Addenda 2008 to variationsQCD2007 (2nd)

On concise hypotheses for the interpretation of a wide scalar resonance as gauge boson binary in QCD → some new analyses

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This work aims to give some answers to questions raised at QCD2008 [1]. Fig. 1 plots the s-wave phase shifts versus $K = (M_{\pi\pi}^2 - 4m_{\pi}^2)^{1/2}$.

- from ref. [2] Colangelo, Gasser and Leutwyler, interpolates

- minimal meromorphic parametrization of the influence of f0(980)

- linear approximations $\delta_{00} = 0.5a_{00}K$, $a_{00}m_{\pi} = 0.22$, $a_{00}m_{\pi} = 0.16$.

ideal in the caption to figure 1 refers to the limit $e = 0$, $m_d = m_u$.

The rapid phase variation induced by f0(980) defines two fringes, denoted low and high, the two regions

low : $0 \leq K \leq \sim 0.9$ GeV ; high : $\sim 1.0$ GeV $\leq K \leq \sim 1.6$ GeV

$2m_{\pi} \leq \sqrt{s} \leq \sim 0.94$ GeV ; $\sim 1.04$ GeV $\leq \sqrt{s} \leq \sim 1.625$ GeV

Fig. 1 : The $\pi\pi$, $I = 0$ ideal elastic s-wave from threshold to $\sim 1.625$ GeV.
The minimal meromorphic parametrization is defined from the complex pole position on the second s-sheet, the K-plane with \(\Im K < 0\) \((s_0 = 4m^2_{\pi})\)

\[
C^2_R = \left( K_R - \frac{1}{2} i \gamma_R \right)^2 = M^2_R - s_0 = \left( M_R - \frac{1}{2} i \Gamma_R \right)^2 - s_0
\]

\(\text{(2)}\)

\[
S_{mmp} (K_R, \gamma_R; K) = \frac{|C_R|^2 - K^2 + i \gamma_R K}{|C_R|^2 - K^2 - i \gamma_R K}
\]

The analytically correct derivations from solving the Roy equations in the range limited by Lehmann ellipses are reviewed in ref. [6]. The combination of scattering data, used through absorptive parts between \(0.8 \text{ GeV} \leq M_{\pi\pi} \leq 2 \text{ GeV}\) with ideal \(\pi\pi\) scattering lengths, accurately determined through chiral expansions, lead to an apparently most definite prediction and evaluation of pole parametres in the \(l=0\), s-wave channel in refs. [7] Caprini, Leutwyler and compared with results obtained in ref. [8] Kaminski, Pelaez and Yndurain in eqs. 3 and 4 below.

While the absolute systematic errors differ by a factor 3 - 4, this is by far not a proof of the correctness of these results, as discussed subsequently, and in any case does not change the apparent excellent agreement of deduced phase shifts as displayed in figure 1.
The evaluations following Caprini yield 4 typical sets compared below with results from ref. [8]

\[
\begin{align*}
M_\sigma &= 446 \pm 6 \text{ (stat)} \pm 40 \text{ (syst)} \text{ MeV} \\
\Gamma_\sigma &= 534 \pm 12 \text{ (stat)} \pm 88 \text{ (syst)} \text{ MeV} \\
M_\sigma &= 455 \pm 6 \text{ (stat)} \pm 31 \text{ (syst)} \text{ MeV} \\
\Gamma_\sigma &= 556 \pm 12 \text{ (stat)} \pm 68 \text{ (syst)} \text{ MeV} \\
M_\sigma &= 458 \pm 6 \text{ (stat)} \pm 36 \text{ (syst)} \text{ MeV} \\
\Gamma_\sigma &= 506 \pm 12 \text{ (stat)} \pm 78 \text{ (syst)} \text{ MeV} \\
M_\sigma &= 463 \pm 6 \text{ (stat)} \pm 31 \text{ (syst)} \text{ MeV} \\
\Gamma_\sigma &= 518 \pm 12 \text{ (stat)} \pm 66 \text{ (syst)} \text{ MeV} \\
\end{align*}
\]

\[(3)\]

\[
\begin{align*}
M_\sigma &= 496 \pm 6 \text{ (stat)} \pm 11 \text{ (syst)} \text{ MeV} \\
\Gamma_\sigma &= 516 \pm 16 \text{ (stat)} \pm 4 \text{ (syst)} \text{ MeV} \\
\end{align*}
\]

\[(4)\]
Fig. 2: The $\pi\pi$, $I = 0$ ideal elastic s-wave from threshold to $\sim 1.526$ GeV.
To figure 2: \( M_{\pi\pi} \equiv \sqrt{s} \) throughout. +: from ref. [3]; \( \delta_{00} = 0.5 \ a_{00} \ K \) as in figure 1.

