Addenda 2008 to variationsQCD2007 (2nd)

On concise hypotheses for the interpretation of a wide scalar resonance as gauge boson binary in QCD $\longrightarrow$ some new analyses

Peter Minkowski

University of Bern

after QCD2008, Montpellier, 6.-13. July 2008

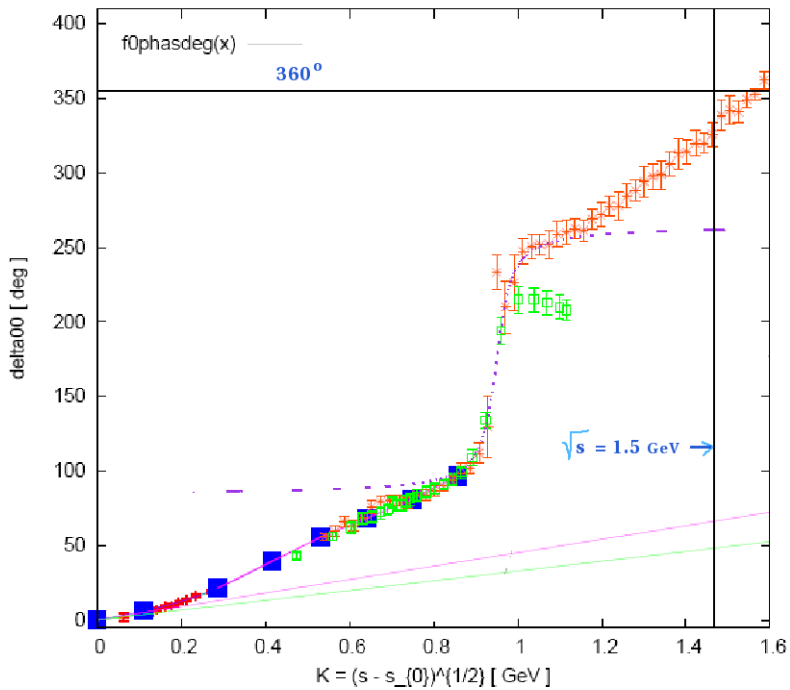

Fig. 1 : The $\pi\pi$, $I = 0$ *ideal* **elastic s-wave from threshold to** $\sim$ 1.625 **GeV .** $\rightarrow$

This work aims to give some answers to questions raised at QCD2008 [1] . Fig. 1 plots the s-wave phase shifts versus $K = \left(M^{\,2}_{\pi\pi} - 4\, m^{\,2}_{\pi} \right)^{1/2}$ **.**

■ : from ref. [2] *Colangelo, Gasser and Leutwyler* **,**

— interpolates ■ , + : from ref. [3] *Na48/2 coll.* **corrected for isospin breaking , I : from ref. [4]** *Protopopescu et al.* **, I : from ref. [5]** *CERN- -Munich coll. ; W. Ochs , thesis 1973* **.**

- - - - : minimal meromorphic parametrization of the influence of f0(980) $\rightarrow$

— , — linear approximations $\delta 00 = 0.5\, a00\, K$ **,** $\leftrightarrow$ $a00\, m_{\pi} = 0.22$ **,** $a00\, m_{\pi} = 0.16$ **.**

ideal **in the caption to figure 1 refers to the limit** $e = 0\,,\ m_d = m_u$ **.**

The rapid phase variation induced by f0(980) defines two fringes, denoted low and high , the two regions

$$\begin{array}{llll} \textbf{low} : & 0 \le K \le\sim 0.9 \text{ GeV} & ; \quad \textbf{high} : & \sim 1.0 \text{ GeV} \le K \le\sim 1.6 \text{ GeV} \\ & 2m_{\pi} \le \sqrt{s} \le\sim 0.94 \text{ GeV} & ; & \sim 1.04 \text{ GeV} \le \sqrt{s} \le\sim 1.625 \text{ GeV} \end{array} \tag{1}$$

