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ON THE HYPOTHESIS THAT SPONTANEOUS CURVATURE FREEZES A SET OF SPACE-LIKE VARIABLES BEYOND THE OBSERVED FOUR AT ENERGIES MUCH BELOW THE PLANCK MASS*

("Früh krümmt sich, was ein Häkchen werden will")

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Abstract

The hypothesis is investigated that the well known interactions are manifestations of a gauge theory on a manifold $\mathcal{M}$ involving a set of space-like coordinates beyond space-time. The gauge group consists of coordinate transformations of $\mathcal{M}$. The scale invariant action couples a family of scalar fields φ_α to the curvature scalar in $\mathcal{M}$:

$$S_{\mathcal{M}} = \int d^M x \sqrt{|g|} \left[\left(\sum_\alpha \frac{Q}{2} \varphi_\alpha \varphi_\alpha \right) R + \frac{1}{2} \sum_\alpha \partial_A \varphi_\alpha \, g^{AB} \partial_B \varphi_\alpha \right]$$

$S_{\mathcal{M}}$ generates a spontaneous ground state solution with constant curvature with respect to the interval coordinates of the order of Planck's length for which $S_{\mathcal{M}} = 0$.

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Strong-, electromagnetic- and weak interactions (called charged interactions in the following) are believed to be described by universal gauge couplings based on a local gauge group G [1]. The local structure of G connects the charge like gauges with the space-time gauges and therefore charged with gravitational interactions.

Pursuing the ideas expressed in ref (2) all mass scales, in particular Newton's constant arise spontaneously [3] and/or through the regularization of quantum fluctuations [4].

An action involving explicit mass scales shall be understood as a low energy effective action, to be replaced by a (classically) scale invariant action inducing spontaneous generation of mass scales.

This situation prevails with respect to the weak- and associated intercations.

We study the following hypotheses:

i) Charge like and space-time gauges are low energy ($E \ll m_{\text{Planck}} = 1.22 \cdot 10^{19}$ GeV) manifestations of common geometric origin.

ii) The unified family of gauges corresponds to coordinate transformations of a manifold $\mathcal{M}$ involving $M > 4$ dimensions.

iii) The ground state shows a spontaneous asymmetry: four dimensional space-time remains flat whereas an M - 4 dimensional subspace acquires constant curvature [FN1] the curvature radius being essentially Planck's length ($l_{Pl} = 1.61 \cdot 10^{-33}$ cm).

iv) The action governing the interactions (in $\mathcal{M}$) is scale invariant and thus involves a family of scalar (hermitian) fields $\varphi_\alpha, \alpha = 1, \cdots, n$ $(n > 1)$ in the following combination with the curvature scalar (in $\mathcal{M}$)

$$S = \int d^M x \sqrt{|g|} \left[Q(\varphi) R_M + \frac{1}{2} g^{AB} \partial_A \varphi_\alpha \partial_B \varphi_\alpha + \mathcal{L}^{matter} \right] \tag{1}$$

FN2)

($\hbar$ = c = 1 units are chosen)

In eq. (1) $Q(\varphi)$ denotes a quadratic function of φ_α subject to restrictions unposed by iii).

We denote coordinates in $\mathcal{M}$ by x^A, A = 0, 1,...M - 1; space-time coordinates by x^μ, μ = A = 0,1,2,3 and the remaining internal coordinates by

z^r, r = A - 3 = 1,2, ..., $\mathcal{D}$ = M - 4.

The vector-index set $\{A\}$ shall similarly be devided

$$\{A\} \doteq \left\{ \begin{array}{lll} \mu & \text{for} & A = 0,1,2,3 \\ r & \text{for} & A = 3+r,\ r = 1, \dots, \mathcal{D} \end{array} \right\}$$

The signature of the metric g_{AB} shall be

$$g_{00} > 0\,,\quad \mathrm{Det} \left\{ \begin{array}{cccc} g_{A_{i_1} A_{i_1}} & g_{A_{i_1} A_{i_2}} & \cdots & g_{A_{i_1} A_{i_m}} \\ g_{A_{i_2} A_{i_1}} & g_{A_{i_2} A_{i_2}} & \cdots & g_{A_{i_2} A_{i_m}} \\ \vdots & & & \\ g_{A_{i_m} A_{i_1}} & g_{A_{i_m} A_{i_2}} & \cdots & g_{A_{i_m} A_{i_m}} \end{array} \right\} < 0 \tag{2}$$

for all subsets (A_{i_1} ---- A_{i_m}), m = 1, ..., M - 1 of space like indices A_{i_K} = 1,,..., M - 1.

