$\mathcal{L}^{\text{matter}}$ contains all fields beyond φ_α and g_{AB}. We note that scale invariance restricts $\mathcal{L}^{\text{matter}}$ (for $M>4$) to contain only kinetic energy terms in the limit of vanishing curvature in $\mathcal{M}$. In particular $\mathcal{L}^{\text{matter}}$ contains a family of $2^{\left[\frac{M}{2}\right]}$ - component fermions $\psi_{(\beta)}$; $\beta = 1,\dots n(\psi)$ denoting different types of fermions. $\left[\frac{M}{2}\right]$ is the largest integer $\leq \frac{M}{2}$:

$$\mathcal{L}_\psi = \sum_{\beta=1}^{n(\psi)} \bar{\psi}_{(\beta)} \gamma_R v^{AR} D_A \psi_{(\beta)} \tag{4}$$

In eq. (4) γ_R denote $2^{\left[\frac{M}{2}\right]} \times 2^{\left[\frac{M}{2}\right]}$ matrices forming the Clifford-Dirac algebra in M dimensions

$$\frac{1}{2}\{\gamma_R, \gamma_S\} = \eta_{RS} \cdot \mathbb{1} \tag{5}$$

$D_A \psi$ denote the covariant derivative

$$D_A \psi = \left[\partial_A + \frac{1}{2} C_A^{[RS]} \Sigma_{[RS]}\right]\psi$$

$$\Sigma_{[RS]} = \frac{1}{4}[\gamma_R, \gamma_S]$$

$$C_A^{[RS]} = -\left[v_B{}^R \left(\partial_A v^{BS}\right) + v_B{}^R T^B_{AD} v^{DS}\right] \tag{6}$$

$$T^B_{AD} = \frac{1}{2} g^{BB'}\left[\partial_A g_{B'D} + \partial_D g_{B'A} - \partial_{B'} g_{AD}\right]$$

We are looking for a static solution to the Euler-Lagrange equations generated by S with all fields composing $\mathcal{L}^{\text{matter}}$ vanishing :

$$Q(\varphi)\left[R_{AB} - \frac{1}{2} g_{AB} R\right] + \left(D_A D_B - g_{AB}\Box\right) Q(\varphi) =$$

$$= -\frac{1}{2} \partial_A \varphi_\alpha \partial_B \varphi_\alpha + \frac{1}{4} g_{AB} g^{CD} \partial_C \varphi_\alpha \partial_D \varphi_\alpha$$

$$Q_{\alpha\beta} \varphi_\beta R = \Box \varphi_\alpha \tag{7}$$

$$Q(\varphi) = \frac{1}{2} \varphi_\alpha Q_{\alpha\beta} \varphi_\beta$$