The minimal meromorphic parametrization is defined from the complex pole position on the second s - sheet, the K - plane with $\Im K < 0 \quad (s_0 = 4 m_\pi^2)$

$$C_R^2 = (K_R - \tfrac{1}{2} i \gamma_R)^2 = \mathcal{M}_R^2 - s_0 = (M_R - \tfrac{1}{2} i \Gamma_R)^2 - s_0$$

(2)
$$S_{mmp} (K_R , \gamma_R ; K) = \frac{| C_R |^2 - K^2 + i \gamma_R K}{| C_R |^2 - K^2 - i \gamma_R K}$$

The analytically correct derivations from solving the Roy equations in the range limited by Lehmann ellipses are reviewed in ref. [6] . The combination of scattering data , used through absorptive parts between $0.8\, GeV \leq M_{\pi\pi} \leq 2\, GeV$ **with** ***ideal*** $\pi\,\pi$ **scattering lengths, accurately determined through chiral expansions, lead to an apparently most definite prediction and evaluation of pole parametres in the I=0 , s-wave channel in refs. [7]** ***Caprini, Leutwyler*** **and compared with results obtained in ref. [8]** ***Kaminski, Pelaez and Yndurain*** **in eqs. 3 and 4 below .**
While the absolute systematic errors differ by a factor 3 - 4 , this is by far not a proof of the correctness of these results, as discussed subsequently , and in any case does not change the apparent excellent agreement of deduced phase shifts as displayed in figure 1 .

The evaluations following *Caprini* yield 4 typical sets compared below with results from ref. [8]

(3)

$$M_\sigma = 446 \pm 6\ \textbf{(stat)}\ {}^{+40}_{-\ 4}\ \textbf{(syst)}\ \textbf{MeV}$$

$$\Gamma_\sigma = 534 \pm 12\ \textbf{(stat)}\ {}^{+88}_{-66}\ \textbf{(syst)}\ \textbf{MeV}$$

$$M_\sigma = 455 \pm 6\ \textbf{(stat)}\ {}^{+31}_{-13}\ \textbf{(syst)}\ \textbf{MeV}$$

$$\Gamma_\sigma = 556 \pm 12\ \textbf{(stat)}\ {}^{+68}_{-86}\ \textbf{(syst)}\ \textbf{MeV}$$

$$M_\sigma = 458 \pm 6\ \textbf{(stat)}\ {}^{+36}_{-11}\ \textbf{(syst)}\ \textbf{MeV}$$

$$\Gamma_\sigma = 506 \pm 12\ \textbf{(stat)}\ {}^{+78}_{-56}\ \textbf{(syst)}\ \textbf{MeV}$$

$$M_\sigma = 463 \pm 6\ \textbf{(stat)}\ {}^{+31}_{-17}\ \textbf{(syst)}\ \textbf{MeV}$$

$$\Gamma_\sigma = 518 \pm 12\ \textbf{(stat)}\ {}^{+66}_{-68}\ \textbf{(syst)}\ \textbf{MeV}$$

---

(4)

$$M_\sigma = 496 \pm 6\ \textbf{(stat)} \pm 11\ \textbf{(syst)}\ \textbf{MeV}$$

$$\Gamma_\sigma = 516 \pm 16\ \textbf{(stat)} \pm 4\ \textbf{(syst)}\ \textbf{MeV}$$