Eq. (2) implies that there exists a symmetric M-bein such that

$$g_{AB} = v_{AR}\, \eta^{RS}\, v_{BS}\,, \quad v_{AR} = v_{RA}$$

$$\eta^{RS} = \begin{pmatrix} 1 & & & \\ & -1 & & 0 \\ & & -1 & \\ & 0 & & \ddots \\ & & & & -1 \end{pmatrix}; \quad A, B, R, S = 0, 1, \dots, M-1 \tag{3}$$

$\mathcal{L}^{\text{matter}}$ contains all fields beyond φ_α and g_{AB}. We note that scale invariance restricts $\mathcal{L}^{\text{matter}}$ (for $M > 4$) to contain only kinetic energy terms in the limit of vanishing curvature in $\mathcal{M}$. In particular $\mathcal{L}^{\text{matter}}$ contains a family of $2^{[\frac{M}{2}]}$- component fermions $\psi_{(\beta)}$; $\beta = 1, \dots n(\psi)$ denoting different types of fermions. $[\frac{M}{2}]$ is the largest integer $\leq \frac{M}{2}$:

$$\mathcal{L}_\psi = \sum_{\beta=1}^{n(\psi)} \overline{\psi}_{(\beta)} \gamma_R v^{AR} \mathcal{D}_A \psi_{(\beta)} \tag{4}$$

In eq. (4) γ_R denote $2^{[\frac{M}{2}]} \times 2^{[\frac{M}{2}]}$ matrices forming the Clifford-Dirac algebra in M dimensions

$$\frac{1}{2}\{\gamma_R, \gamma_S\} = \eta_{RS} \cdot \mathbb{1} \tag{5}$$

$D_A \psi$ denote the covariant derivative

$$\mathcal{D}_A \psi = \left[\partial_A + \frac{1}{2} C_A^{[RS]} \Sigma_{[RS]}\right] \psi$$

$$\Sigma_{[RS]} = \frac{1}{4} [\gamma_R, \gamma_S] \tag{6}$$

$$C_A^{[RS]} = -\left[v_B{}^R \left(\partial_A v^{BS}\right) + v_B{}^R \Gamma^B_{AD} v^{DS}\right]$$

$$\Gamma^B_{AD} = \frac{1}{2} g^{BB'} \left[\partial_A g_{B'D} + \partial_D g_{B'A} - \partial_{B'} g_{AD}\right]$$

We are looking for a static solution to the Euler-Lagrange equations generated by S with all fields composing $\mathcal{L}^{\text{matter}}$ vanishing :

$$Q(\varphi)\left[R_{AB} - \frac{1}{2} g_{AB} R\right] + \left(D_A D_B - g_{AB} \Box\right) Q(\varphi) =$$

$$= -\frac{1}{2} \partial_A \varphi_\alpha \partial_B \varphi_\alpha + \frac{1}{4} g_{AB} g^{CD} \partial_C \varphi_\alpha \partial_D \varphi_\alpha$$

$$Q_{\alpha\beta} \varphi_\beta R = \Box \varphi_\alpha \tag{7}$$

$$Q(\varphi) = \frac{1}{2} \varphi_\alpha Q_{\alpha\beta} \varphi_\beta$$

In eq. (7) $\Box$ denotes the Laplace-Beltrami operator in $\mathcal{M}$

$$\Box = \frac{1}{\sqrt{|g|}} \partial_A \, g^{AB} \sqrt{|g|} \, \partial_B$$

The Ansatz corresponding to flat space-time and constant curvature in the internal space $\mathcal{D}$ $(z^1, \dots, z^D)$ shall be:

$$\partial_\mu \varphi_\alpha = 0, \quad \partial_\mu g_{AB} = 0$$

$$R_{[\mu A][BC]} = 0 \quad ; \quad D_r R_{[st][uv]} = 0 ; \; s,t,u,v,r = 1, \dots, D \tag{8}$$

$R_{[AB][CD]}$: Riemann tensor in $\mathcal{M}$.

