

# Exceptional Confinement and Deconfinement in $G(2)$ Gauge Theory

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- The exceptional group  $G(2)$
- Confinement and deconfinement in  $G(2)$  Yang-Mills theory
- Higgsing  $G(2)$  to  $SU(3)$
- Deconfinement and Size of the Gauge group
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K. Holland, P. Minkowski, M. Pepe, U.-J.W., Nucl. Phys. B668

(2003) 207

# The Exceptional Group $G(2)$

Group embeddings:  $SO(7) \supset G(2) \supset SU(3)$

Constraints:  $\Omega_{ab} \Omega_{ac} = \delta_{bc}$ ,  $T_{abc} = T_{def} \Omega_{da} \Omega_{eb} \Omega_{fc}$

$$T_{127} = T_{154} = T_{163} = T_{235} = T_{264} = T_{374} = T_{576} = 1$$

Fundamental representation:  $\{7\} = \{3\} \oplus \{\bar{3}\} \oplus \{1\}$

Adjoint representation:  $\{14\} = \{8\} \oplus \{3\} \oplus \{\bar{3}\}$

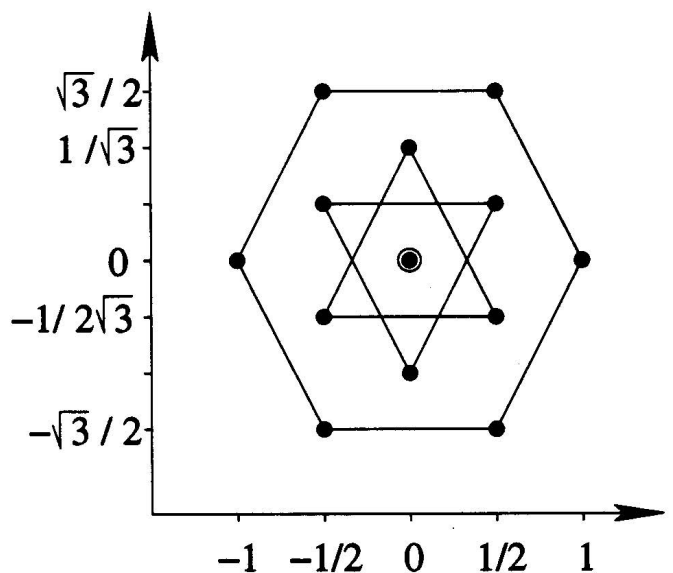
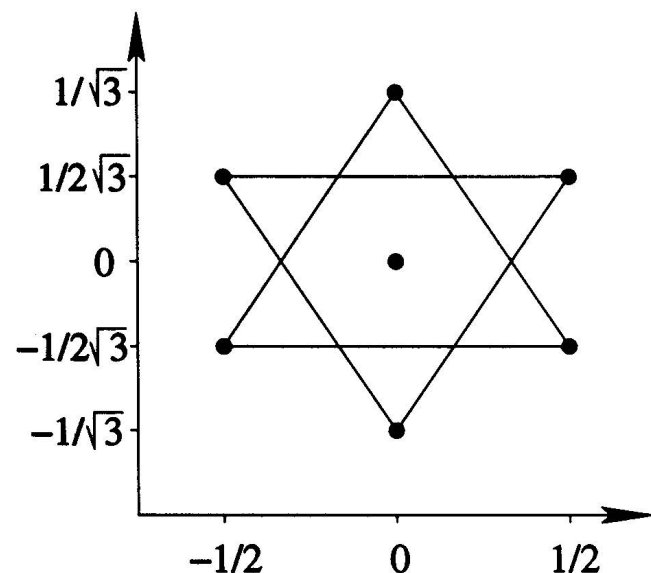
Center:  $G(2)$  has no triality

Reductions:  $\{7\} \otimes \{7\} = \{1\} \oplus \{7\} \oplus \{14\} \oplus \{27\} \Rightarrow$  Mesons

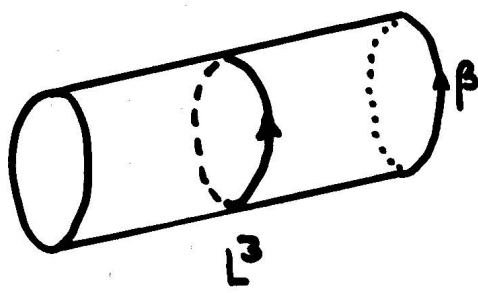
$\{7\} \otimes \{7\} \otimes \{7\} = \{1\} \oplus 4\{7\} \oplus 2\{14\} \oplus 3\{27\} \oplus 2\{64\} \oplus \{77\} \Rightarrow$  Baryons

