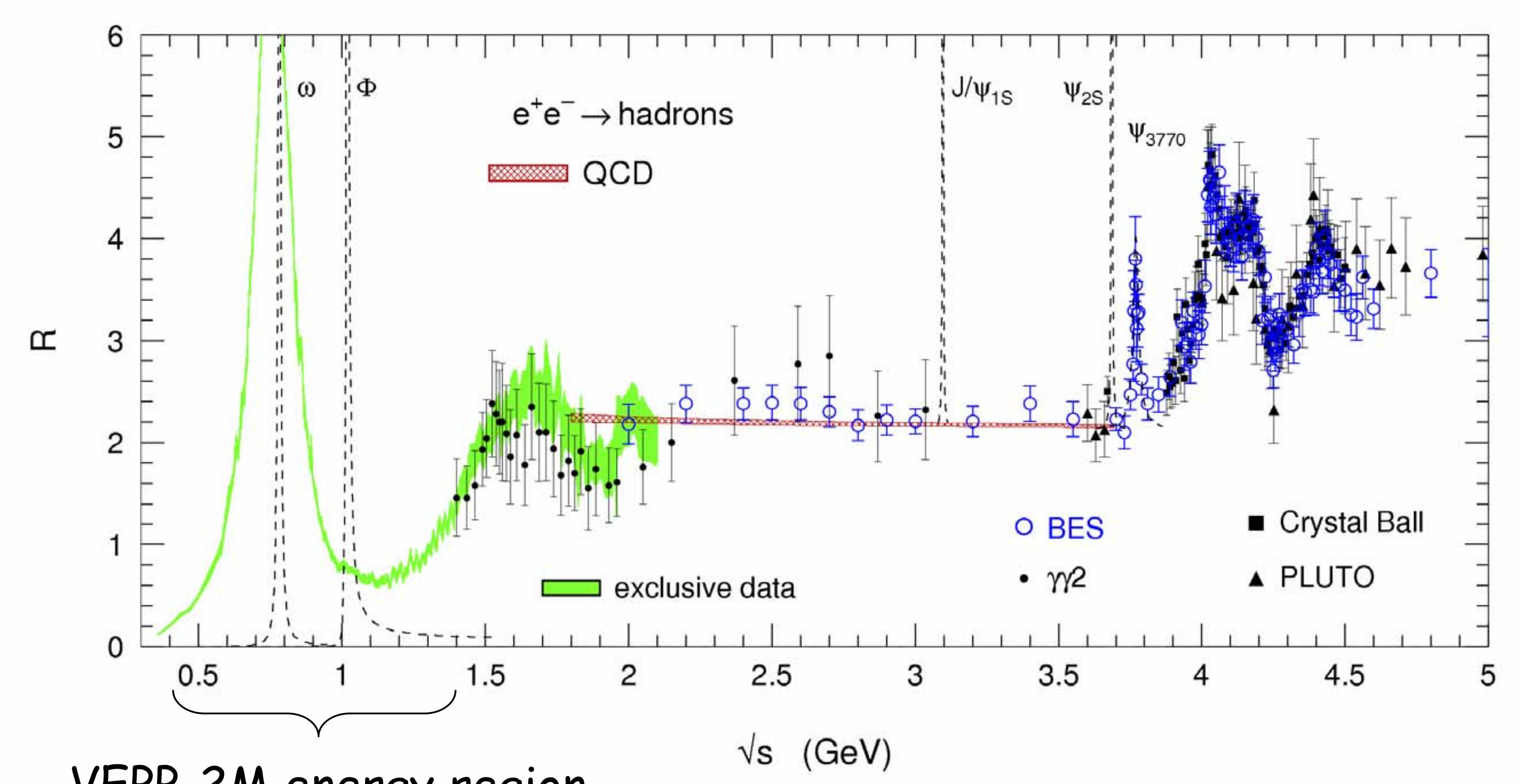
$\Delta\alpha_{had}^{(5)}(M_z^2)$ =

0.02800 (70)	Eidelman, Jegerlehner'95
0.02761 (36)	Burkhardt, Pietrzyk 2001
0.02755 (23)	Hagivara et al., 2004
0.02758 (35)	Burkhardt, Pietrzyk 6-05



Hagivara et al., PRD69 (2004) 093003

R: current status (Logashenko@HEP'05)



VEPP-2M energy region