Eq. (8) implies

$$R_{\mu A} = 0 \; , \quad R_{st} = \frac{1}{D} g_{st} R \tag{9}$$ FN3)

We can integrate for $g_{\mu A}$ to obtain

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} ; \quad g_{\mu r} = 0 \tag{10}$$

The first equation in (7) yields two equations, one each for (AB) = $(\mu\nu)$ and (AB) = (rs):

$$(AB) = (\mu\nu) : \quad Q(\varphi) R + 2 \Box \, Q(\varphi) = -\frac{1}{2} g^{rs} \partial_r \varphi_\alpha \partial_s \varphi_\alpha$$

$$(AB) = (rs) : \quad Q(\varphi) \left[R_{st} - \frac{1}{2} g_{st} R \right] + \left(D_s D_t - g_{st} \Box \right) Q(\varphi) =$$

$$= -\frac{1}{2} \partial_s \varphi_\alpha \partial_t \varphi_\alpha + \frac{1}{4} g_{st} g^{uv} \partial_u \varphi_\alpha \partial_v \varphi_\alpha \tag{11}$$

Multiplying the second equation in (11) by g^{st} one obtains

$$\left[Q(\varphi)\cdot R + \frac{1}{2} g^{rs} \partial_r \varphi_\alpha \partial_s \varphi_\alpha\right]\left(1-\frac{D}{2}\right) + (1-D)\,\Box\, Q(\varphi) = 0 \tag{12}$$

Combining eq. (11) and (12) we have

$$\Box\ Q(\varphi) = 0$$

$$\Longrightarrow R.Q(\varphi) + \frac{1}{2}\ g^{rs}\ \partial_r \varphi_\alpha \partial_s \varphi_\alpha = 0 \tag{13}$$

Eq. (13) yields a significant result; for the solutions in question the action of eq. (1) vanishes.

The Ansatz of eq. (8) renders the internal space $\mathcal{D}$ a symmetric space [5)] i.e. $\mathcal{D}$ corresponds to a pair G, H where G is a Lie group which we identify with the (global) charge like gauge group of strong- eletromagnetic and weak interactions, H is a subgroup of G equivalent to the stability group of an arbitrary point $z \in \mathcal{D}$ under the motions induced by G:

$$\mathcal{D} = G/H\ , \quad D = \dim G - \dim H \tag{14}$$

We choose R positive. This corresponds to a compact group G and a compact space $\mathcal{D}$ (with respect to the metric g_{st}).

Eqs. (8) and (10) reduce the Laplace-Beltami operator in $\mathcal{M}$ to the corresponding (compact) operator in $\mathcal{D}$. As a consequence eq. (13) implies

$$Q(\varphi) = \text{constant (on } \mathcal{D}) \tag{15}$$

Excluding the physically uninteresting possiblity that the quadratic form $Q_{\alpha\beta}$ is degenerate, the second equation in (7) and (11) imply

$$Q_{\alpha\beta} = Q\,\delta_{\alpha\beta}\,; \quad Q > 0$$
$$\Box\varphi_\alpha = (QR)\,\varphi_\alpha\,; \quad QR > 0 \tag{16}$$

Hence φ_α are (scalar) spherical functions on $\mathcal{D}$ transforming according to a nontrivial (real) unitary representation of G:

$$\varphi_\alpha(a.z) = \mathcal{D}_{\alpha\beta}(a)\,\varphi_\beta(z)\;;\; \alpha,\beta = 1,\cdots,n\,;\; n > 1$$
$$\mathcal{D}(a_1)\,\mathcal{D}(a_2) = \mathcal{D}(a_1 a_2)\,;\; \mathcal{D}(a)\,\mathcal{D}^T(a) = \mathbb{1}$$
$$a,\, a_1,\, a_2 \in G \tag{17}$$

From eq. (17) we deduce that eq. (15) is valid

$$Q(\varphi) = \frac{1}{2}\,Q\,\sum_\alpha \varphi_\alpha(z)\,\varphi_\alpha(z) = \text{constant (on } \mathcal{D})$$

