ON THE ANOMALOUS DIVERGENCE OF THE DILATATION CURRENT IN GAUGE THEORIES

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Abstract

It is shown that gauge theories involving massless fermions and for which local gauge invariance is preserved (e.g. massless quantum electrodynamics (QED) and/or massless quantum chromodynamics (QCD)) exhibit a universal anomaly in the Ward identity for the dilatation current.

Scaling laws at short distances continue to challenge the understanding of quantum field theory [1]. The short distance behaviour of fields and composite operators and the renormalization constants reciprocally determine each other in a nonperturbative way, even for asymptotically free nonabelian gauge theories [2].

In deep inelastic lepton-hadron scattering or e^+e^--annihilation into hadrons the short distances are prepared by weak- or electromagnetic probes whereby the correspondingly rescaled strong interaction coupling constant becomes physically observable (in principle). The phenomenon of approximate Björken scaling [3] interpreted within QCD (quantum chromodynamics) [4] reflects the small value of this coupling constant [5].

The true asymptotic behaviour of current products near the light cone reveals the breakdown of scale invariance yet to be clearly established or disproved by experiment [6]. Soft scale breaking terms i.e. mass terms do not determine the short distance behaviour except for providing an absolute meaning to the numerical value of coupling constants e.g. the charge in QED.

The sliding scale renormalization introducing a redundant normalization mass M and the dimensionless coupling constants

$$g_{1M}, \dots g_{nM} \; ; \; \varepsilon_1 = \frac{m_1}{M}, \dots , \varepsilon_\nu = \frac{m_\nu}{M} .$$

where $m_1, \dots m_\nu$ denote mass parameters appearing in the Lagrangian, enables one to reach the limit $\varepsilon_i \to 0$.

The renormalization group equation eliminates the redundancy of the arbitrary scale M. In the limit $\varepsilon_i \to 0$ rescaling is governed by the sliding-scale effective coupling constants $\bar{g} = (\bar{g}_1 \ldots \bar{g}_n)$ satisfying the Callan-Symanzik equation [1)]

$$M \frac{d}{dM} \bar{g} = \beta(\bar{g}) \; ; \; \beta = (\beta_1, \ldots \beta_n)$$

$$\bar{g}(M_0, g_{M_0}) = g_{M_0} \tag{1}$$

Let the plane of trajectories orthogonal to the vector field β, passing through a given point $g(0)$ be represented by

$$F(g_1, \ldots, g_n) = \rho \qquad \rho = F(g(0)) \tag{2}$$

and $\tilde{g} = \tilde{g}_1, \ldots \tilde{g}_{n-1}$ denote a set of n-1 variables describing a point on the plane $F(g) = \rho$.

$\tilde{g}$ are scale invariants of the massless theory, they satisfy the equation

$$\beta_i \frac{\partial}{\partial g_i} \tilde{g} = 0$$

if reexpressed as functions of $g_1 \ldots g_n$.

Eq. (1) reduces to the one dimensional rescaling equation

$$M \frac{d}{dM} \bar{\rho} = \tilde{\beta}(\bar{\rho}, \tilde{g}) = F_{,g_i} \beta_i \Big/_{F(g) = \bar{\rho}}$$

$$\bar{\rho}(M_0, \rho_{M_0}) = \rho_{M_0} \tag{3}$$

Eq. (3) permits to define a renormalization group invariant mass M*
(unless $\beta(\rho_M, \tilde{g}) = 0$)

$$M^* = M \exp - \int_{\rho_M}^{\rho^*} d\rho' \left[\tilde{\beta}(\rho', \tilde{g})\right]^{-1} \tag{4}$$

The very fact that M* can be defined implies a breakdown of scale invariance. For the latter to hold it is necessary that there exists a zero of β:

$$\beta_i(g) = 0 \quad \text{for} \quad g = (g_1^*, \cdots g_n^*) \tag{5}$$

Then the condition

$$g_i(M) = g_i^*$$

independent of M, defines a scale invariant theory [1] with no connection to perturbation theory except for $g^* = 0$.

For $g(M) \neq g^*$ (and $g(M) \neq 0$) scale invariance is broken even though the classical field equations do exhibit it provided mass terms are absent.