$\{14\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \{7\} \oplus 5\{14\} \oplus 3\{27\} \oplus 2\{64\} \oplus 4\{77\} \oplus 3\{77'\} \Rightarrow$

Three  $G(2)$  "gluons" can screen a quark  $\Rightarrow$  string breaking



# Deconfinement in $G(2)$ Yang- Mills Theory



Polyakov loop:

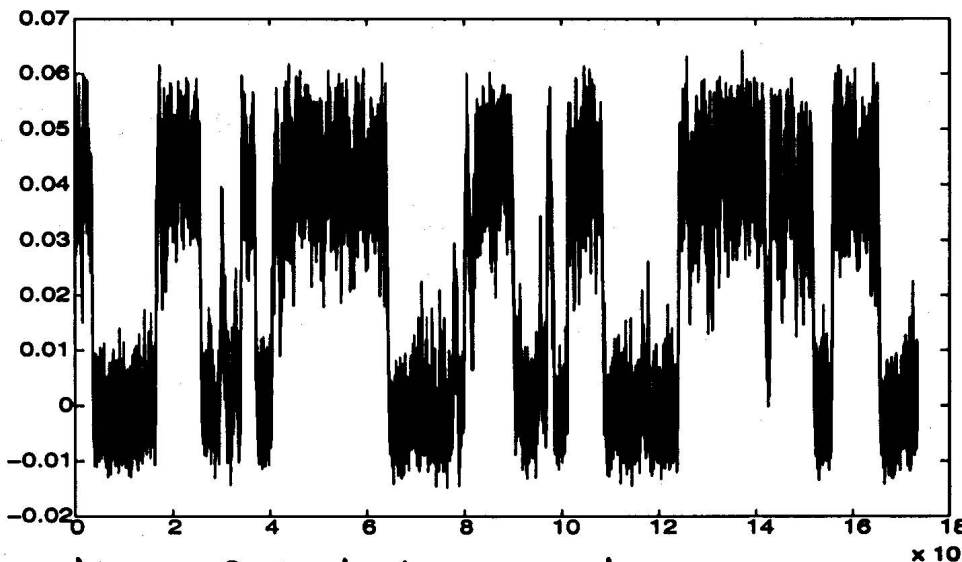
$$\Phi(\vec{x}) = \text{Tr} \mathcal{P} \exp \int_0^\beta dt A_0(\vec{x}, t)$$

Free energy of a static quark:

