QCD – mass and gauge in a field theory
QCD glue mesons

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Topics

Honoring Murray Gell-Manns 80th birthday, Singapore 24.-26. February 2010

1 Meeting you, MGM , while a student at ETH-Zurich

It was a 'strange' feeling and the article(s) I found reading Physical Review papers, with your name as author(s) – Murray Gell-Mann – promised something I should learn more about.

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Symmetries of Baryons and Mesons*

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Fig 1 : from reference [1-1961]

It was an intense struggle to eliminate alternatives through the crossroad

- within a renormalizable local field theory base fields starting from quarks must explicitely display all quantum numbers, also color , in a canonical way accompanied by gauge fields , with associated quantum numbers

I would like to present some aspects of gluonic mesons here.

1-1 QCD – the two central anomalies : scale and chiral U1

Premises

We face the theoretical abstraction of QCD with $N_{fl} = 6$, representing strong interactions – adaptable to two or three light flavors (u , d , s) of quarks and antiquarks. $\leftrightarrow$

quarks : color is counted in $\pi^0 \to \gamma\gamma$ and $R = \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

spin and flavor are clearly seen in $q\overline{q}$ and $3q$, $3\overline{q}$ spectroscopy.

$$\mathcal{L} = \left[\overline{q}^{\,\dot{c}'}_{\dot{\mathcal{B}}\,\dot{f}} \left\{ \begin{array}{c} \frac{i}{2} \overleftrightarrow{\partial}_{\mu} \, \delta_{c'c} \\ - v^{A}_{\mu} \left(\frac{1}{2} \lambda^{A} \right)_{c'\dot{c}} \end{array} \right\} \gamma^{\mu}_{\dot{\mathcal{B}}\mathcal{A}} \, q^{c}_{\mathcal{A}\,f} \quad - m_f \, \overline{q}^{\,\dot{c}}_{\dot{\mathcal{A}}\,\dot{f}} \, q^{c}_{\mathcal{A}\,f} \right] \tag{1}$$

$$- \frac{1}{4\,g^2} F^{\mu\nu\,A} F^{A}_{\mu\nu} + \Delta\mathcal{L}$$

quarks : c' , $c = 1, 2, 3$ color , $f = 1, \cdots, 6$ flavor

$\mathcal{B}, \mathcal{A} = 1, \cdots, 4$ spin , m_f mass

$\longrightarrow$

gauge bosons :

$$F^{A}_{\mu\nu} = \partial_{\nu} v^{A}_{\mu} - \partial_{\mu} v^{A}_{\nu} - f_{ABC}\, v^{B}_{\nu}\, v^{C}_{\mu}$$

(2) $$A, B, C = 1, \cdots, dim\,(G = SU3_{c}) = 8$$

Lie algebra labels, $\left[\frac{1}{2} \lambda^{A} , \frac{1}{2} \lambda^{B} \right] = i f_{ABC} \frac{1}{2} \lambda^{C}$

perturbative rescaling :

$$v^{A}_{\mu} = g\, v^{A}_{\mu\, pert} , \quad F^{A}_{\mu\nu} = g\, F^{A}_{\mu\nu\, pert}$$

Degrees of freedom are seen in jets , in (e.g.) the energy momentum sum rule in deep inelastic scattering but not clearly in spectroscopy.

Completing $\Delta\,\mathcal{L}$ in Fermi gauges

$$\Delta\,\mathcal{L} = \left\{ \begin{array}{c} - \frac{1}{2\,\eta\,g^{2}} \left(\partial_{\mu} v^{\mu\, A} \right)^{2} \\ + \partial^{\mu} \overline{c}^{A} \left(D_{\mu}\, c \right)^{A} \end{array} \right\} \; ; \quad \eta \; : \text{ gauge parameter}$$

(3)

ghost fermion fields : $c\,,\, \overline{c}\;\; ; \quad \left(D_{\mu}\, c \right)^{A} = \partial_{\mu}\, c^{A} - f_{ABD}\, v^{B}_{\mu}\, c^{D}$

gauge fixing constraint : $C^{A} = \partial_{\mu}\, v^{\mu\, A}$

$\longrightarrow$

Gauge boson binary bilocal and adjoint string operators

One goal is, to identify – not just some candidate resonance – gluonic mesons, binary and higher modes, and to relate them to the base quantities within QCD.

(4)
$$\begin{aligned} & B_{[\mu_1 \nu_1], [\mu_2 \nu_2]}(x_1, x_2) = \\ & = F_{[\mu_1 \nu_1]}(x_1; A)\, U(x_1, A; x_2, B)\, F_{[\mu_2 \nu_2]}(x_2; B) \\ & A, B, \cdots = 1, \cdots, 8 \;; \quad \text{no flavor but spin} \end{aligned}$$

$F_{[\mu \nu]}(x; A)$ **denote the color octet of field strengths.**
The quantity $U(x, A; y, B)$ **in eq. (4) denotes the octet string operator, i. e. the path ordered exponential over a straight line path** $\mathcal{C}$ **from y to x**

(5)
$$\begin{aligned} & U(x, A; y, B) = P \exp \left(\int_y^x \Big|_{\mathcal{C}} dz^{\mu} \tfrac{1}{i} v_{\mu}(z, D) \mathcal{F}_D \right)_{A B} \\ & (\mathcal{F}_D)_{AB} = i f_{A D B} \end{aligned}$$

with the local limit $\longrightarrow$

(6) $$B_{[\mu_1 \nu_1], [\mu_2 \nu_2]}(x_1 = x_2 = x) = (:) F^A_{[\mu_1 \nu_1]}(x) F^A_{[\mu_2 \nu_2]}(x)(:)$$

no flavor but spin

The same procedure involving a triplet string applies to $\overline{q}\, q$ bilinears

(7) $$B^q_{[\dot{\mathcal{A}} \dot{f}_1, \mathcal{B} f_2]}(x_1, x_2) = \overline{q}^{\dot{c}_1}_{\mathcal{B} \dot{f}_2}(x_1)\, U(x_1, c_1; x_2, \dot{c}_2)\, q^c_{\mathcal{A} f_1}(x_2)$$
$$U(x_1, c_1; x_2, \dot{c}_2) = P \exp\left(\int_y^x \Big|_{\mathcal{C}} dz^\mu \tfrac{1}{i} v_\mu(z, D) \tfrac{1}{2} \lambda_D \right)_{c_1 \dot{c}_2}$$

flavor *and* spin

with the local limit

(8) $$B^q_{[\dot{\mathcal{B}} \dot{f}_2, \mathcal{A} f_1]}(x_1 = x_2 = x) = (:) \overline{q}^{\dot{c}}_{\mathcal{B} \dot{f}_2}(x)\, q^c_{\mathcal{A} f_1}(x)\,(:)$$

The symbols $(:)$ in eqs. 6 and 8 should indicate that normal ordering of regulating the local limits is required and further that such normal ordering is *not* unique , and dependent on quark masses in the case of the $\overline{q}\, q$ bilinears . $\longrightarrow$