Furthermore eq. (13) is verified by the identity

$$\frac{1}{2}(\partial_s\varphi_\alpha)\,g^{st}(\partial_t\varphi_\alpha) = \frac{1}{2}\left\{\begin{array}{l}\frac{1}{\sqrt{|g|}}\,\partial_s\left[\varphi_\alpha\sqrt{|g|}\,g^{st}\,\partial_t\varphi_\alpha\right]\\ -\,\varphi_\alpha\,\Box\,\varphi_\alpha\end{array}\right\} =$$

$$= -\,R\;Q(\varphi) \tag{18}$$

It remains to show that

$$\frac{1}{D}\, Q(\varphi) . R\, g_{st} + \frac{1}{2}\, \partial_s \varphi_\alpha\, \partial_t \varphi_\alpha = 0 \qquad (19)$$

$\partial_s \varphi_\alpha\, \partial_t \varphi_\alpha$ is proportional to g_{st} because the metric tensor is the unique tensor spherical function not depending on further representation indices with respect to G. This proves eq. (19) FN4); 8), 9), 10).

It was noted in ref. 8), 9) that the metric

$$g_{AB} = \left(\begin{array}{c|c} \eta_{\mu\nu} & 0 \\ \hline 0 & g_{rs}(z) \end{array} \right) \qquad (20)$$

where g_{rs} is the metric on the symmetric space $\mathcal{D}$ is not the most general metric compatible with the internal structure of $\mathcal{D}$. One can allow spacetime dependent local coordinate transformations on $\mathcal{D}$, generated by group elements $a(x^\mu) \in G$ depending in an arbitrary way on $x^\mu (\mu = 0,1,2,3)$.

The general metric involves the gauge fields of the local gauge group $G(x^\mu)$: W^f_μ, f = 1,...., δ = dim G, and the Killing vector fields generated by the motions of G in $\mathcal{D}$:

$$a \in G ; \quad a : z \longrightarrow a.z \longleftrightarrow F^r(a, z) \,, r = 1, \cdots, d \qquad (21)$$

Infinitesimal $a : a = \varepsilon^f I_f$ correspond to a basis in the Lie algebra of G

$$I_1, \ldots, I_\delta\ ;\ [I_\alpha, I_\beta] = c_{\alpha\beta\gamma} I_\gamma \qquad (22)$$

$c_{\alpha\beta\gamma}$: structure constants of G.

The Killing vectors on $\mathcal{D}$ are given by

$$h^r_f(z) = \frac{\partial T^r(a^f = \varepsilon^f; z)}{\partial \varepsilon^f}\Big/_{\varepsilon = 0} \qquad (23)$$ FN5)

The Killing vectors h^r_f transform as vector fields under coordinate transformations on $\mathcal{D}$, they are vector spherical functions on $\mathcal{D}$. Under the action of G on $\mathcal{D}$ they transform in the following way:

$$h^r_f(a.z) = \psi^r_s(a,z)\, h^s_{f'}(z)\, [Ad(a)]^{f'}_{\ f} \qquad (24)$$

In eq. (24) ψ^r_s denotes the Jacobian of the coordinate transformation $z \rightarrow a \cdot z$:

$$\psi^r_s(a,z) = \frac{\partial T^r(a,z)}{\partial z^s}$$

$$(\psi^{-1})^s_r(a,z) = \psi^s_r(a^{-1}, a.z) \qquad (25)$$

Ad(a) denotes the adjoint representation of G by real, orthogonal matrices.

The generalized metric is of the form

$$g_{AB}\, dx^A dx^B = \eta_{\mu\nu}\, dx^\mu dx^\nu + g_{rs}\, d\xi^r d\xi^s$$

$$d\xi^r = dz^r + \gamma^r_\mu(x^\nu, z)\, dx^\mu$$

$$\gamma^r_\mu = W^f_\mu(x^\nu)\, h^r_f(z) \qquad (26)$$ 8), 9), 10), FN5)

