In eq. (7) $\Box$ denotes the Laplace-Beltrami operator in $\mathcal{M}$

$$\Box = \frac{1}{\sqrt{|g|}}\, \partial_A\, g^{AB} \sqrt{|g|}\, \partial_B$$

The Ansatz corresponding to flat space-time and constant curvature in the internal space $\mathcal{D}$ $(z^1, \ldots, z^D)$ shall be:

$$\partial_\mu \varphi_\alpha = 0, \quad \partial_\mu g_{AB} = 0$$

$$R_{[\mu A][BC]} = 0 \quad ; \quad D_r R_{[st][uv]} = 0; \; s,t,u,v,r = 1, \ldots, D \tag{8}$$

$R_{[AB][CD]}$: Riemann tensor in $\mathcal{M}$.

Eq. (8) implies

$$R_{\mu A} = 0 \;, \quad R_{st} = \frac{1}{D}\, g_{st}\, R \tag{9}$$ FN3)

We can integrate for $g_{\mu A}$ to obtain

$$g_{\mu\nu} \doteq \eta_{\mu\nu} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & \ddots \\ & & & -1 \end{pmatrix} ; \quad g_{\mu r} = 0 \tag{10}$$

The first equation in (7) yields two equations, one each for (AB) = $(\mu\nu)$ and (AB) = (rs):

$$(AB)=(\mu\nu): \quad Q(\varphi) R + 2 \Box Q(\varphi) = -\frac{1}{2}\, g^{rs}\, \partial_r \varphi_\alpha\, \partial_s \varphi_\alpha$$

$$(AB)=(rs): \quad Q(\varphi)\left[R_{st} - \frac{1}{2} g_{st} R\right] + \left(D_s D_t - g_{st} \Box\right) Q(\varphi) =$$

$$= -\frac{1}{2}\, \partial_s \varphi_\alpha\, \partial_t \varphi_\alpha + \frac{1}{4}\, g_{st}\, g^{uv}\, \partial_u \varphi_\alpha\, \partial_v \varphi_\alpha$$

(11)