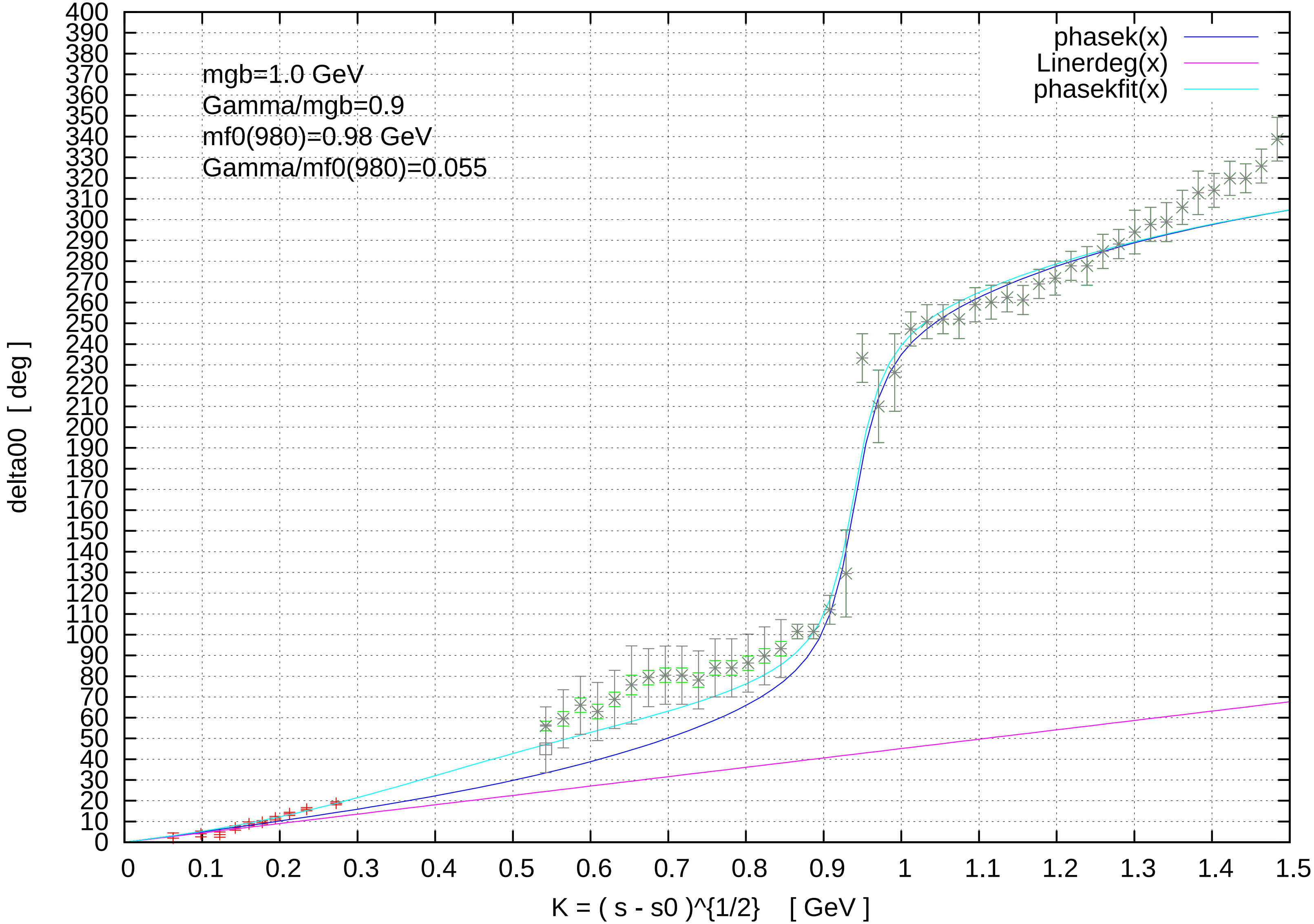

Fig. 2 : The $\pi\pi$, $I = 0$ *ideal* elastic s-wave from threshold to $\sim\ 1.526$ GeV . $\rightarrow$

To figure 2 : $M_{\pi\pi} \equiv \sqrt{s}$ **throughout . + : from ref. [3] ;** — $\delta 00 = 0.5\, a00\, K$ **as in figure 1 .**

: **from ref. [5] as in figure 1 , with enlarged errors for systematics .** $\rightarrow$ [1]

: **from ref. [9] with statistical errors , lowest** $M_{\pi\pi}$ **bin only .**

— : **minimal meromorphic phase from the superposition of f0(980) and gb with masses and widths as indicated in the figure .**

— : **background relative to the minimal meromorphic phase , chosen to follow the lower boundary along the low fringe permitted by** *and* **to maintain optimal agreement in the threshold- and high fringe regions .** $\rightarrow$ [2]