The U1-axial central anomaly involves the local chiral current projections from $B^{\,q}_{\,[\,\dot{\mathcal{B}}\,\dot{f}_{\,2}\,,\,\mathcal{A}\,f_{\,1}\,]}\,(x\,)$ **in eq. 8**

$$\left(j^{\,\pm}_{\,\mu} \right)_{\,\dot{f}_{\,2}\,f_{\,1}}\,(\,x\,) \;=\; B^{\,q}_{\,[\,\dot{\mathcal{B}}\,\dot{f}_{\,2}\,,\,\mathcal{A}\,f_{\,1}\,]}\,(x\,)\,\left(\gamma_{\,\mu}\,\tfrac{1}{2}\,(\,\P\,\pm\,\gamma_{\,5}\,)\right)_{\,\mathcal{B}\,\dot{\mathcal{A}}}$$

(9)

$$=\; (:)\,\overline{q}^{\,\dot{c}}_{\,\dot{f}_{\,2}}\,\gamma^{\,\pm}_{\,\mu}\,q^{\,c}_{\,f_{\,1}}\,(\,x\,)\,(:)$$

$$\gamma_{\,5} \,=\, \gamma_{\,5\,R} \,=\, \tfrac{1}{i}\,\gamma_{\,0}\,\gamma_{\,1}\,\gamma_{\,2}\,\gamma_{\,3} \;\;;\;\; \gamma^{\,\pm}_{\,\mu} \,=\, \gamma_{\,\mu}\,\tfrac{1}{2}\,(\,\P\,\pm\,\gamma_{\,5}\,)$$

The equations of motion for the fermion fields are *and superficially imply* **(upon** $f_{\,1}\;\leftrightarrow\;f_{\,2}$ **)**

$$\begin{aligned} \partial\!\!\!/\,q^{\,c}_{\,f_{\,2}} &= \tfrac{1}{i}\left(\,v\!\!\!/^{\,c\,\dot{c}'} + \delta^{\,c\,\dot{c}'}\,m_{\,f_{\,2}}\right)q^{\,c'}_{\,f_{\,2}} \\ \overline{q}^{\,\dot{c}}_{\,\dot{f}_{\,1}}\,\overleftarrow{\partial\!\!\!/} &= \overline{q}^{\,\dot{c}'}_{\,\dot{f}_{\,1}}\,\tfrac{1}{i}\left(-\,v\!\!\!/^{\,c'\,\dot{c}} - \delta^{\,c'\,\dot{c}}\,m_{\,f_{\,1}}\right) \end{aligned} \quad ; \quad \textbf{no sums over } \dot{f}_{\,1}\,,\,f_{\,2} \;\longrightarrow$$

$$\partial^{\,\mu}\left(j^{\,\pm}_{\,\mu}\right)_{\,\dot{f}_{\,1}\,f_{\,2}} = \tfrac{1}{2\,i}\left(\left(m_{\,f_{\,2}} - m_{\,f_{\,1}}\right)S_{\,\dot{f}_{\,1}\,f_{\,2}} \mp \left(m_{\,f_{\,2}} + m_{\,f_{\,1}}\right)P_{\,\dot{f}_{\,1}\,f_{\,2}}\right)$$

$$S_{\,\dot{f}_{\,1}\,f_{\,2}} = (:)\,\overline{q}^{\,\dot{c}}_{\,\dot{f}_{\,1}}\,q^{\,c}_{\,f_{\,2}}\,(:) \;\;,\;\; P_{\,\dot{f}_{\,1}\,f_{\,2}} = (:)\,\overline{q}^{\,\dot{c}}_{\,\dot{f}_{\,1}}\,\gamma_{\,5}\;q^{\,c}_{\,f_{\,2}}\,(:)$$

(10)

In eq. 10 $m_{\,f}$ **denotes the** *real* , *nonnegative* **quark mass for flavor f.** $\longrightarrow$

From eq. 10 the relations for vector and axial vector currents *superficially* follow

(11)

$$(j_\mu)_{\dot{f}_1 f_2} = \left(j_\mu^+\right)_{\dot{f}_1 f_2} + \left(j_\mu^-\right)_{\dot{f}_1 f_2}$$

$$(j_\mu^5)_{\dot{f}_1 f_2} = \left(j_\mu^+\right)_{\dot{f}_1 f_2} - \left(j_\mu^-\right)_{\dot{f}_1 f_2}$$

$$\partial^\mu (j_\mu)_{\dot{f}_1 f_2} = \frac{1}{i} \left(m_{f_2} - m_{f_1}\right) S_{\dot{f}_1 f_2}$$

$$\partial^\mu (j_\mu^5)_{\dot{f}_1 f_2} = \left(m_{f_2} + m_{f_1}\right) i P_{\dot{f}_1 f_2}$$

As it follows from the original derivation by Adler and Bell and Jackiw [2-1969] in QED, the vecor current Ward identities in eq. 11 can be implemented also in QCD , leaving the axial current ones reduced to the flavor non-singlet case , leaving the U1 axial current divergent anomalous

$$\partial^\mu (j_\mu)_{\dot{f}_1 f_2} = \frac{1}{i} \left(m_{f_2} - m_{f_1}\right) S_{\dot{f}_1 f_2} \quad \checkmark$$

(12)

$$\left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}^{NS}_{\dot{f}_1 f_2} = \left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}_{\dot{f}_1 f_2} - \frac{1}{N_{fl}} \delta_{\dot{f}_1 f_2} \sum_f \left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}_{\dot{f} f}$$

$\longrightarrow$

and similarly

(13) $$\left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}^S_{\dot f_1 f_2} = \sum_f \left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}_{\dot f f}$$

Quark masses and splittings : m_f and $\Delta m_f = m_f - \langle m \rangle$

In the subtitle above $\langle m \rangle$ stands for the mean quark mass

(14) $$\langle m \rangle = \frac{1}{N_{fl}} \sum_f m_f$$

The identities for vector currents in eqs. 11 and 12 can be extended separating the conributions proportional to Δm_f and $\langle m \rangle$

(15) $$\begin{array}{l} \partial^\mu \, (j_\mu)_{\dot f_1 f_2} = \frac{1}{i} \, (\Delta m_{f_2} - \Delta m_{f_1}) \, S_{\dot f_1 f_2} \ \surd \\ \partial^\mu \, (j_\mu^5)^{NS}_{\dot f_1 f_2} = \quad (\Delta m_{f_2} + \Delta m_{f_1}) \, i P^{NS}_{\dot f_1 f_2} \ \surd \\ \partial^\mu \, (j_\mu^5)^S_{\dot f_1 f_2} = 2 \, \langle m \rangle \, i P^S \ \surd\!\!/ \ [\longrightarrow + \delta_5] \end{array}$$

[a] $$\delta_5 = (2 N_{fl}) \frac{1}{32\pi^2} F^A_{\mu\nu} \widetilde{F}^{\mu\nu A} \Big|_{\to ren.gr.inv} \quad ; \quad \widetilde{F}^A_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\sigma\tau} F^{\mu\nu A}$$ $\rightarrow$

[a] δ_5 was – as far as I know from Yair Zarmi and Anthony Hay – introduced by Murray Gell-Mann in lectures $\sim$ 1970 in Hawaii .