\( \square \): from ref. [5] as in figure 1, with enlarged errors for systematics.

\( \blacksquare \): from ref. [9] with statistical errors, lowest \( M_{\pi\pi} \) bin only.

\( \longrightarrow \): minimal meromorphic phase from the superposition of \( f_0(980) \) and gb with masses and widths as indicated in the figure.

\( \longrightarrow \): background relative to the minimal meromorphic phase, chosen to follow the lower boundary along the low fringe permitted by \( \square \) and to maintain optimal agreement in the threshold- and high fringe regions.

\( \rightarrow 1 \) The systematic error with respect to ref. [5] is chosen by multiplying the quoted statistical error by the factor 2.5, below \( K = 0.9 \ \text{GeV} \); \( M_{\pi\pi} = 0.94 \ \text{GeV} \). This is justified here considering the difference between the nominal data and the minimal meromorphic phase as shown in figure 2 and from the detailed discussion of errors in ref. [10].

\( \rightarrow 2 \) The minimal meromorphic superposition of \( N \) resonances with identical \textit{ideal} quantum numbers – in any two body channel – corresponds to the multiplication of the individual \( S_{mmp}(K_{R_{\alpha}}, \gamma_{R_{\alpha}}; K) \) factors for resonance \( R_{\alpha}; \alpha = 1, \ldots, N \) as defined in eq. 2.

\( S_{mmp}^{N}(K) = \prod_{\alpha=1}^{N} S_{mmp}(K_{R_{\alpha}}, \gamma_{R_{\alpha}}; K) \)
The background introduced above for $J^{PC} = 0^{++}$, $I = 0$; $\pi\pi \to \pi\pi$ is defined relative to $S_{mmp}^N$ given in eq. (5)

$$S = S_{bg}^N S_{mmp}^N ; \quad S_{bg}^N (K) = \eta_{bg}^N (K) \exp \left( 2i \delta_{bg}^N (K) \right)$$

It follows from the meromorphic structure of $S_{mmp}^N$, that the presence in $T = \frac{1}{2i} (S - 1)$ of an Adler 0, for $-K^2 = \kappa^2 = 4m^2_\pi - s > 0$; $\kappa > 0$ requires a nontrivial background $\to S_{bg}^N \not\equiv 1$. For $\to$ we use the parametrization

$$\delta_{bg}^2 (K) = (K / K_1)^3 e^{-B K^2} ; \quad K_1 = 0.59 \text{ GeV}, \quad B = 4.2 \text{ GeV}^{-2}$$

$$\eta_{bg}^2 (K) = 1 ; \text{ with modifications particularly for } N = 2 \to N = 3$$

concentrating on the low fringe region, and coming back to inelasticities in the high fringe below in conjunction with the third resonance f0(1500) and figure 3. I follow the hypotheses and derivations presented in refs. [11] and concerning the role of f0(1500) in the decays $B \to K \pi\pi$, $K \overline{K}K$ [12] in collaboration with Wolfgang Ochs.

To figure 3: This is an extension of figure 2 to include the influence of three resonances f0(980), gb and f0(1500).
Fig. 3: The $\pi\pi$, $I = 0$ ideal elastic s-wave from threshold to $\sim 1.526$ GeV.
To figure 3 (continued): +, —: as in figure 1.

□: as in figure 2 except for the color.

- - - -: minimal meromorphic phase from the superposition of gb

and f0(980) but with different f0 mass $m_f0 = 0.99 \text{ GeV}$,

same ratio $\frac{\Gamma_{f0}}{m_f0} = 0.055$.

- - - -: background phase added with same mass and width

parameters as for — and $K_1 = 0.62 \text{ GeV}$ (eq. 7).