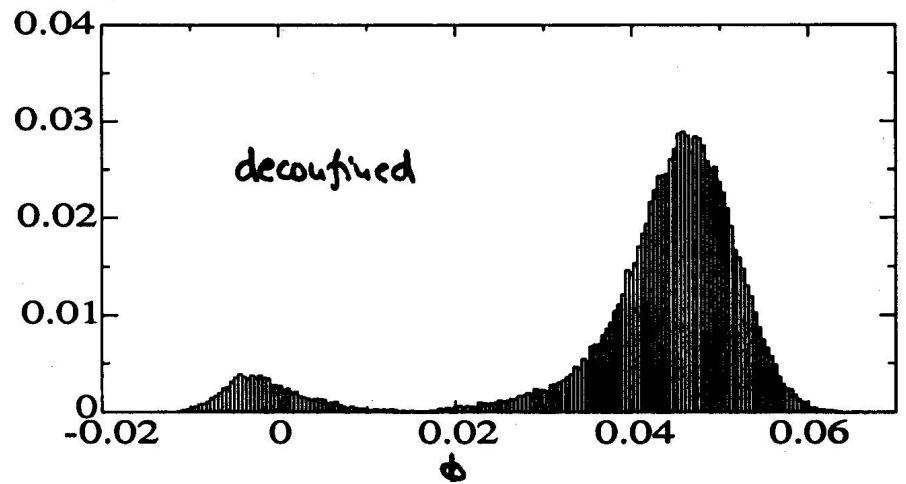
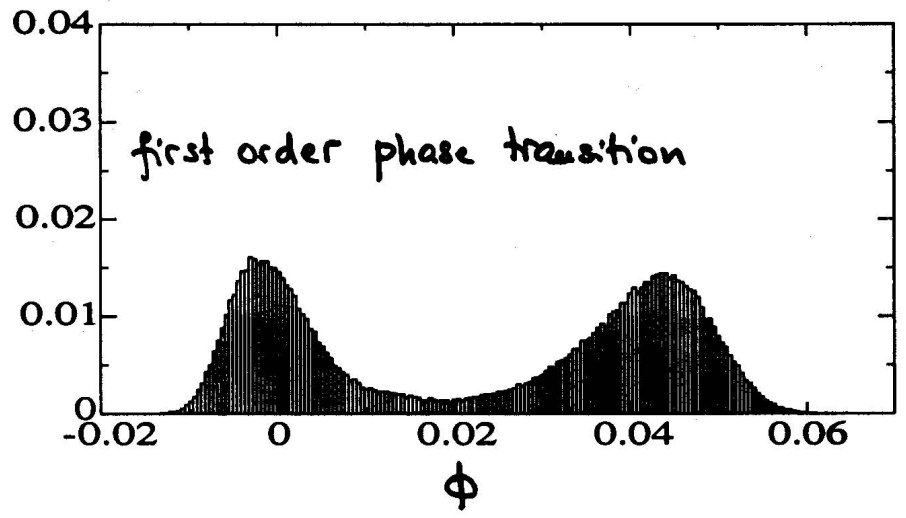
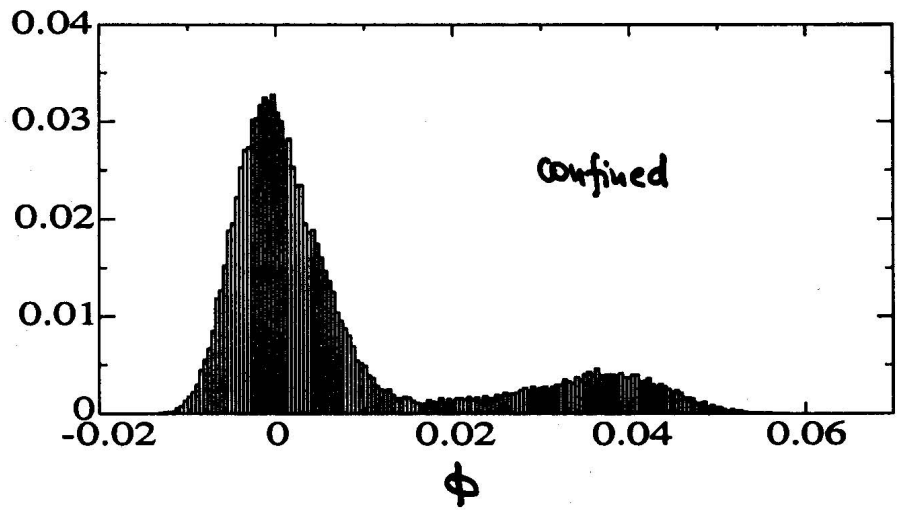
$$\langle \Phi \rangle \sim \exp(-\beta F)$$

Although the center is trivial,  $G(2)$  Yang-Mills theory still has a first order phase transition.

M. Pepe, PoS LAT2005, 017



Monte Carlo history of  $\phi$



# Higgsing $G(2)$ to $SU(3)$

$G(2)$  gauge-Higgs model with fundamental Higgs  $\{7\}$  :