The singlet axial current anomaly

We shall return to the question of how the local operator $ch_2 \, (\, F \,) \, \equiv \, \frac{1}{32\pi^{\,2}} \, (:) \, F^{\,A}_{\,\mu \, \nu} \, \widetilde{F}^{\,\mu \, \nu \, A} \, (:)$ is to be normalized and rendered renormalization group invariant [3-1991] . Here we just assume this to have been achieved and denote the U1-axial anomaly, the first of the central two, in its general form (eq. 15)

(16)
$$\left\{ \, \partial^{\,\mu} \, \left(\, j^{\,5}_{\,\mu} \, \right)^{\,S} \, = \, 2 \, \langle \, m \, \rangle \, i \, P^{\,S} \, + \, \delta_{\,5} \, \right\} \, (\, x \,)$$
$$\delta_{\,5} \, = \, \left(\, 2 \, N_{\,fl} \, \right) \, \frac{1}{32\pi^{\,2}} \, (:) \, F^{\,A}_{\,\mu \, \nu} \, \widetilde{F}^{\,\mu \, \nu \, A} \, (:) \, \Big|_{\,\rightarrow ren.gr.inv}$$

From here it is conceptually clear how the scale- (or trace-) anomaly arises but strictly within QCD . The renormalizability of a field theory in the limit of uncurved space-time gives rise to a local , symmetric and *conserved* **energy momentum tensor , implying exact Poincaré invariance**

(17)
$$\{ \, \vartheta_{\,\mu \, \nu} \, = \, \vartheta_{\,\nu \, \mu} \, \} \, (\, x \,)$$
$$\partial^{\,\nu} \, \vartheta_{\,\mu \, \nu} \, = \, 0$$

In connection with the normal ordering questions it is important to admit in the precise form of the energy momentum tensor a nontrivial vacuum expected value , which $\longrightarrow$

in view of exact Poincaré invariance must be of the form

$$\langle \Omega | \vartheta_{\mu\nu}(x) | \Omega \rangle = \frac{1}{4} \eta_{\mu\nu} \tau$$

$$\left\{ \begin{array}{c} \eta_{\mu\nu} = diag(1, -1, -1, -1) \\ \tau \end{array} \right\}$$ independent of x $\longrightarrow$

(18)

$$\Delta \vartheta_{\mu\nu}(x) = \vartheta_{\mu\nu}(x) - \langle \Omega | \vartheta_{\mu\nu}(x) | \Omega \rangle \times \left\{ \begin{array}{l} \widehat{\P} \\ \text{or} \quad | \Omega \rangle \langle \Omega | \end{array} \right.$$

with $\partial^{\nu} \Delta \vartheta_{\mu\nu}(x) = 0 \;;\; \langle \Omega | \Delta \vartheta_{\mu\nu}(x) | \Omega \rangle = 0$

In eq. 18 $\widehat{\P}$ denotes the unit operator in the entire Hilbert space of states , while $P_{\Omega} = | \Omega \rangle \langle \Omega |$ stands for the projector on the ground state .

Furthermore from the two local , *conserved* tensors in eq. 18 only $\Delta \vartheta_{\mu\nu}(x)$ with vanishing vacuum expected value is acceptable as representing the conserved 4 momentum *operators* in the integral form

(19)
$$\widehat{P}_{\mu} = \int_t d^3 x \, \Delta \vartheta_{\mu\, 0}(t, \vec{x})$$

$\longrightarrow$

All these arguments *notwithstanding* to subtract any eventual vacuum expected values of local operators , often put forward as mathematical prerequisites , it is wise *not to do so* in the presence of spontaneous parameters , the dynamical origin of spontaneous symmetry breaking, e.g. chiral symmetries in the limit or neighbourhood of some $m_f \rightarrow 0$.
Using the (classical) equations of motion pertaining to the Lagrangean in eqs. 1 - 3

$$\left(D_{\nu} F^{\mu\nu} \right)^{A} = j^{\mu\ A} (\overline{q} , q) \ ; \ \ F \rightarrow F_{pert}$$

$$\left(D_{\varrho} F^{\mu\ \nu} \right)^{A} = \partial_{\varrho} F^{\mu\ \nu\ A} - f_{ABD} v_{\varrho}^{B} F^{\mu\ \nu\ D}$$

(20)
$$j_{\mu}^{A} (\overline{q} , q) = g \overline{q}^{\dot{c}}_{\dot{\mathcal{A}}\ \dot{f}} (\gamma_{\mu})_{\dot{\mathcal{A}}\ \mathcal{B}} \tfrac{1}{2} \left(\lambda^{A} \right)_{c\dot{c}'} q^{c'}_{\mathcal{A}\ f}$$

$$i \left(\gamma^{\mu} D_{\mu} q \right)^{c}_{\mathcal{A}\ f} = m_{f}\, q^{c}_{\mathcal{A}\ f} \quad \textbf{and} \quad q \rightarrow \overline{q}$$

$$\left(D_{\mu} q \right)^{c}_{\mathcal{A}\ f} = \left[\partial_{\mu} \delta_{c\dot{c}'} + i g v_{\mu}^{D} \tfrac{1}{2} \left(\lambda^{D} \right)_{c\dot{c}'} \right] q^{c'}_{\mathcal{A}\ f}$$

the associated form of the energy momentum becomes $\longrightarrow$

(21) $$\vartheta^{(cl)}_{\mu\,\nu} = \left[\begin{array}{l} F^{\ A}_{\mu\,\varrho}\, F^{\varrho\ A}_{\ \nu} - \frac{1}{4}\,\eta_{\,\mu\,\nu}\, F^{\ A}_{\sigma\,\varrho}\, F^{\varrho\,\sigma\,A} + \\ + \frac{1}{2} \left[\overline{q}_{\,\dot{f}}\, \gamma_{\,\mu}\, \frac{i}{2}\, \overleftrightarrow{D}_{\nu}\, q_{\,f} + \mu \leftrightarrow \nu \right] \end{array} \right]$$

and using once more the fermion part of the equations of motion the trace of the classical energy momentum tensor becomes [a]

(22) $$\vartheta^{\,\mu\ (cl)}_{\ \ \mu} = \sum_{f} m_{\,f}\, S_{\,\dot{f}\,f} \ \ ; \ \ S_{\,\dot{f}_{\,1}\,f_{\,2}} = (:)\, \overline{q}^{\ \dot{c}}_{\,\dot{f}_{\,1}}\, q^{\ c}_{\,f_{\,2}}\, (:)$$

The scale- or trace- anomaly

From the classical soft fermionic contribution to the trace of the energy momentum tensor there is a clear conjecture , also by Murray Gell-Mann , of the anomalous contribution , which subsequently became the scale- or trace- anomaly within QCD