The trace of the energy momentum tensor develops an anomalous part [8], [9] of the general form

$$\vartheta^\mu_\mu = \sum_i \left(\beta_i / g_i\right) \cdot \vartheta_i \quad + \text{mass terms} \tag{6}$$

The renormalization group equations for gauge invariant operators

$$\left[M\frac{\partial}{\partial M} + \beta_a \frac{\partial}{\partial g_a} + \sum \gamma_{O_i}\right] \langle\Omega| T^*\left(\vartheta^\mu_\mu(0)\, O_1(x_1) \ldots O_n(x_n)\right)|\Omega\rangle = 0$$

$$\left[M\frac{\partial}{\partial M} + \beta_a \frac{\partial}{\partial g_a} + \sum \gamma_{O_i}\right] \langle\Omega| T^*\left(\vartheta_J(0)\, O_1(x_1) \cdots O_n(x_n)\right)|\Omega\rangle$$

$$= - \gamma_{KJ}(\vartheta) \langle\Omega| T^*\left(\vartheta_K(0)\, O_1(x_1) \cdots O_n(x_n)\right)|\Omega\rangle \tag{7}$$

imply the characteristic form for the anomalous (logarithmic) dimension matrix

$$\sum_i \beta_i \frac{\partial}{\partial g_i}\left(\frac{\beta_k}{g_k}\right) = \sum_i \beta_i \frac{\partial}{\partial g_i} \gamma_{ki}(g) \tag{8}$$

For definiteness we consider the gauge theory of tricolored electrically charged quarks coupled to the bosons forming the unbroken gauge group $SU3^c \times U1^{e.m.}$ corresponding to the Lagrangian

$$\mathcal{L} = \mathcal{L}_{inv} + \mathcal{L}_{fix} + \mathcal{L}_{ghost}$$

$$\mathcal{L}_{inv} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \sum_{flavors} \bar{q} \gamma^\mu \overleftrightarrow{D}_\mu q$$

$$- \sum_{flavors} \bar{q} M_q q$$

$$\mathcal{L}_{fix} = -\frac{1}{2\alpha_G} \partial_\mu V^{\mu a} \partial_\nu V^{\nu a} - \frac{1}{2\alpha_F} \partial_\mu A^\mu \partial_\nu A^\nu$$

$$\mathcal{L}_{ghost} = \partial_\mu \xi^{*a} \partial^\mu \xi^a + g \left(\partial_\mu \xi^{a*}\right) f_{abc} V^{\mu c} \xi^b \tag{9}$$

In eq. (9) all quantities are unrenormalized. $F_{\mu\nu}$, $G_{\mu\nu}^a$ denote the field strengths

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu \quad ; \quad G^a_{\mu\nu} = \partial_\nu V^a_\mu - \partial_\mu V^a_\nu - g f^{abc} V^b_\nu V^c_\mu$$

$$a = 1, \dots, 8$$

and D_μ the gauge covariant derivatives

$$D_\mu q = \partial_\mu q + i \left[g \frac{\lambda^a}{2} V^a_\mu + e Q_q A_\mu \right] \cdot q$$

$$\mathrm{Tr}\, \lambda_a \lambda_b = 2 \delta_{ab} \; , \; [\lambda_a, \lambda_b] = 2 i f_{abc} \lambda_c$$

e,g: electric- and strong coupling constants.

We are particularly interested in the chiral limit of vanishing quark masses $M_q \to 0$.

The renormalized propagators and vertices are normalized at specific symmetric euclidean momenta characterized by the (redundant) reference scale M [5), 11)]:

gluon propagator:

$$\mathcal{D}_{0\,\mu\nu}^{ab}(p, g_M, e_M, M)\Big|_{p^2=-M^2} = Z_3^V \delta_{ab}\left\{\left[-g_{\mu\nu}+\frac{p_\mu p_\nu}{-M^2}\right]\frac{1}{-M^2} - \alpha_G^M \frac{p_\mu p_\nu}{M^4}\right\}$$

ghost propagator:

$$\tilde{\mathcal{D}}_0^{ab}\Big|_{p^2=-M^2} = -Z_3^S \delta_{ab}\frac{1}{M^2}$$

photon propagator:

$$\mathcal{D}_{0\,\mu\nu}\Big|_{p^2=-M^2} = Z_3^A\left[-g_{\mu\nu}+\frac{p_\mu p_\nu}{-M^2}\right]\frac{1}{-M^2} - Z_3^A \alpha_\gamma^M \frac{p_\mu p_\nu}{M^4} \qquad (10)$$