Gauge invariance determines the transformation properties of W_μ^f under a local gauge transformation a(x):

$$a(x):\quad W_\mu^{\ f}(x^\nu) \longrightarrow [Ad(a)]^{-1\,f}{}_{f'}\, W_\mu^{\ f'}(x^\nu) - \\ - \left(a(x^\nu)\,\partial_\mu\, a^{-1}(x^\nu)\right)^f \tag{27a}$$

and for infinitesimal a(x) = ε (x):

$$W_\mu^{\ f}(x^\nu) \longrightarrow W_\mu^{\ f}(x^\nu) + \partial_\mu\, \varepsilon^f(x^\nu) - c_{fmn}\, W_\mu^{\ m}(x^\nu)\, \varepsilon^n(x^\nu) \tag{27b}$$

The symbol $\left(a(x^\nu)\,\partial_\mu\, a^{-1}(x^\nu)\right)^f$ in eq. (27a) can be obtained from any linear representation of G:

$$\mathcal{D}\left(a(x^\nu)\right)\left[\partial_\mu\, \mathcal{D}^{-1}\left(a(x^\nu)\right)\right] = \left(a(x^\nu)\,\partial_\mu\, a^{-1}(x^\nu)\right)^f i_f(\mathcal{D})$$

$$\mathcal{D}(a=\varepsilon) = \mathbb{1} + \varepsilon^f\, i_f(\mathcal{D}) \tag{28}$$

The scalar curvature R in $\mathcal{M}$ contains the gauge covariant field strengths

$$W_{\mu\nu}^{\ f} = \partial_\nu W_\mu^{\ f} - \partial_\mu W_\nu^{\ f} - c_{fmn}\, W_\nu^{\ m} W_\mu^{\ n} \tag{29}$$

in the characteristic combination

$$-\frac{1}{4}\left(W_{\mu\nu}^{\ f}\, W_{\mu'\nu'}^{\ f'}\, \eta^{\mu\mu'}\eta^{\nu\nu'}\right)\cdot\left(h_f^{\ r}(z)\, g_{rs}(z)\, h_{f'}^{\ s}(z)\right) \tag{30}$$

Effective actions in four dimensions result from integrating appropriate approximations to S in eq. (1) over $\mathcal{D}$. In order to do this we introduce the dimensionless (angular) coordinates on $\mathcal{D}$:

$$Z^r = \sqrt{R}\, z^r \implies \Box_{\mathcal{D}} = -R\,\Delta_{\mathcal{D}}$$

$$\Delta_{\mathcal{D}} = -\frac{1}{\sqrt{|g|}}\,\partial_{Z^r}\, g^{rs}\sqrt{|g|}\,\partial_{Z^s} \tag{31}$$

$$\sqrt{R} = L^{-1}$$

$\Delta_{\mathcal{D}}$ has universal eigenvalues on $\mathcal{D}$ which depend only on the structural invariants of G and H. The quadratic Casimir operator

$$C^{(2)} = - \sum_f I_f I_f \tag{32}$$

is normalized by the structure constants c_{kem} in eq. (22)

$$C_{\mathcal{D}}^{(2)} = - h_f^r \partial_{z^r} h_f^s \partial_{z^s} = - k_1 \Delta_{\mathcal{D}} \tag{33}$$

$k_1 > 0$ and in particular Q are structural constants of G, H. Only for the correct discrete set of values of Q spontaneous curvature can arise. We conclude

$$Q = 0(1)$$

unless the number of scalar fields becomes very large.

From (33) it follows

$$h_f^r h_f^s = k_1 R^{-1} g^{rs}$$

Furthermore

$$\int d^D z \sqrt{|g|} = L^D \cdot k_2 \quad (< \infty) \tag{34}$$

k_2 : structural constant of G, H.

Thus we obtain the effective action of the vector bosons:

$$S_{eff}^{(W)} = - \frac{1}{4} \left[Q(\varphi) L^{D+2} \frac{D}{\delta} k_1 k_2 \right] \int d^4x \left(W_{\mu\nu}^f W^{\mu\nu f} \right) \tag{35}$$

Besides R (or L) there exists a second spontaneous constant determining the normalization of the spherical functions φ_α , which due to the homogeneity of the equations of motion with respect to φ remains arbitrary in the absence of quantum effects. We set

$$\varphi_\alpha = (L)^{-\left(\frac{D}{2}+1\right)} N \phi_\alpha$$

$$\int d^D z \sqrt{|g|}\, \phi_\alpha \phi_\beta = \delta_{\alpha\beta} \tag{36}$$

N: second spontaneous parameter besides R.