(23) $$\begin{array}{l} \vartheta^{\,\mu}_{\ \ \mu} = \sum_{f} m_{\,f}\, S_{\,\dot{f}\,f} + \delta_{\,0} \\ \delta_{\,0} = - \left(-2\,\beta\,(\,g\,)\,/\,g^{\,3} \right) \left[\frac{1}{4}\,(:)\, F^{\ A}_{\mu\,\nu}\, F^{\,\mu\,\nu\,A}\,(:) \right]_{\rightarrow\, ren.gr.inv} \end{array}$$

$\longrightarrow$

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[a] **The classical energy momentum (density-) tensor can be obtained from the coupling to an external gravitational field .**

The two central anomalies alongside : scale- or trace- and U1-axial anomaly

We collect the two anomalous identities in eqs. 23 and 16

$$\left\{ \vartheta^{\mu}{}_{\mu} = \sum_f m_f S_{\dot{f} f} + \delta_0 \right\} (x)$$
$$\left\{ \partial^{\mu} \left(j^{5}_{\mu} \right)^{S} = 2 \langle m \rangle \, i P^{S} + \delta_5 \right\} (x)$$

(24)
$$\delta_0 = - \left(-2 \beta(g) / g^3 \right) \left[\tfrac{1}{4} (:) F^{A}_{\mu\nu} F^{\mu\nu A} (:) \right]_{\to ren.gr.inv}$$
$$\delta_5 = \left(2 N_{fl} \right) \frac{1}{8\pi^2} \left[\tfrac{1}{4} (:) F^{A}_{\mu\nu} \widetilde{F}^{\mu\nu A} (:) \right]_{\to ren.gr.inv}$$

$$- \beta / g^3 = \frac{1}{16\pi^2} b_0 + O(Y) \; ; \; Y = g^2 / (16 \pi^2$$

$\beta(g)$: **Callan-Symanzik rescaling function in QCD**

The qualification 'central' for the anomalies in eq. 24 stands for the property that in rendering the square coupling constant and the associated $\vartheta -$ parameter in the gauge boson *renormalized* Lagrangean density x dependent

(25)
$$\mathcal{L}_{g.b.} = - \frac{1}{g^2} \frac{1}{4} (:) F^{A}_{\mu\nu} F^{\mu\nu A} (:) + \vartheta \frac{1}{8\pi^2} \frac{1}{4} (:) F^{A}_{\mu\nu} \widetilde{F}^{\mu\nu A} \longrightarrow$$
$$g^2 \to g^2(x) \; ; \; \vartheta \to \vartheta(x)$$

maintains perturbative renormalizability and acts together with suitable boundary conditions $\rightarrow$

as external sources for the scalar and pseudoscalar local field strength bilinears, rendered renormalization group invariant by absorbing the g dependent part of the rescaling function $\left(\beta(g)/g^3\right) \,/\, \left(\beta(g')/g'^3\right)_{g' \to 0}$ into the field strength bilinear and likewise normalizing the second Chern character at zero distance $\leftrightarrow \ \mu = \infty$. The field strength bilinears in brackets without subscript qualifiers denote the renormalization group invariant operators.

$$\left[\tfrac{1}{4}\,(:)\,F^{\;A}_{\mu\,\nu}\,F^{\,\mu\,\nu\,A}\,(:)\right] = \frac{\left(\beta(g)/g^3\right)}{\left(\beta(g')/g'^3\right)_{g' \to 0}}\left[\tfrac{1}{4}\,(:)\,F^{\;A}_{\mu\,\nu}\,F^{\,\mu\,\nu\,A}\,(:)\right]_{\to ren.gr.inv}$$

(26) $$\left[\tfrac{1}{4}\,(:)\,F^{\;A}_{\mu\,\nu}\,\widetilde{F}^{\,\mu\,\nu\,A}\,(:)\right] \ : \ \text{normalized at zero distance}$$

With these renormalizations the central anomalies (eq. 24) take the form [a]

$$\begin{aligned} \left\{\vartheta^{\,\mu}{}_{\mu} \right. &= \left. \textstyle\sum_f m_f\, S_{\dot{f}\,f} + \delta_0 \right\}(x) \\ \left\{\partial^{\,\mu}\,\left(j^{\,5}_{\,\mu}\right)^{S} \right. &= \left. 2\,\langle m \rangle\, i\, P^{\,S} + \delta_5 \right\}(x) \end{aligned}$$

(27)

$$\begin{aligned} \delta_0 &= -\,b_0\,\tfrac{1}{8\pi^2} \quad \left[\tfrac{1}{4}\,(:)\,F^{\;A}_{\mu\,\nu}\,F^{\,\mu\,\nu\,A}\,(:)\right] \quad ; \quad b_0 = \tfrac{1}{3}\left(33 - 2\,N_{fl}\right) \\ \delta_5 &= \left(2\,N_{fl}\right)\,\tfrac{1}{8\pi^{\,2}} \quad \left[\tfrac{1}{4}\,(:)\,F^{\;A}_{\mu\,\nu}\,\widetilde{F}^{\,\mu\,\nu\,A}\,(:)\right] \end{aligned}$$

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[a] Credit is due to H. Kluberg-Stern and J. B. Zuber segregating gauge variant and invariant twist 4 operators [4-1975] .

2 Resonances composed of gauge bosons – glueballs

here restricting to mainly binaries with $J^{PC} = 0^{++}, 0^{-+}, 2^{++}$ [a]

Color neutral gauge boson binaries can be described by a wave function in a similar way as photon binaries, as originally derived by L.D Landau and C. N. Yang [5-1948] , [6-1950] . We choose a common time and at that time fix gauge potentials and transform to the c.m. frame and spacelike relative momenta

$$v_0^A = 0 \quad , \quad \partial_m v^{m\,A}(t\,;\,\vec{x}) = 0 \quad m = 1,2,3 \quad \longrightarrow$$

(28) $$\Psi_{mn}^{AB}(t\,;\,\vec{k}) = \delta^{AB} \exp(-iMt)\,\psi_{mn}(\vec{k}) \quad ; \quad k_m \psi_{mn} = k_m \psi_{nm} = 0$$

$$\psi_{mn}(\vec{k}) = \psi_{nm}(-\vec{k})$$ **Bose symmetry**

In the wake of excitement after the discovery of the J/ψ $c\bar{c}$ vector-meson in November 1974 [7-1974] , [8-1974] we took a new look at *pure* gluemesons with Harald Fritzsch [9-1975] .

d) <u>Phenomenology of Glue Mesons</u>

Thus far no glue mesons have been identified in the hadronic spectrum, although there are several candidates of I = 0 -mesons which may consist predominantly of glue. Among those are the broad ε-enhancement in the 0^+-wave at ~ 600 MeV, and the E(1420)-meson. Both may be two-gluon resonances.

Fig 2 : Quote from ref. [9-1975] $\longleftrightarrow$

$\longrightarrow$

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[a] These lowest states we called "the three musketeers" with Wolfgang Ochs .