quark propagator

quark propagator

$$S_0(\not p, g_M, e_M, M)\Big|_{\not p = iM} = \frac{Z_2^q}{iM - m_q(M)}$$

For (one particle irreducible) vertex functions we only specify the normalization conditions unrelated by Slavnov identities [12)] three gluon vertex function:

$$T_0^{(3V)\,abc}{}_{\mu_1\mu_2\mu_3}[p_1 p_2 p_3; g_M, e_M, M]\Big/_{\substack{p_i^2=-M^2\\ p_i\cdot p_j=\frac{1}{2}M^2,\, i\neq j}} =$$

$$(Z_3^V)^{-3/2}\, g_M\, \frac{1}{i} f^{abc}\left[(p_1-p_2)_{\mu_3}\, g_{\mu_1\mu_2} + \text{cyclic } (123)\right]$$

quark-photon vertex function

$$T_0^{(\bar{q}Aq)}{}_{\mu}\Big/_{\substack{p_i^2=-M^2\\ p_i\cdot p_j=\frac{1}{2}M^2,\, i\neq j}} = (Z_3^A)^{-1/2}(Z_2^q)^{-1}\times\left[e_M\gamma_\mu Q_q + \begin{smallmatrix}\text{terms}\\ \text{transverse to } p_i\end{smallmatrix}\right] \tag{11}$$

The renormalization constants are $Z_3^V, Z_3^S, Z_3^A, Z_1^V, Z_1^S$, Z_1^A, Z_2^q relating the coupling constants

$$g_M = \left(\frac{(Z_3^V)^{3/2}}{Z_1^V}\right) g_0 \;,\quad \alpha_G^M = (Z_3^V)^{-1}\alpha_G$$

$$e_M = \left((Z_3^A)^{1/2}\,\frac{Z_2^q}{Z_1^A}\right) e_0 \;,\quad \alpha_F^M = (Z_3^A)^{-1}\alpha_F$$

$$Z_3^V/Z_1^V = Z_3^S/Z_1^S \;;\quad Z_2^q = Z_1^A \tag{12}$$

The renormalization group equation for an n-point (one particle irreducible) vertex function is of the form

$$\left\{ M\frac{\partial}{\partial M} + \psi_1\, g_M \frac{\partial}{\partial g_M} + \psi_2\, e_M \frac{\partial}{\partial e_M} - 2\gamma_V\, \alpha^M_G \frac{\partial}{\partial \alpha^M_G} - 2\gamma_A\, \alpha^M_A \frac{\partial}{\partial \alpha^M_A} - \sum_{\substack{\text{exter-}\\\text{nal}\\\text{fields}}} \gamma_\nu \right\} \Gamma^{(n)} = 0$$

$$\gamma_\nu = M\frac{\partial}{\partial M}\left(\frac{1}{2}\log Z_\nu\right) \qquad \nu = \{V^a_\mu, A_\mu, q\}$$

$$\psi_1 = \frac{\beta_1}{g_M} = M\frac{\partial}{\partial M}\log\left(\left(Z_3^V\right)^{3/2} \Big/ Z_1^V\right)$$

$$= -\left(11 - \frac{2}{3}n_{fe}\right)\frac{g_M^2}{16\pi^2} + O\left(g_M^4,\, g_M^2 e_M^2\right)$$

$$\psi_2 = \frac{\beta_2}{e_M} = M\frac{\partial}{\partial M}\log\left(\left(Z_3^A\right)^{1/2}\right) = \frac{e_M^2\sum Q_q^2}{24\pi^2} + O\left(e_M^4,\, e_M^2 g_M^2\right) \tag{13}$$

The operators ϑ_1, ϑ_2 of eq. (6) corresponding to this theory are multiples of

$$\vartheta_G = \frac{1}{4}\,\overline{G^a_{\mu\nu}\,G^{\mu\nu a}} \quad , \quad \vartheta_F = \frac{1}{4}\,\overline{F_{\mu\nu}\,F^{\mu\nu}} \tag{14}$$

The bar in eq. (14) denotes the gauge invariant renormalized operators constructed from the normal products of the corresponding fields.