Eq. (36) implies

$$S_{eff}^{(W)} = -\frac{1}{4 g_{ch}^2} \int d^4x \left(W_{\mu\nu}^f W^{\mu\nu f} \right)$$

$$(g_{ch})^{-2} = \frac{N^2 \cdot C^{(2)}(\varphi) \cdot D}{2\,\delta} \tag{37}$$

In eq. (36) g_{ch} denotes the coupling constant of the charge like gauge group G. $C^{(2)}(\varphi)$ is the value of the quadratic Casimir operator for the representation of G formed by φ_α .

Shifting the fields φ_α, g_{AB} by the above solution in the ground state

$$\varphi_\alpha \rightarrow \tilde{\varphi}_\alpha = \varphi_\alpha + \hat{\varphi}_\alpha \;;\; g_{AB} \rightarrow \tilde{g}_{AB} = g_{AB} + \hat{g}_{AB} \tag{38}$$

with φ_α and g_{AB} denoting the ground state field configuration, we observe that R generates a mass term (in four dimensions) for the scalars $\hat{\varphi}_\alpha$ (with $\partial_{z^\nu} \hat{\varphi}_\alpha = 0$)

$$m(\hat{\varphi}_\alpha) = \frac{\sqrt{Q}}{L} \tag{39}$$

Finally Newton's constant is given by

$$(16 \pi G_N)^{-1} = \int d^D z \sqrt{|g|}\, Q(\varphi) = \frac{N^2 Q}{2 L^2} \tag{40}$$

$$\varkappa = 8 \pi G_N = \frac{L^2}{N^2 Q} \;;\; \varkappa\, m^2(\hat{\varphi}_\alpha) = \frac{1}{N^2}$$

From eq. (37) we infer

$$\left|\frac{1}{g_{ch}}\right| \simeq O(1) \quad \left(\frac{1}{e} \simeq 3\right) \Longrightarrow N \simeq O(1)$$

and

$$L = O(\ell_{Planck})$$

We note that the compact structure of $\mathcal{D}$ and G (R > 0) imply that Newton's constant (in our interpretation) is positive i.e. that gravity is attractive.

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It is a pleasure to thank my colleagues in Bern for their contributions to the seminar on gauge theories [FN5] and for many interesting discussions. Most of all I should like to thank H. Leutwyler for his beautiful demonstration of the connection between charge-like gauges and symmetric spaces.

Footnotes

FN1) The intrinsic connection of Riemannian spaces with constant curvature

$$\mathcal{D}_{\kappa} R_{[AB][CD]} = 0$$

with the motions induced by a Lie transformation group has been demonstrated by E. Cartan [5].

FN2) The unique structure of the scale invariant coupling $Q(\varphi) \cdot R$ (in 4 dimensions) has been noted by F. Gürsey [6] in discussing Mach's principle. It has been used (with an unphysical sign) by C. Callan, S. Coleman and R. Jackiw [7] to construct a conserved energy momentum tensor for scalars, finite under renormalization.

FN3) For simplicity we consider the case where $\mathcal{D}$ is irreducible i.e. cannot be decomposed into a direct product of mutually orthogonal subspaces each one with constant curvature scalar.

FN4) A geometric interpretation of the electromagnetic gauges assuming G = U1(e.m.) in 5 dimensions is due to Th. Kaluza [8] and O. Klein [9].

FN5) The method sketched here is due to H. Leutwyler, who presented it as part of a series of seminars on gauge theories at the University of Bern (1977). Even though it is believed that the geometric interpretation of vector gauge fields is widely known I feel that his outstanding contribution deserves special mention.

FN6) The fact that the curvature scalar contains the field strength's quadratically (as noted already by Th. Kaluza and O. Klein [8], [9], [10], [FN5]) may answer a question raised by R.P. Feynman whether one could understand why the Yang-Mills Lagrangian just contains the quadratic invariant $W^{f}_{\mu\nu} W^{\mu\nu f}$.

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