There exist 3 series with respect to total angular momentum, all with C=+

series	helicity transfer	P	J	candidates
1	0	+	0 ,2,4 $\cdots$	$\longrightarrow$ discussed below
2	0	-	0 ,2,4 $\cdots$	η (1405)
3	$\pm$ 2	+	2 ,3,4 $\cdots$	f2 (1640) f2 (1810) $\cdots$ f2 (2150)

Table 1

We list the local gauge boson bilinear operators, which can connect the ground state to the lowest spin states in the three series in Table 1

(29)
$$\begin{aligned}
\langle\,\Omega\,|\ \left[\,\tfrac{1}{4}\,(:)\,F^{\,A}_{\,\mu\,\nu}\,F^{\,\mu\,\nu\,A}\,(:)\,\right]\,\left|\,0^{\,++}\,,\,p\,\right\rangle &= f_{+}\,m^{\,2}\,(\,0^{\,++}\,) \\
\langle\,\Omega\,|\ \left[\,\tfrac{1}{4}\,(:)\,F^{\,A}_{\,\mu\,\nu}\,\widetilde{F}^{\,\mu\,\nu\,A}\,(:)\,\right]\,\left|\,0^{\,-+}\,,\,p\,\right\rangle &= f_{-}\,m^{\,2}\,(\,0^{\,-+}\,) \\
\langle\,\Omega\,|\qquad \vartheta_{\,\mu\nu}\,(\,F\,F\,)\qquad \left|\,2^{\,++}\,,\,p\,,\,\varepsilon\,\right\rangle &= f_{2}\,m^{\,2}\,(\,2^{\,++}\,)\,\varepsilon_{\,\mu\nu}\,(\,S_{\,z}\,)
\end{aligned}$$

$$\varepsilon_{\,\mu\nu} = \varepsilon_{\,\nu\mu}\ ,\quad \varepsilon^{\,\mu}_{\,\mu} = 0\ ,\quad p^{\,\mu}\,\varepsilon_{\,\mu\nu} = 0\ ,\quad \varepsilon^{\,\mu\nu}\,\varepsilon_{\,\mu\nu} = 1\ ,\quad p^{\,2} = m^{2}$$

3 A meson composed of glue among scalar resonances – an unresolved question

partial aspects in elastic $\pi\pi$ scattering and $p\ (\ \pi\pi\ )\ p$ central production

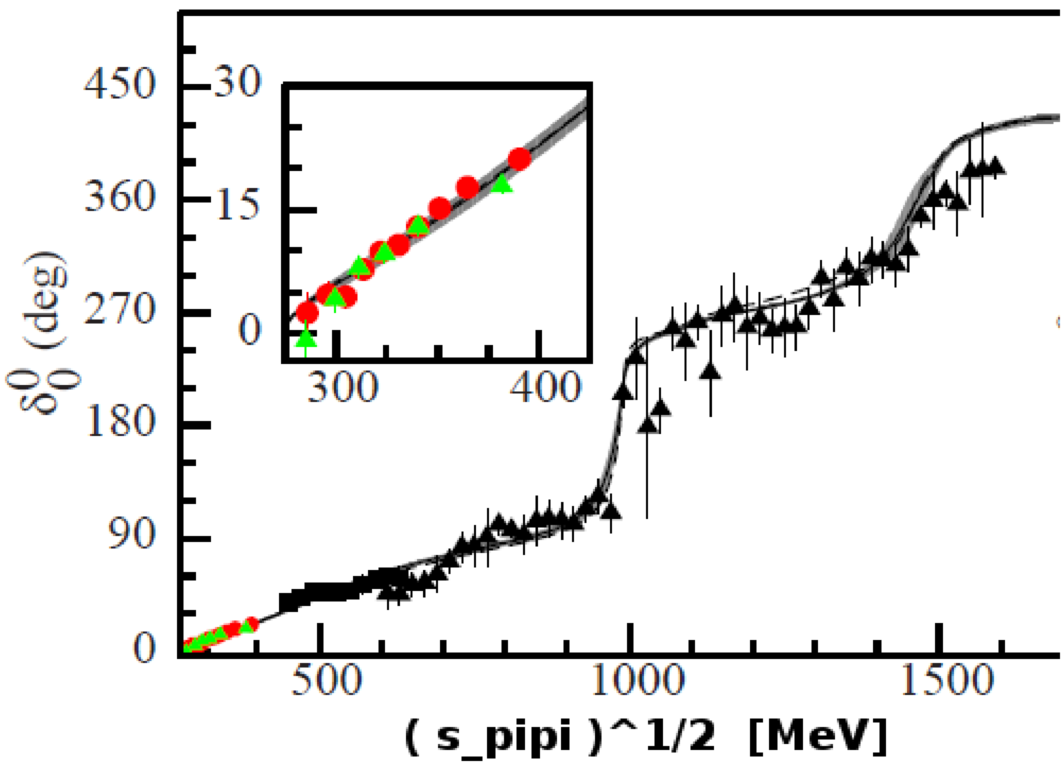

Fig 3 : $\pi\pi\ ;\ I\ =\ 0$ **phase shifts from [11-2008]**

The data points near $\pi\pi$ threshold are the result of the precision measurement *and analysis* of the Na48/2 collaboration [12-2008] (red points) without applying a correction for isospin violations →

illustrated in figure 4 taken from ref. [12-2008] below and yielding excellent agreement with the determination of the isospin exact extrapolation of the pi pi scattering length by Gilberto Colangelo, Jürg Gasser and Heinrich Leutwyler [13-2001] .

$$a_0 = 0.220 \pm 0.005_{\text{theo}}$$
$$a_2 = -0.0444 \pm 0.0010_{\text{theo}}$$