The unrenormalized operator $\vartheta_G^{(0)}$ is given by

$$\vartheta_G^{(0)} = Z_3^V \frac{1}{4} \tilde{G}^a_{\mu\nu} \tilde{G}^{\mu\nu a}$$

$$\tilde{G}^a_{\mu\nu} = \partial_\nu \tilde{V}^a_\mu - \partial_\mu \tilde{V}^a_\nu - \frac{Z_1^V}{Z_3^V} g_M f^{abc} \tilde{V}^b_\nu \tilde{V}^b_\mu \tag{15}$$

From eq. (15) where $\tilde{V}$, $\tilde{G}$ denote renormalized fields we see that the renormalized operator ϑ_G is only directly related to the renormalized coupling constant g_M if we choose the special gauge

$$Z_1^V = Z_3^V \quad , \quad Z_1^\xi = Z_3^\xi \longrightarrow \alpha_G^M = -3 \text{ (to lowest order)}$$

$$\gamma_V = \psi_1 \tag{16}$$

Defining the renormalized three gluon vertex function at an arbitrary symmetric point

$$\Gamma^{(3V)\,abc}_{\mu_1\mu_2\mu_3}(p_1 p_2 p_3\; g_M e_M, M)\Big/_{\substack{p_i^2 = -\bar{M}^2 \\ p_i p_j = \frac{1}{2}\bar{M}^2 \; (i \neq j)}} =$$

$$= g_M \; \alpha^{(3V)}(\bar{M}^2, g_M, e_M, M)$$

and the transversephoton propagator

$$D^{(A)\,tr}_{\mu\nu}(p, g_M, e_M, M)\Big/_{p^2 = -\bar{M}^2} = \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{-\bar{M}^2}\right)\left(\frac{1}{-\bar{M}^2}\right) \times \alpha^{(2A)}[\bar{M}^2, g_M, e_M, M]$$

The renormalization group equation yields the relations

$$\bar{M}\frac{\partial}{\partial\bar{M}}\,\alpha^{(3V)}\Big/_{\bar{M}=M} = 2\psi_1$$

$$\bar{M}\frac{\partial}{\partial\bar{M}}\,\alpha^{2A}\Big/_{\bar{M}=M} = -2\psi_2 \quad ; \quad \psi_i = \frac{\beta_i}{g_i} \tag{17}$$

We will establish the relation

$$\vartheta^\mu_\mu = 2\psi_1\vartheta_G + 2\psi_2\vartheta_F \tag{18}$$

by comparing matrix elements of the operators $\vartheta^\mu_\mu, \vartheta_G, \vartheta_F$ evaluated under identical conditions to those governing the renormalization procedure. Thus we are interested in the matrix elements

$$D^{(2A)}_{\alpha\beta}\begin{pmatrix}\vartheta^\mu_\mu\\ \vartheta_F\end{pmatrix} = \int dx_1\,dx_2\,\exp i(q_1x_1+q_2x_2)$$

$$\langle\Omega/\,T^*\begin{pmatrix}\vartheta^\mu_\mu(0)\\ \vartheta_F(0)\end{pmatrix}\tilde{A}_\alpha(x_1)\,\tilde{A}_\beta(x_2)/\Omega\rangle$$

$$[\,q_1+q_2=0,\ q_i^2=-M^2$$

$$D^{(3V)a_1a_2a_3}_{\mu_1\mu_2\mu_3}\begin{pmatrix}\vartheta^\mu_\mu\\ \vartheta_G\end{pmatrix} = \int\prod_{j=1}^{3}dx_j\,\exp i(q_jx_j)$$

$$\langle\Omega/\,T^*\begin{pmatrix}\vartheta^\mu_\mu(0)\\ \vartheta_G(0)\end{pmatrix}\tilde{V}^{a_1}_{\mu_1}(x_1)\,\tilde{V}^{a_2}_{\mu_2}(x_2)\,\tilde{V}^{a_3}_{\mu_3}(x_3)/\Omega\rangle$$

$$[\,q_1+q_2+q_3=0,\ q_i^2=-M^2,\ q_i\cdot q_j=\tfrac{1}{2}M^2 \tag{19}$$

In order to obtain $D^{(2A)}$, $D^{(3V)}$ we consider the Ward identity for the dilatation current [1), 8)]

$$d_\mu(y) = y^\nu \vartheta_{\mu\nu}(y)$$

involving the (full) Green functions extended to include matrix elements of d_μ and ϑ^μ_μ : $G^{(n)}_\mu(d)_{ext}$, $G^{(n)}(\vartheta^\mu_\mu)_{ext}$, $G^{(n)}$:

$$G^{(n)}_\mu(d)_{ext}[q;\underline{p}](2\pi)^4\,\delta^{(4)}\Big(q+\sum_{\alpha=1}^n p_\alpha\Big) =$$