.-.-.: as — in figure 2,

with $m_f0 = 0.98 \text{ GeV}$, $\frac{\Gamma_{f0}}{m_f0} = 0.055$.

- - - -: minimal meromorphic phase from the superposition of

gb, f0(980) and f0(1500) with mass and width parameters

$m_f0(1500) = 1.51 \text{ GeV}$, $\frac{\Gamma_{f0(1500)}}{m_f0(1500)} = 0.07$,

and background parameters $\eta_{bg}^3 = 1$ to keep qualitative features

of f0(1500) only and $K_1 = 0.62 \text{ GeV}$, $B = 4.2$ (eq. 7).

The rise of the s-wave phase towards the end of the

high fringe region was remarked in ref. [8].

It formed the entry point of the discussion in ref. [1].
Fig. 4: The imaginary part of the $\pi\pi$, $I = 0$ ideal elastic s-wave from threshold to $\sim 1.526$ GeV.
To figure 4: Here we present aspects of the absorptive part \( \Im t_{00} \) with

\[
t_{00} = \left( \frac{M_{\pi\pi}}{K} \right) \frac{1}{2i} \left( S^N_{bg} S^N_{mmp} - 1 \right) ; N = 2, 3
\]

and compare with the analyses of refs. [13] Au, Morgan and Pennington


Ananthanarayan, Colangelo, Gasser and Leutwyler.

The resonance parameters used for \( N = 2 \) and \( N = 3 \) are

\[
\begin{align*}
mf_0 & = 0.99 \text{ GeV} & \Gamma_{f0} / mf_0 & = 0.055 \\
mgb & = 1.0 \text{ GeV} & \Gamma_{gb} / mgb & = 0.9 \\
mf_0(1500) & = 1.51 \text{ GeV} & \Gamma_{f0(1500)} / mf_0(1500) & = 0.07
\end{align*}
\]

The inelasticity is extended to include (for \( \pi\pi \) elastic) two \( I = 0 \) \( \pi\pi \) and \( K\bar{K} \) two-body channels

\[
\eta^{2,3}_{bg}(K) = \vartheta(K_{th} - K) + \vartheta(K - K_{th}) a e^{-b K'/K_{15}}
\]

\[
K'(K) = (K^2 - K_{th}^2)^{1/2} ; K_{th} = (4m_k^2 - 4m_\pi^2)^{1/2}
\]

(8)

\[
K_{15}' = (mf_0(1500)^2 - 4m_k^2)^{1/2} ; m_k = 0.49565 \text{ GeV}
\]

with parameters fixed at \( a = 1 ; b = -\log 0.6 = 0.5108 ; m_\pi = 0.13957 \text{ GeV} \)

No data is used to determine the elasticity parameter – \( \eta^{2,3}_{bg}(M_{\pi\pi} = mf_0(1500)) \sim 0.6 \). →
To figure 3 (continued) : from $S = S_{bg}^{2,3} S_{mmp}^{2}$ with parameters as specified above (and eqs. 7, 8) as in figure 3 except: upper curve $\rightarrow K_1 = 0.59 \text{ GeV}$ as in figure 2 lower curve $\rightarrow K_1 = 0.67 \text{ GeV}$.

: from $S = S_{bg}^{2,3} S_{mmp}^{3}$ with parameters as specified above (and eqs. 7, 8) as in figure 3 except: upper curve $\rightarrow K_1 = 0.59 \text{ GeV}$ as in figure 2 lower curve $\rightarrow K_1 = 0.67 \text{ GeV}$.

: $t_{00}$ from ref. [13].

: $t_{00}$ from ref. [2].

Concluding remarks

1) the main analyses of reactions $\pi N \rightarrow \pi\pi N$ ($\Delta$) in refs. [4], [5] cannot be taken at face value for the derived elastic $\pi\pi$ s-waves within the quoted errors, in both low and high fringe regions (defined in eq. 1),

2) derivations and hypotheses discussed in refs. [11], [12] are basically correct,

3) claims of a scalar resonance pole in the region within a radius of at least 150 MeV around the position $\sqrt{s} = 500 - \frac{i}{2} 500 \text{ MeV}$ on the second s-sheet of elastic $\pi\pi$ scattering are incorrect.

I wish to dedicate this work to the memories of Jan Stern, Francisco Yndurain and Peter Schlein.
References


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