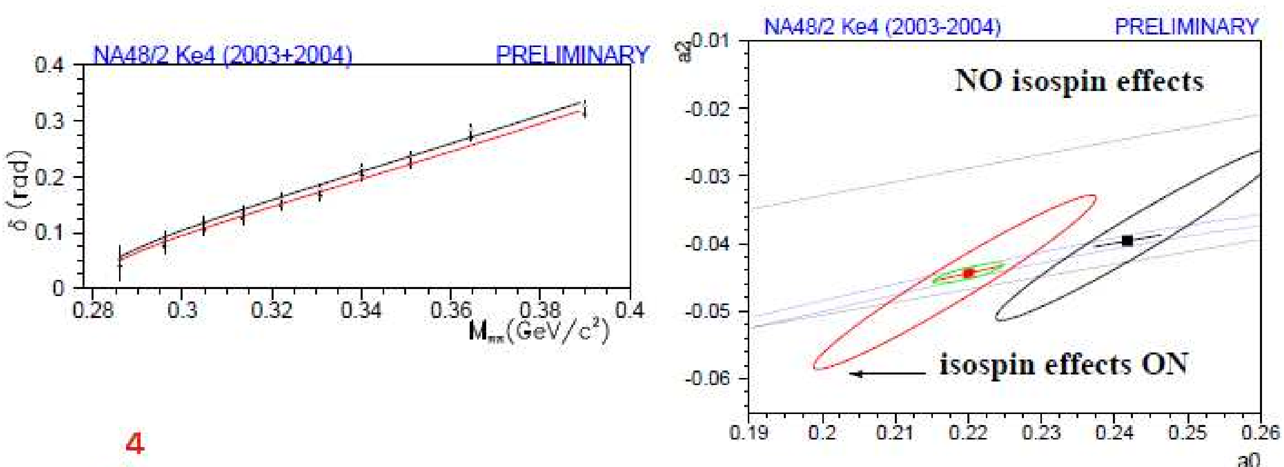

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Figure 3: Left: Phase shift (δ) measurements without mass effects (top black line) and with mass effects included (bottom red line). In each case the line corresponds to the 2-parameter fit. Right: Fits of the NA48/2 K_{e4} data in the (a_0,a_2) plane without (black) and with (red) isospin mass effects. The symbols are the result of the one-parameter fit imposing the ChPT constraint. The small (green) ellipse corresponds to the most accurate prediction from ChPT.

→

Fig. 3 (from Oller et al. [11-2008]) shall illustrate that the new and precise data from Na48/2 does not pose difficulties to an *interpolation* to the phase shift analysis of the CERN-MUNICH collaboration , as documented in Fig. 5 below

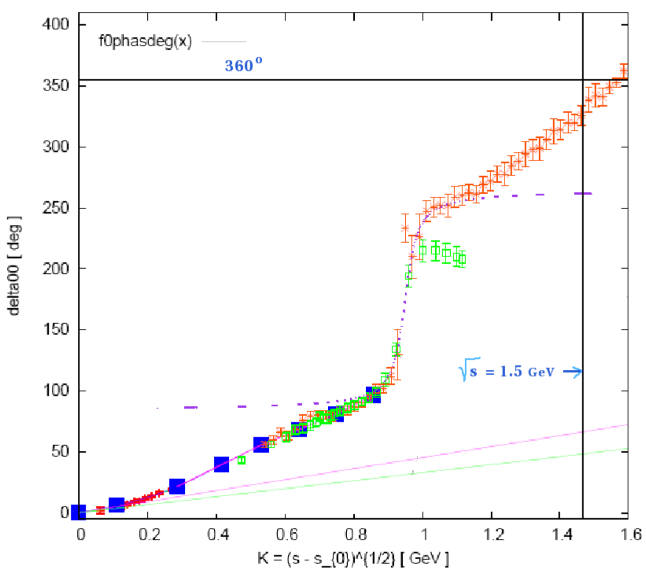


Fig. 5 : The $\pi\pi$, $I = 0$ *ideal* elastic s-wave from threshold to $\sim\ 1.625$ GeV.

Fig. 5 plots the s-wave phase shifts versus $K\ =\ (M^{\,2}_{\pi\pi}\ -\ 4\,m^{\,2}_{\pi})^{1/2}$.

■ : from ref. [13-2001] *Colangelo, Gasser and Leutwyler* **,**

— interpolates ■ , + : from ref. [14-2007] *Na48/2 coll.* **corrected for isospin breaking ⌊,⌉ : from ref. [15-1973]** *Protopopescu et al.* **⌊,⌉ : from ref. [16-1973]** *CERN- -Munich coll. ; W. Ochs , thesis 1973* **.**

– – : minimal meromorphic parametrization of the influence of f0(980) $\longrightarrow$

— , — linear approximations $\delta 00\ =\ 0.5\,a00\,K$,

$\longleftrightarrow$ $a00\,m_{\,\pi}\ =\ 0.22$, $a00\,m_{\,\pi}\ =\ 0.16$.

ideal **in the caption to figure 5 refers to the limit $e\ =\ 0\,,\ m_{\,d}\ =\ m_{\,u}$.**

$\longrightarrow$

The rapid phase variation induced by f0(980) defines two fringes, denoted low and high the two regions

(30)
$$\textbf{low} : \quad 0 \le K \le \sim 0.9 \textbf{ GeV} \quad ; \quad \textbf{high} : \quad \sim 1.0 \textbf{ GeV} \le K \le \sim 1.6 \textbf{ GeV}$$
$$2m_\pi \le \sqrt{s} \le \sim 0.94 \textbf{ GeV} \quad ; \quad \sim 1.04 \textbf{ GeV} \le \sqrt{s} \le \sim 1.625 \textbf{ GeV}$$

The minimal meromorphic parametrization is defined from the complex pole position on the second s - sheet, the K - plane with $\Im K < 0 \quad (s_0 = 4 m_\pi^2)$

$$C_R^2 = \left(K_R - \tfrac{1}{2} i \gamma_R \right)^2 = \mathcal{M}_R^2 - s_0 = \left(M_R - \tfrac{1}{2} i \Gamma_R \right)^2 - s_0$$

(31)
$$S_{mmp} (K_R , \gamma_R ; K) = \frac{|C_R|^2 - K^2 + i \gamma_R K}{|C_R|^2 - K^2 - i \gamma_R K}$$

The minimal meromorphic superposition of N resonances with identical *ideal* quantum numbers – in any two body channel – corresponds to the multiplication of the individual $S_{mmp} (K_{R_\alpha} , \gamma_{R_\alpha} ; K)$ **factors for resonance** $R_\alpha ; \alpha = 1, \cdots, N$ **as defined in eq. 31 .**

(32)
$$S^N_{mmp} (K) = \prod_{\alpha=1}^N S_{mmp} (K_{R_\alpha} , \gamma_{R_\alpha} ; K)$$
$$S = S^N_{bg} S^N_{mmp} \quad ; \quad S^N_{bg} (K) = \eta^N_{bg} (K) \exp\left(2 i \delta^N_{bg} (K) \right)$$

The background introduced above for $J^{PC} = 0^{++} , I = 0 ; \pi\pi \to \pi\pi$ **is defined *relative* to** S^N_{mmp} **given in eq. 32 .** $\longrightarrow$