$$= \int dy \exp iqy \langle\Omega|T^* d_\mu(y) \prod_{\alpha=1}^n \varphi_\alpha(x_\alpha) \exp ip_\alpha x_\alpha|\Omega\rangle$$

$$G^{(n)}(\vartheta^\mu_\mu)_{ext}[q;\underline{p}](2\pi)^4\,\delta^{(4)}\Big(q+\sum_{\alpha=1}^n p_\alpha\Big) =$$

$$= \int dy \exp iqy \langle\Omega|T^* \vartheta^\mu_\mu(y) \prod_{\alpha=1}^n \varphi_\alpha(x_\alpha) \exp ip_\alpha x_\alpha|\Omega\rangle$$

$$G^{(n)}[q;\underline{p}](2\pi)^4\,\delta^{(4)}\Big(q+\sum_{\alpha=1}^n p_\alpha\Big) =$$

$$= \sum_{\alpha=1}^n \frac{\partial}{\partial p^\mu_\alpha} p^\mu_\alpha \langle\Omega|T^* \int dx_1 \exp(ip_1x_1)\,\varphi_1(x_1) \cdots\cdots$$

$$\int dx_\alpha \exp[i(p_\alpha+q)x_\alpha]\,\varphi_\alpha(x_\alpha) \cdots\cdots$$

$$\cdots \int dx_n \exp(ip_nx_n)\,\varphi_n(x_n)|\Omega\rangle$$

$$\underline{p} = p_1, \cdots p_n \quad ; \quad \varphi = \{V^a_\mu, A_\mu, q\} \qquad (20)$$

Teh Ward identity then takes the form

$$\frac{1}{i}\, q^{\mu}\, G^{(n)}_{\mu}(d)_{ext}\, [q;\underline{P}] =$$
$$= G^{(n)}(\vartheta)_{ext}[q;\underline{P}] + G^{(n)}[q;\underline{P}] \qquad (21)$$

Due to the finiteness of the matrix elements of the energy momentum tensor [8] and momentum conservation the limit of eq. (21) for $q \to 0$ is regular and yields

$$\langle\Omega|\, T^{*}\{\vartheta^{\mu}_{\mu}(0) \prod_{\alpha=1}^{n} dx_{\alpha} \exp(iP_{\alpha}x_{\alpha})\, \varphi_{\alpha}(x_{\alpha})\}|\Omega\rangle \Big/_{\sum_{\nu}^{n} P_{\nu}=0}$$
$$= i\left[M\frac{\partial}{\partial M} + d_n\right] G^{(n)}[q=0;\underline{P}] \qquad (22)$$

d_n denotes the (naive) physical dimension of $G^{(n)}$. Evaluating eq. (22) for $G^{(2A)}$, $G^{(3V)}$ at the symmetric respective normalization points $(\overline{M} = M)$ eq. (17) multiplies

$$D^{(2A)}_{\alpha\beta}(\vartheta) = \frac{g_{\alpha\beta}}{M^2}\left(2\psi_2\right) + P_{\alpha}P_{\beta}(\cdots)$$

$$D^{(3V)\,a_1 a_2 a_3}_{\mu_1\mu_2\mu_3}(\vartheta) = g_M \frac{1}{i} f^{a_1 a_2 a_3}\left[(P_1-P_2)_{\mu_3}\, g_{\mu_1\mu_2} + \text{cyclic}(123)\right] \times \left(\frac{4\psi_1}{i}\right) \qquad (23)$$

The matrix elements of ϑ^{a}_{F}, ϑ_{G} for zero momentum transfer are generated by the following variation of the functional integral

$$W(\varepsilon_G, \varepsilon_F) = \int \prod_{\alpha}(\mathcal{D}\varphi_{\alpha})_{f} \exp i[S(\varepsilon_G, \varepsilon_F) + S_{\text{gauge fixing}}]$$

$$S(\varepsilon_G, \varepsilon_F) = \int dx\, [-\varepsilon_G \frac{1}{4} G^{a}_{\mu\nu}(x)\, G^{\mu\nu a}(x) - \varepsilon_F \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x)$$
$$+ \bar{q}(x)\, \gamma^{\mu} \frac{i}{2} \overleftrightarrow{D}_{\mu}\, q(x) - \sum_{\beta} J_{\beta}(x)\varphi_{\beta}(x)]$$

