It remains to show that

$$\frac{1}{D}\, Q(\varphi).R\, g_{st} + \frac{1}{2}\, \partial_s \varphi_\alpha\, \partial_t \varphi_\alpha = 0 \qquad (19)$$

$\partial_s \varphi_\alpha \partial_t \varphi_\alpha$ is proportional to g_{st} because the metric tensor is the unique tensor spherical function not depending on further representation indices with respect to G. This proves eq. (19) [FN4); 8), 9), 10).]

It was noted in ref. 8), 9) that the metric

$$g_{AB} = \left(\begin{array}{c|c} \eta_{\mu\nu} & 0 \\ \hline 0 & g_{rs}(z) \end{array}\right) \qquad (20)$$

where g_{rs} is the metric on the symmetric space $\mathcal{D}$ is not the most general metric compatible with the internal structure of $\mathcal{D}$. One can allow spacetime dependent local coordinate transformations on $\mathcal{D}$, generated by group elements $a(x^\mu) \in G$ depending in an arbitrary way on $x^\mu (\mu = 0,1,2,3)$.

The general metric involves the gauge fields of the local gauge group $G(x^\mu)$: W_μ^f, $f = 1,\ldots,$ δ = dim G, and the Killing vector fields generated by the motions of G in $\mathcal{D}$:

$$a \in G;\quad a: z \longrightarrow a.z \longleftrightarrow F^r(a, z)\,, \quad r = 1, \cdots, \alpha \qquad (21)$$