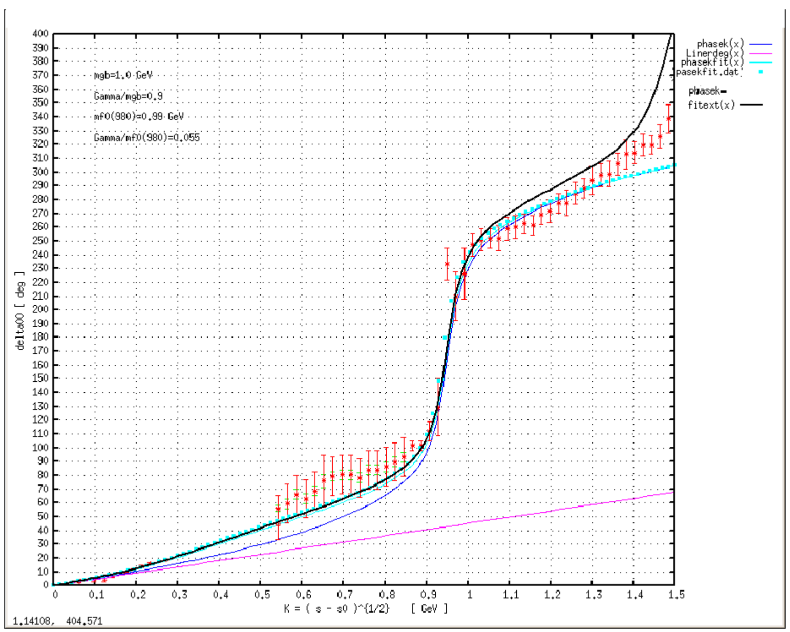
3 a 3 resonances – 1 channel : f0 ($\sim$990) , gb ($\sim$1000) , f0 ($\sim$1500)

elastic $\pi\pi$ scattering and $p\,(\,\pi\pi\,)\,p$ central production

In the following we display results using the form of the $\pi\pi$, $I = 0\,,\,0^{++}$ elastic amplitude with three resonances as given in eq. 32 with mass and width parameters in MeV

$$R_1\ :M\ =\ 980\,;\ \Gamma\ =\ 54\,|\,R_2\ :M\ =\ 1000\,;\ \Gamma\ =\ 900\,|\,R_3\ :M\ =\ 1510\,;\ \Gamma\ =\ 106$$

(33)



To figure 6 **:**

— **: minimal meromorphic phase from two resonances gb and f0(980) with $mf0\ =\ 0.99$ GeV ,** **same ratio $\Gamma_{f0}/mf0 = 0.055$.**

— **: background phase added with same parameters as for —**

▪ ▪ ▪ **: for only 2 resonances with $mf0 = 0.98$GeV , $\Gamma_{f0}/mf0 = 0.055$.**

— **: full phase from three resonances f0(980) , gb *and* f0(1500) (eq. 33) .**

Fig 6 : $\pi\pi\ ;\ I\ =\ 0$ phase shifts from [16-1973] , [17-2008] →

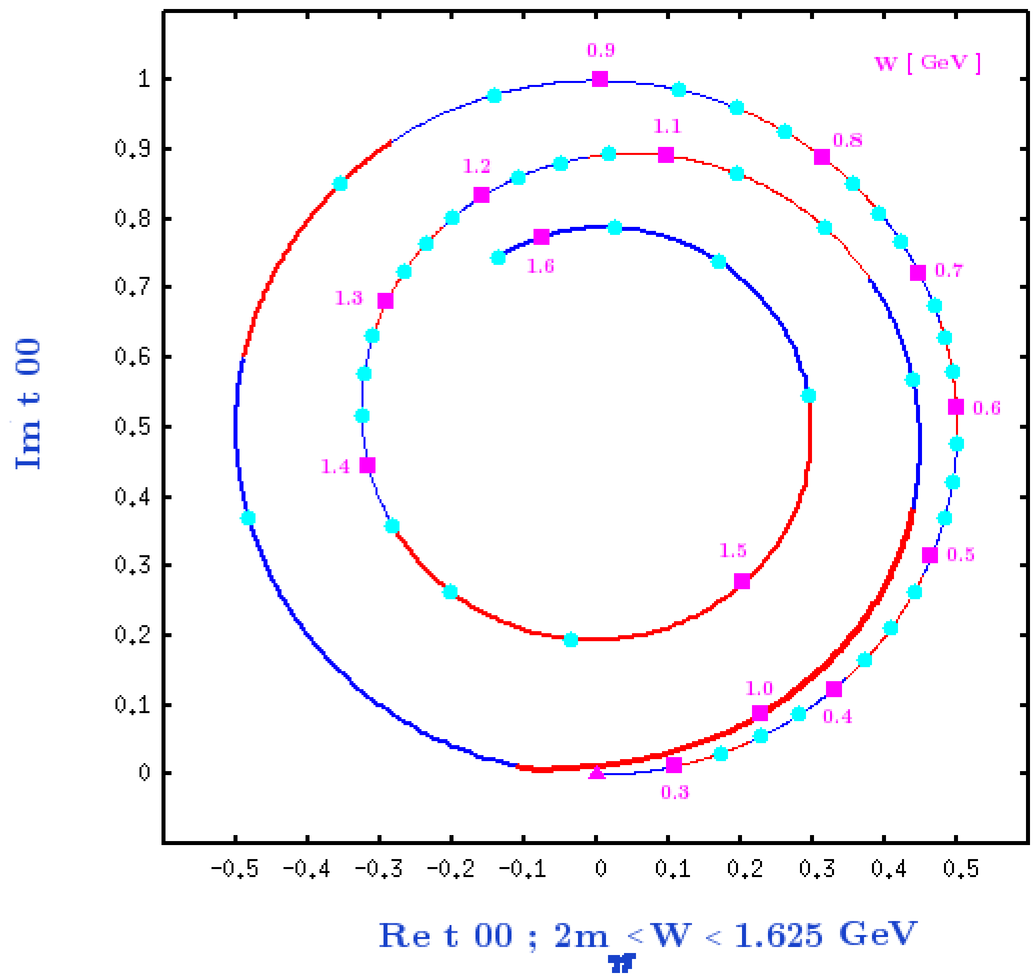


Fig 7 : $\pi\pi$; $I = 0$ **Argand diagram from material used in ref. [17-2008]**