$\rightarrow$ [1] **The systematic error with respect to ref. [5] is chosen by multiplying the quoted statistical error by the factor 2.5 , below** $K = 0.9$ **GeV** ; $M_{\pi\pi} = 0.94$ **GeV . This is justified here considering the difference between the nominal data and the minimal meromorphic phase as shown in figure 2** *and* **from the detailed discussion of errors in ref. [10] .**

$\rightarrow$ [2] **The minimal meromorphic superposition of N resonances with identical** *ideal* **quantum numbers – in any two body channel – corresponds to the multiplication of the individual** $S_{mmp}\,(\,K_{R_\alpha}\,,\,\gamma_{R_\alpha}\,;\,K\,)$ **factors for resonance** $R_\alpha\,;\ \alpha = 1, \cdots, N$ **as defined in eq. 2 .**

(5) $$S^{N}_{mmp}\,(\,K\,) = \prod_{\alpha=1}^{N} S_{mmp}\,(\,K_{R_\alpha}\,,\,\gamma_{R_\alpha}\,;\,K\,)$$

$\rightarrow^{2}$; $\cdots$ **The background introduced above for** $J^{PC} = 0^{++}$, $I = 0$; $\pi\pi \rightarrow \pi\pi$ **is defined** *relative* **to** S^{N}_{mmp} **given in eq. 5**

(6) $$S = S^{N}_{bg} S^{N}_{mmp} \quad ; \quad S^{N}_{bg}(K) = \eta^{N}_{bg}(K) \exp\Big(2\, i\, \delta^{N}_{bg}(K) \Big)$$

It follows from the meromorphic structure of S^{N}_{mmp} **, that the presence in** $T = \frac{1}{2i}(S - 1)$ **of an Adler 0 , for** $-K^{2} = \kappa^{2} = 4\, m^{2}_{\pi} - s > 0$; $\kappa > 0$ **requires a nontrivial background** $\rightarrow S^{N}_{bg} \not\equiv 1$ **. For** — **we use the parametrization**

(7) $$\delta^{2}_{bg}(K) = (K / K_{1})^{3}\, e^{-B K^{2}} \quad ; \quad K_{1} = 0.59 \text{ GeV}, \; B = 4.2 \text{ GeV}^{-2}$$

$$\eta^{2}_{bg}(K) = 1 ; \text{ with modifications particularly for } N = 2 \rightarrow N = 3$$

concentrating on the low fringe region , and coming back to inelasticities in the high fringe below in conjunction with the third resonance f0(1500) and figure 3 . I follow the hypotheses and derivations presented in refs. [11] and concerning the role of f0(1500) in the decays $B \rightarrow K\pi\pi$, $K\overline{K}K$ **[12] in collaboration with Wolfgang Ochs .**

To figure 3 **:** **This is an extension of figure 2 to include the influence of three resonances f0(980) , gb and f0(1500) .** $\rightarrow$