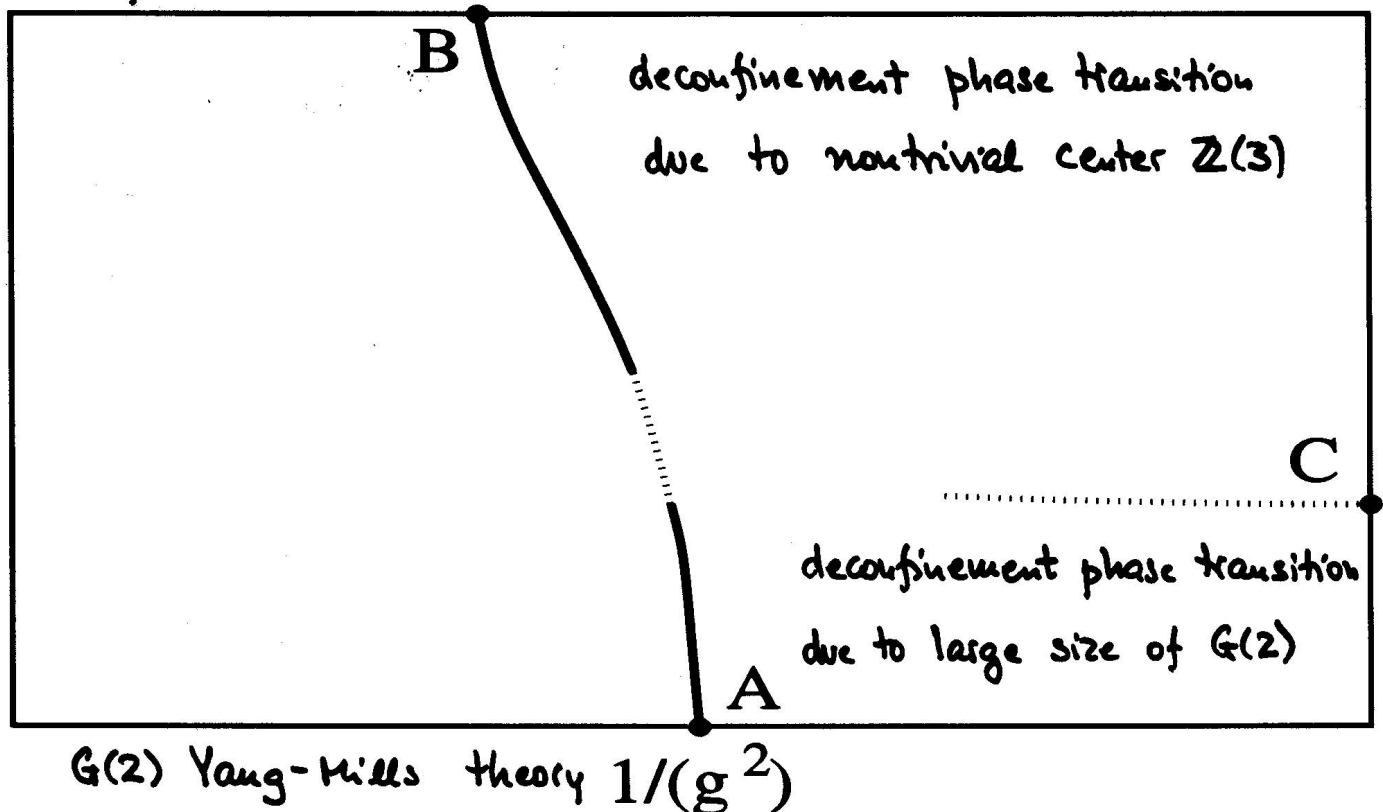
$$S[A_\mu, X] = \int d^4x \left\{ \frac{1}{2g^2} \text{Tr} F_{\mu\nu} F_{\mu\nu} + D_\mu X D_\mu X + V(X) \right\}$$

Mexican hat potential :  $V(X) = \lambda (X^2 - v^2)^2$

Spontaneous symmetry breaking :  $SO(7) \rightarrow SO(6)$   
6 Goldstone bosons

Gauging of  $G(2) \subset SO(7)$  :  $G(2) \rightarrow SU(3)$   
6 additional "gluons" pick up a mass

$SU(3)$  Yang-Mills theory



# Not so Grand Unification

Standard Model with  $N_c = 3$ :

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R, \begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R, \begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R, \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \nu_R, e_R$$

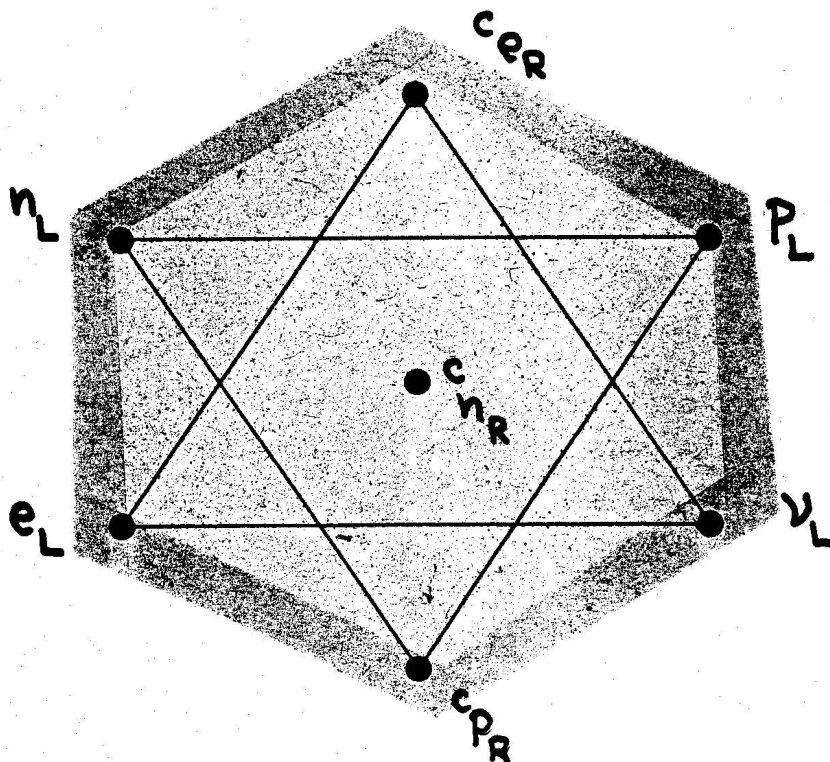
fits into  $\{16\}$  of  $SO(10) \supset SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