J_{β} : external source for the field φ_{β}

$$\int \prod_{\alpha} \mathcal{D}\varphi_\alpha \int dy \begin{Bmatrix} \vartheta_F(y) \\ \vartheta_G(y) \end{Bmatrix} \exp i\left[S'(\varepsilon_F = \varepsilon_G = 1) + S_{\substack{gauge \\ fixing}}\right] =$$

$$= i \begin{Bmatrix} \frac{\partial}{\partial \varepsilon_F} \\ \frac{\partial}{\partial \varepsilon_G} \end{Bmatrix} W(\varepsilon_F, \varepsilon_G) \Big/_{\varepsilon_F = \varepsilon_G = 1} \qquad (24)$$

The variation with respect to ε_F, ε_G in eq. (24) is equivalent to the following substitutions

$$\delta\varepsilon_F : \quad A_\mu \to (\varepsilon_F)^{-1/2} A_\mu, \; e_M \to (\varepsilon_F)^{-1/2} e_M$$
$$J_\mu^A \to (\varepsilon_F)^{-1/2} J_\mu^A, \; \alpha_F^M \to (\varepsilon_F)^{-1} \alpha_F^M$$

$$\delta\varepsilon_G : \quad V_\mu^a \to (\varepsilon_G)^{-1/2} V_\mu^a, \; g_M \to (\varepsilon_G)^{-1/2} g_M$$
$$J_{\mu a}^{(V)} \to (\varepsilon_G)^{-1/2} J_{\mu a}^{(V)}, \; \alpha_G^M \to \varepsilon_G^{-1} \alpha_G^M \qquad (25)$$

Then one obtains the matrix elements at zero momentum transfer of the operators ϑ_F, ϑ_G:

$$\langle \Omega | T^* \{\vartheta_F(0) \prod_\alpha dx_\alpha \exp(i P_\alpha x_\alpha)\, \varphi_\alpha(x_\alpha)\} | \Omega \rangle \Big/_{\Sigma P_\nu = 0} =$$

$$= \frac{1}{i} \left[\frac{1}{2} e_M \frac{\partial}{\partial e_M} + \alpha_F^M \frac{\partial}{\partial \alpha_F^M} + \frac{n^{(A)}}{2}\right] G^{(n)}[q=0; P]$$

$$\langle \Omega | T^* \{\vartheta_G(0) \prod_\alpha dx_\alpha \exp(i P_\alpha x_\alpha)\, \varphi_\alpha(x_\alpha)\} | \Omega \rangle \Big/_{\Sigma P_\nu = 0} =$$

$$= \frac{1}{i} \left[\frac{1}{2} g_M \frac{\partial}{\partial g_M} + \alpha_G^M \frac{\partial}{\partial \alpha_G^M} + \frac{n^{(V)}}{2}\right] G^{(n)}[q=0, P] \qquad (26)$$

$n^{(A)}$, $n^{(V)}$ in eq. (26) denote the number of electromagnetic potentials (A) or color gauge fields (V) forming the external legs of $G^{(n)}$

Evaluating eq. (26) at the symmetric normalization points for the extended matrix elements $\mathcal{D}^{(2A)}_{\alpha\beta}(\vartheta_F)$, $\mathcal{D}^{(3V)\,a_1a_2a_3}_{\mu_1\mu_2\mu_3}(\vartheta_G)$ it follows

$$\mathcal{D}^{(2A)}_{\alpha\beta}(\vartheta_F) = \frac{g_{\alpha\beta}}{M^2} + p_\alpha p_\beta(\cdots)$$

$$\mathcal{D}^{(3V)\,a_1a_2a_3}_{\mu_1\mu_2\mu_3}(\vartheta_G) = g_M \frac{1}{i} f^{a_1a_2a_3}\left[\begin{matrix}(p_1-p_2)_{\mu_3} g_{\mu_1\mu_2} \\ + \text{cyclic}(123)\end{matrix}\right]\cdot\left(\frac{2}{i}\right)$$

(27)

Comparing equations (23) and (21) the relation of eq. (18)

$$\vartheta^\mu_\mu = \left\{\begin{array}{l} 2\psi_1\vartheta_G + 2\psi_2\vartheta_F = \\ = \frac{1}{2}\left(\frac{\beta_g}{g}\right)_M \overline{G^a_{\mu\nu}G^{\mu\nu a}} + \frac{1}{2}\left(\frac{\beta_e}{e}\right)_M \overline{F_{\mu\nu}F^{\mu\nu}} \end{array}\right\}$$

follows.

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