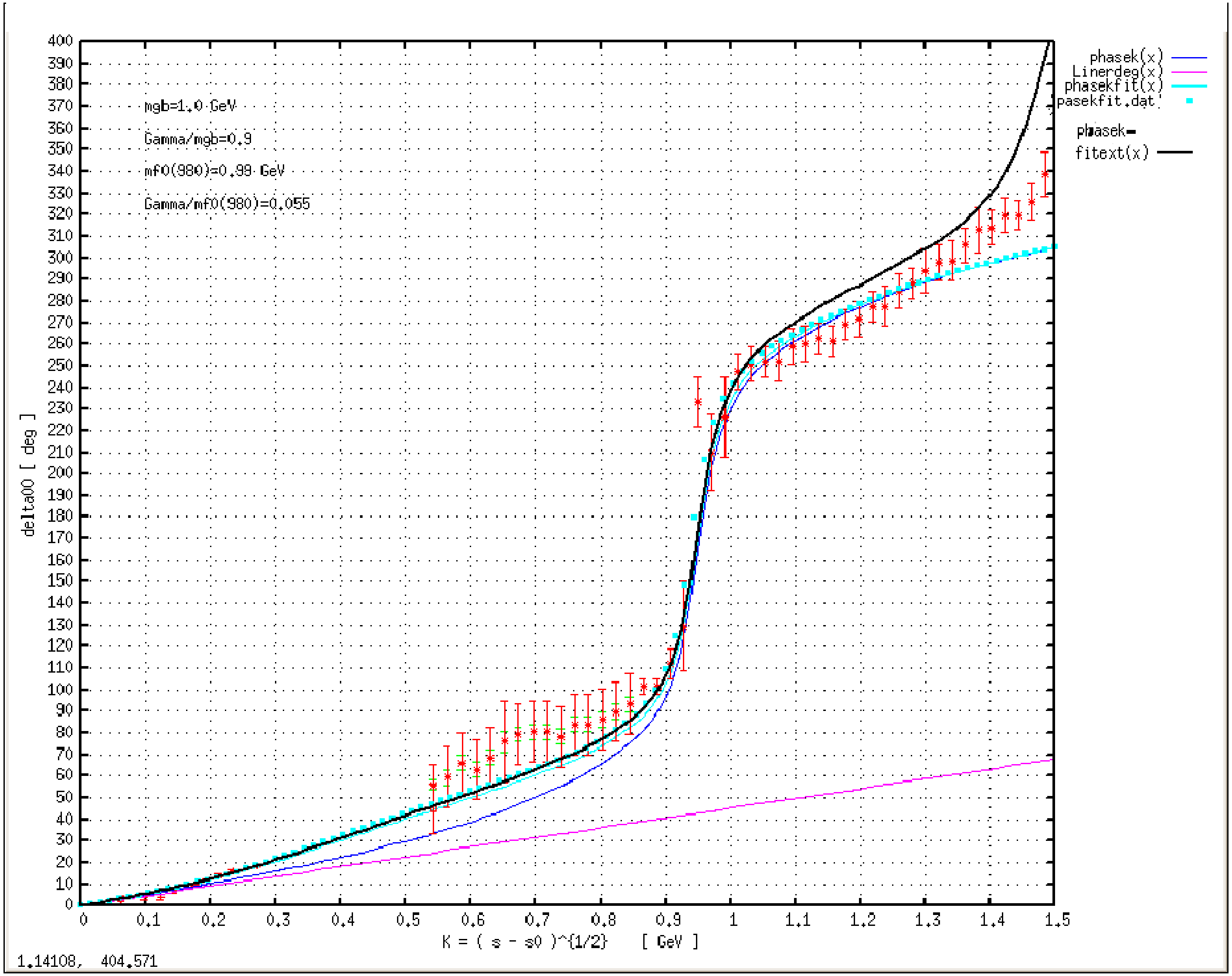

Fig. 3 : The $\pi\pi$, $I = 0$ *ideal* elastic s-wave from threshold to $\sim$ 1.526 GeV . $\leftrightarrow$

To figure 3 : **+ , —** : **as in figure 1 .**

(continued) **, : as in figure 2 except for the color .**

— **: minimal meromorphic phase from the superposition of gb and f0(980) but with different f0 mass** $mf0 = 0.99$ **GeV , same ratio** $\Gamma_{f0} / mf0 = 0.055$ **.**

— **: background phase added with same mass and width parameters as for — and** $K_1 = 0.62$ **GeV (eq. 7) .**

· · · : as — in figure 2 , with $mf0 = 0.98$ **GeV ,** $\Gamma_{f0} / mf0 = 0.055$ **.**

— **: minimal meromorphic phase from the superposition of gb , f0(980)** *and* **f0(1500) with mass and width parameters** $mf0(1500) = 1.51$ **GeV ,** $\Gamma_{f0(1500)} / mf0(1500) = 0.07$ **, and background parameters** $\eta_{bg}^{3} = 1$ **to keep qualitative features of f0(1500) only and** $K_1 = 0.62$ **GeV** , $B = 4.2$ **(eq. 7) .**

The rise of the s-wave phase towards the end of the high fringe region was remarked in ref. [8] .