Non-Standard Model with  $N_c = 1$ :

$$\begin{pmatrix} p_L \\ n_L \end{pmatrix}, p_R, n_R, \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, e_R, \nu_R$$

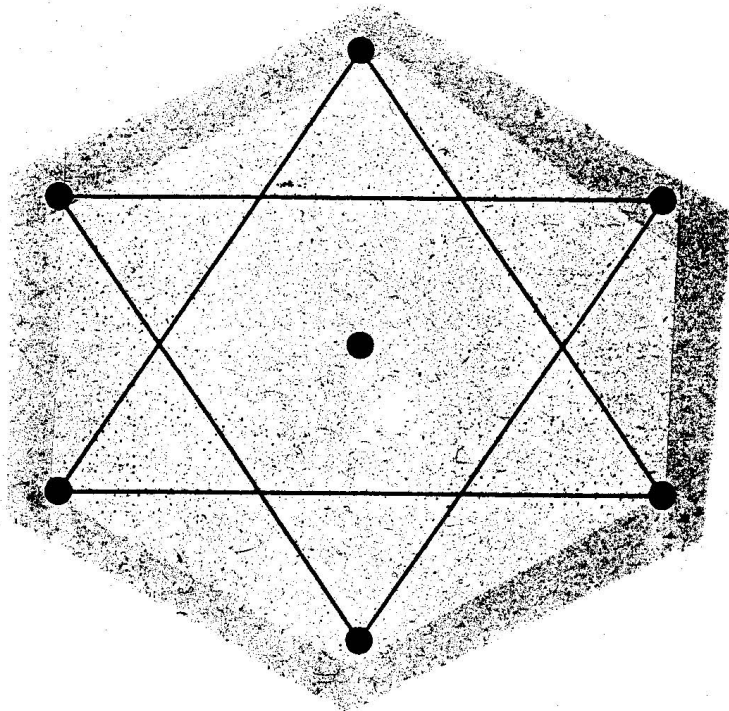
fits into  $\{7\}$  of  $G(2) \supset SU(2)_L \otimes U(1)_Y$

or into  $\{8\}$  of  $SO(7) \supset SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$

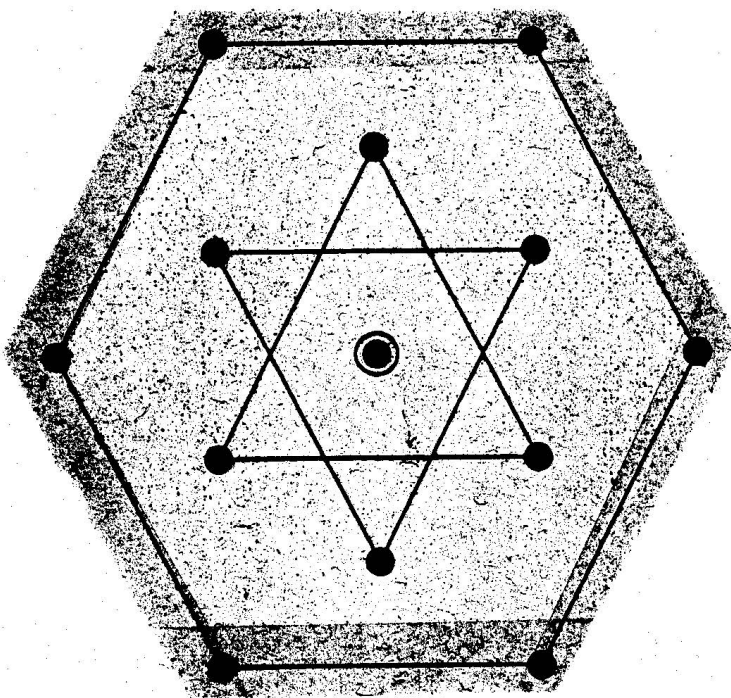


Not a grand idea, because  $G(2)$  and  $SO(7)$  have no chiral representations  $\Rightarrow$  fermion masses run to GUT scale.

Is  $G(2)$  realized in Nature?



{7}



{14}



