Multiplying the second equation in (11) by g^{st} one obtains

$$\left[Q(\varphi) \cdot R + \frac{1}{2} g^{rs} \partial_r \varphi_\alpha \partial_s \varphi_\alpha \right] \left(1 - \frac{D}{2}\right) + (1-D) \Box Q(\varphi) = 0 \qquad (12)$$

Combining eq. (11) and (12) we have

$$\Box \; Q(\varphi) = 0$$

$$\Longrightarrow R \cdot Q(\varphi) + \frac{1}{2} g^{rs} \partial_r \varphi_\alpha \partial_s \varphi_\alpha = 0 \qquad (13)$$

Eq. (13) yields a significant result: for the solutions in question the action of eq. (1) vanishes.

The Ansatz of eq. (8) renders the internal space $\mathcal{D}$ a symmetric space [5] i.e. $\mathcal{D}$ corresponds to a pair G, H where G is a Lie group which we identify with the (global) charge like gauge group of strong- eletromagnetic and weak interactions, H is a subgroup of G equivalent to the stability group of an arbitrary point $z \in \mathcal{D}$ under the motions induced by G:

$$\mathcal{D} = G/H \;, \quad D = \dim G - \dim H \qquad (14)$$

We choose R positive. This corresponds to a compact group G and a compact space $\mathcal{D}$ (with respect to the metric g_{st}).

Eqs. (8) and (10) reduce the Laplace-Beltami operator in $\mathcal{M}$ to the corresponding (compact) operator in $\mathcal{D}$. As a consequence eq. (13) implies

$$Q(\varphi) = \text{constant (on } \mathcal{D}) \qquad (15)$$