It formed the entry point of the discussion in ref. [1] .

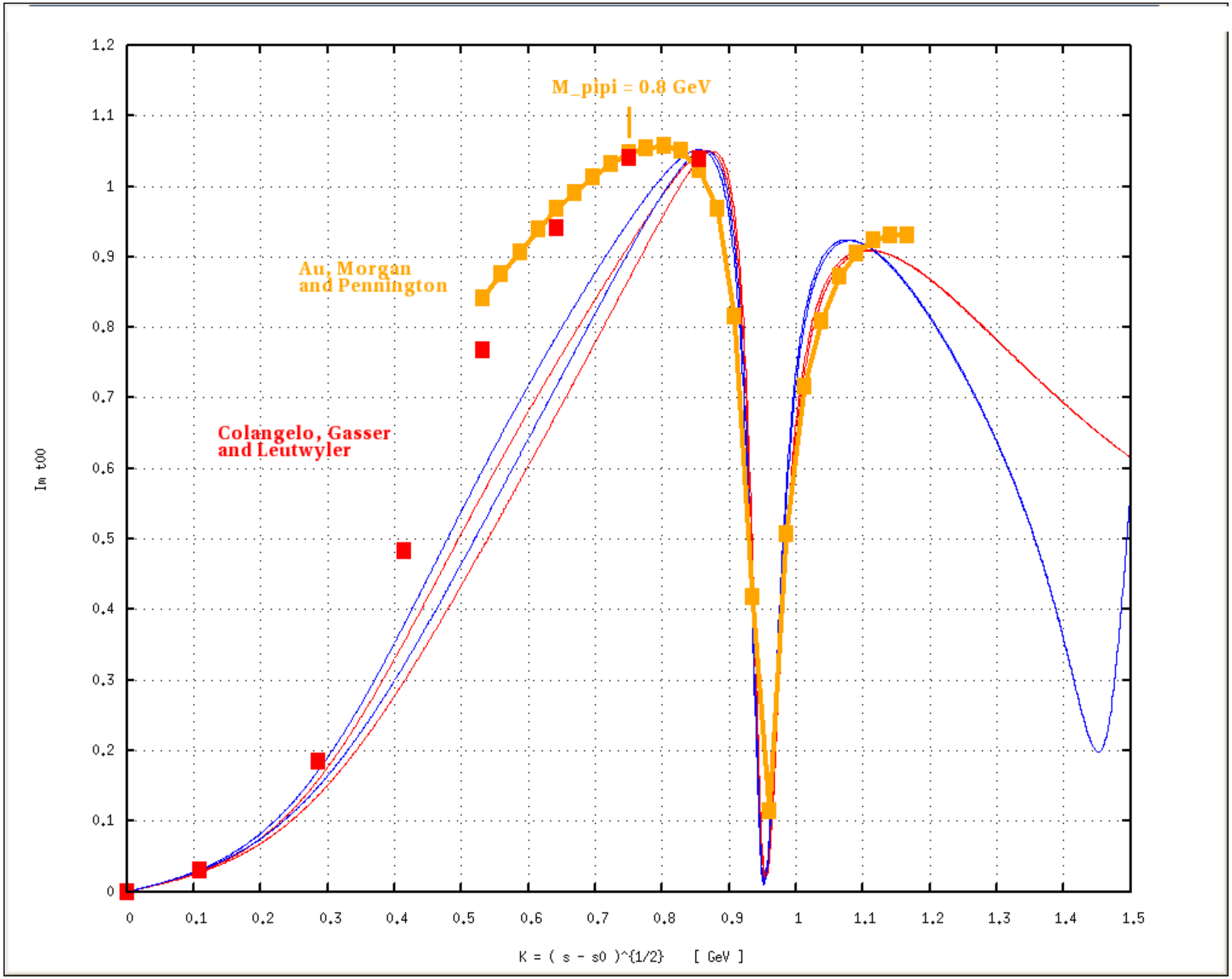

Fig. 4 : The imaginary part of the $\pi\pi$, $I = 0$ ***ideal*** **elastic s-wave from threshold to** $\sim\ 1.526$ **GeV .** $\rightarrow$

To figure 4 : **Here we present aspects of the absorptive part** $\Im t_{00}$
with $t_{00} = (M_{\pi\pi} / K) \frac{1}{2i} \left(S_{bg}^{N} S_{mmp}^{N} - 1 \right) ; N = 2, 3$
and compare with the analyses of refs. [13] *Au, Morgan and Pennington* **[2]** *Colangelo, Gasser and Leutwyler* **and the solution to the Roy equations [14]** *Ananthanarayan, Colangelo, Gasser and Leutwyler* **.**

The resonance parameters used for $N = 2$ **and** $N = 3$ **are**

$mf0$	=	**0.99**	**GeV**	$\Gamma_{f0} / mf0$	=	**0.055**
mgb	=	**1.0**	**GeV**	Γ_{gb} / mgb	=	**0.9**
$mf0(1500)$	=	**1.51**	**GeV**	$\Gamma_{f0(1500)} / mf0(1500)$	=	**0.07**

The inelasticity is extended to include (for $\pi\pi$ **elastic) two** $I = 0\ \pi\pi$ **and** $K\overline{K}$ **two-body channels**

$$\eta_{bg}^{2,3}(K) = \vartheta(K_{th} - K) + \vartheta(K - K_{th})\, a\, e^{-b K' / K'_{15}}$$

$$K'(K) = \left(K^{2} - K_{th}^{2}\right)^{1/2} \quad ; \quad K_{th} = \left(4 m_{K}^{2} - 4 m_{\pi}^{2}\right)^{1/2} \tag{8}$$

$$K'_{15} = \left(mf0(1500)^{2} - 4 m_{K}^{2}\right)^{1/2} \quad ; \quad m_{K} = 0.49565 \text{ GeV}$$

with parameters fixed at $a = 1 \;;\; b = -\log 0.6 = 0.5108 \;;\; m_{\pi} = 0.13957$ **GeV**

No data is used to determine the elasticity parameter – $\eta_{bg}^{2,3}(M_{\pi\pi} = mf0(1500)) \sim 0.6$ **.** $\rightarrow$

To figure 3 (continued) :

— : from $S = S_{bg}^{2,3} \, S_{mmp}^{2}$
with parameters as specified above (and eqs. 7 , 8) as in figure 3
except : upper curve $\longrightarrow$ $K_1 = 0.59$ GeV as in figure 2
lower curve $\longrightarrow$ $K_1 = 0.67$ GeV .

— : from $S = S_{bg}^{2,3} \, S_{mmp}^{3}$
with parameters as specified above (and eqs. 7 , 8) as in figure 3
except : upper curve $\longrightarrow$ $K_1 = 0.59$ GeV as in figure 2
lower curve $\longrightarrow$ $K_1 = 0.67$ GeV .

: $\Im \, t_{00}$ from ref. [13] .

: $\Im \, t_{00}$ from ref. [2] .

Concluding remarks

1) the main analyses of reactions $\pi \mathcal{N} \longrightarrow \pi \pi \mathcal{N} \; (\Delta)$ in refs. [4] , [5] cannot be taken at face value for the derived elastic $\pi \pi$ s-waves within the quoted errors , in both low and high fringe regions (defined in eq. 1) ,

2) derivations and hypotheses discussed in refs. [11] , [12] are basically correct ,

3) claims of a scalar resonance pole in the region within a radius of at least 150 MeV around the position $\sqrt{s} = 500 - \frac{i}{2} \, 500$ MeV on the second s-sheet of elastic $\pi \pi$ scattering are incorrect .

I wish to dedicate this work to the memories of Jan Stern, Francisco Ynduraín and Peter Schlein .

References

[1] P. Minkowski, ’On concise hypotheses for the interpretation of a wide scalar resonance as gauge boson binary in QCD’, contribution to QCD2008 , Montpellier , 6.-13. July 2008 , URL : http://www.mink.itp.unibe.ch in variationsQCD2008.pdf .

[2] G. Colangelo, J. Gasser and H. Leutwyler ’$\pi\pi$ interactions’, Nucl.Phys.B603 (2001) 125-179 , hep-ph/0103088 .

[3] The NA48/2 Collaboration (J.R. Batley et al.) , CERN-PH-EP-2007-035, Oct 2007, 24pp, ’New high statistics measurement of K_{e4} decay form factors and $\pi\pi$ scattering phase shifts’ , Eur.Phys.J.C54 (2008) 411-423 .

References

[4] S.D. Protopopescu, M. Alston-Garnjost, A. Barbaro-Galtieri, Stanley M. Flatte, J.H. Friedman, T.A. Lasinski, G.R. Lynch, M.S. Rabin, F.T. Solmitz, ' $\pi\pi$ partial wave analysis from reactions $\pi^+ p \to \pi^+\pi^-\Delta^{++}$ and $\pi^+ p \to K^+K^-\Delta^{++}$ at 7.1-GeV/c', Phys.Rev.D7 (1973) 1279 .

[5] B. Hyams, C. Jones and P. Weilhammer, W. Blum, H. Dietl, G. Grayer, W. Koch, E. Lorenz, G. Lütjens, W. Männer, J. Meissburger, W. Ochs, U. Stierlin and F. Wagner, '$\pi\pi$ phase-shift analysis from 600 - 1900 MeV' , Nuclear Physics B64 (1973) 134-162 .
W. Ochs, University of Munich thesis 1973, unpublished.

References

[6] I. Caprini, G. Colangelo and H. Leutwyler, 'Theoretical aspects of the pion-pion interaction', Int.J.Mod.Phys.A21 (2006) 954 , hep-ph/0509266 .

[7] I. Caprini, 'Finding the sigma pole by analytic extrapolation of $\pi\pi$ scattering data', Phys.Rev.D77 (2008) 114019 , arXiv:0804.3504 [hep-ph] .
H. Leutwyler, 'Model independent determination of the sigma pole', arXiv:0804.3182 [hep-ph] .

[8] R. Kaminski, J. Pelaez and F. Yndurain, 'The Pion-pion scattering amplitude. III. Improving the analysis with forward dispersion relations and Roy equations', Phys.Rev.D77 (2008) 054015 , arXiv:0710.1150 [hep-ph] .

References

[9] R. Kaminski, L. Lesniak and K. Rybicki, 'Separation of S wave pseudoscalar and pseudovector amplitudes in π^-p (polarized) $\longrightarrow \pi^+\pi^-n$ reaction on polarized target', INP-1730-PH, June 1996, 35pp., Z.Phys.C74 (1997) 79-91 , hep-ph/9606362 .

[10] G. Grayer et al., CERN-Munich collaboration, 'High statistics study of $\pi^-p \longrightarrow \pi^+\pi^-n$: apparatus, method of analysis and general features of results at 17 GeV/c ', Nucl. Phys. B75 (1974) 189 .

References

[11] P. Minkowski and W. Ochs, ’Identification of the glueballs and the scalar meson nonet of lowest mass’ , BUTP-98-27, MPI-PHT-98-89, Nov 1998. 65pp., Published in Eur.Phys.J.C9 (1999) 283-312 , hep-ph/9811518 .

[12] P. Minkowski and W. Ochs, ’B decays into light scalar particles and glueball’, Eur.Phys.J.C39 (2005) 71, hep-ph/0404194 .

[13] K.L. Au, D. Morgan and M.R. Pennington, ’Meson Dynamics Beyond the Quark Model: A Study of Final State Interactions’, RAL-86-076, DTP-86-9, (Received Sep 1986). 81pp., Phys.Rev.D35 (1987) 1633 .

References

[14] B. Ananthanarayan, G. Colangelo, J. Gasser and H. Leutwyler, 'Roy equation analysis of pi pi scattering', Phys.Rept.353 (2001) 207-279, hep-ph/0005297 .