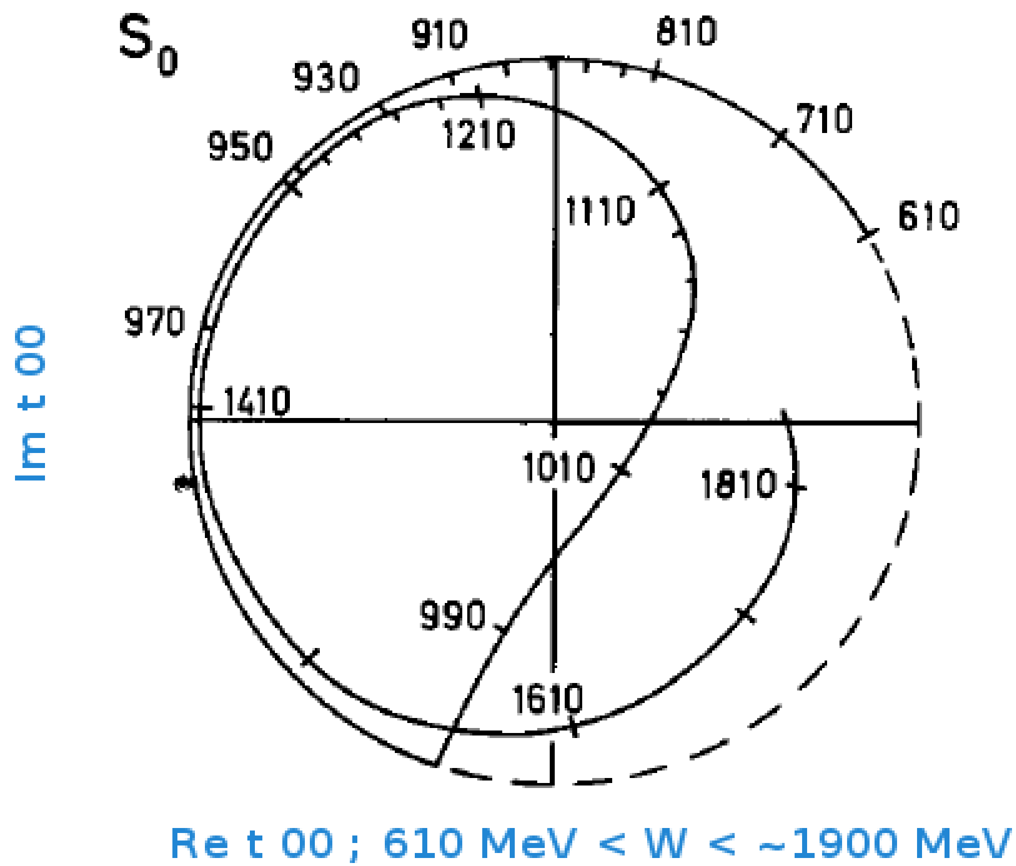


Fig 8 : $\pi\pi\ ;\ I\ =\ 0$ **Argand diagram from the CERN-MUNICH collaboration [16-1973]**

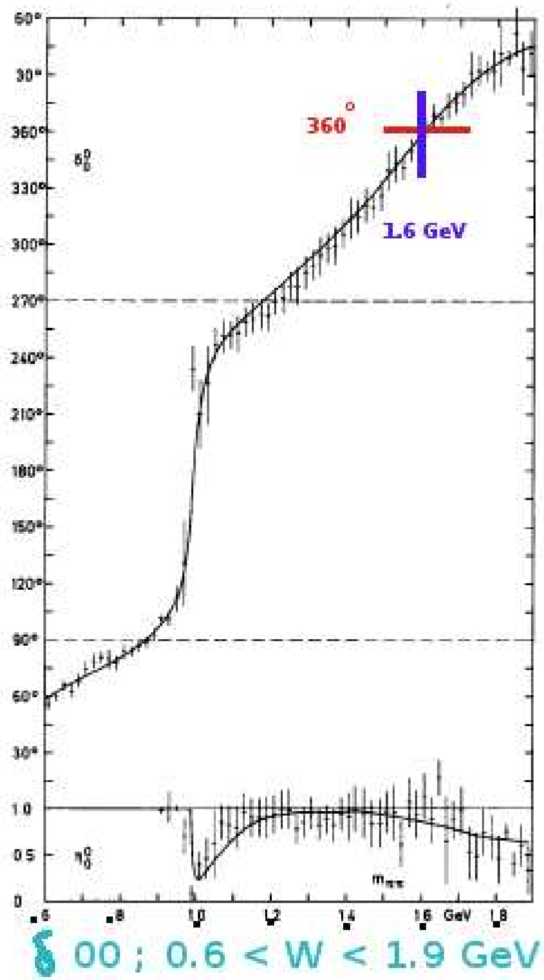


Fig 9 : $\pi\pi$; $I = 0$ **phase shifts from the CERN-MUNICH collaboration [16-1973]**

a tribute to Wolfgang Ochs

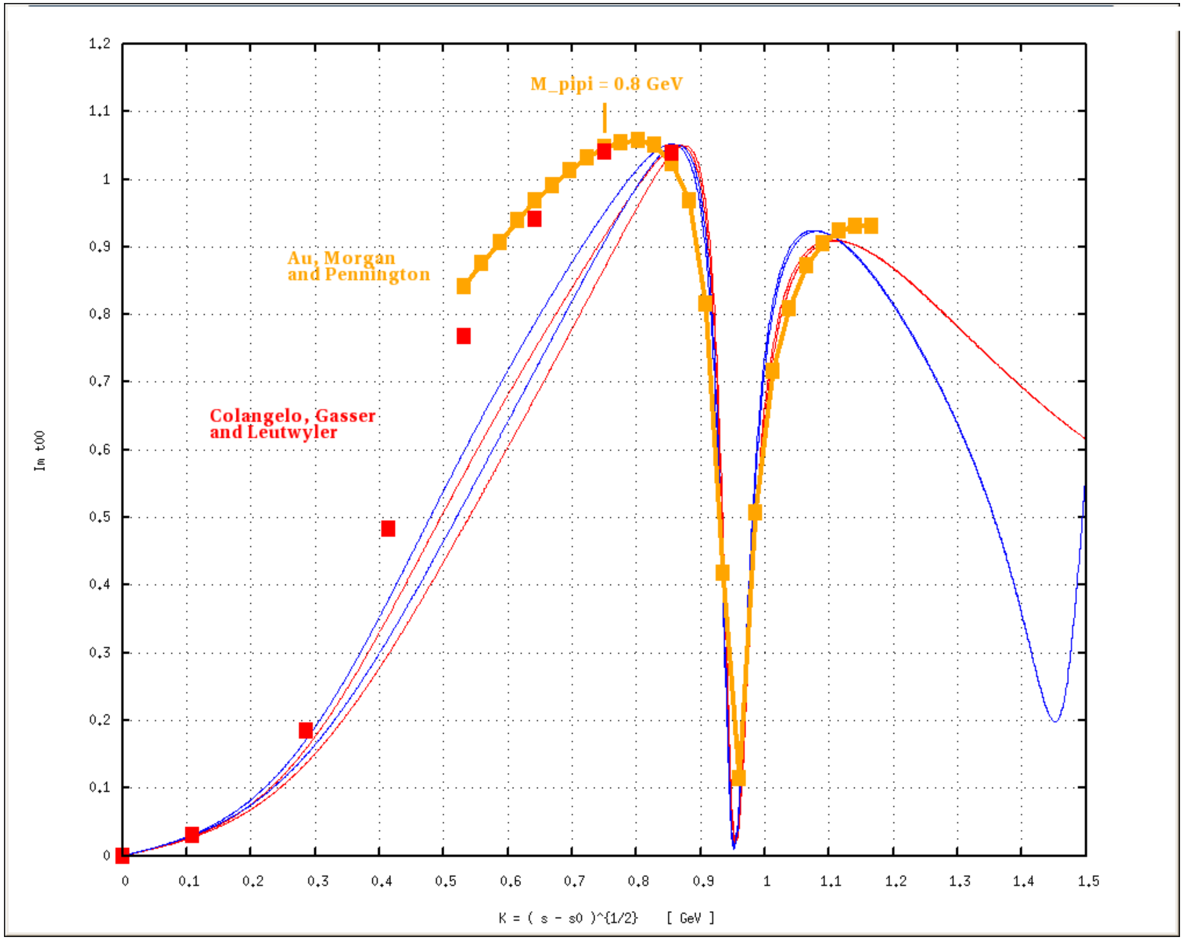
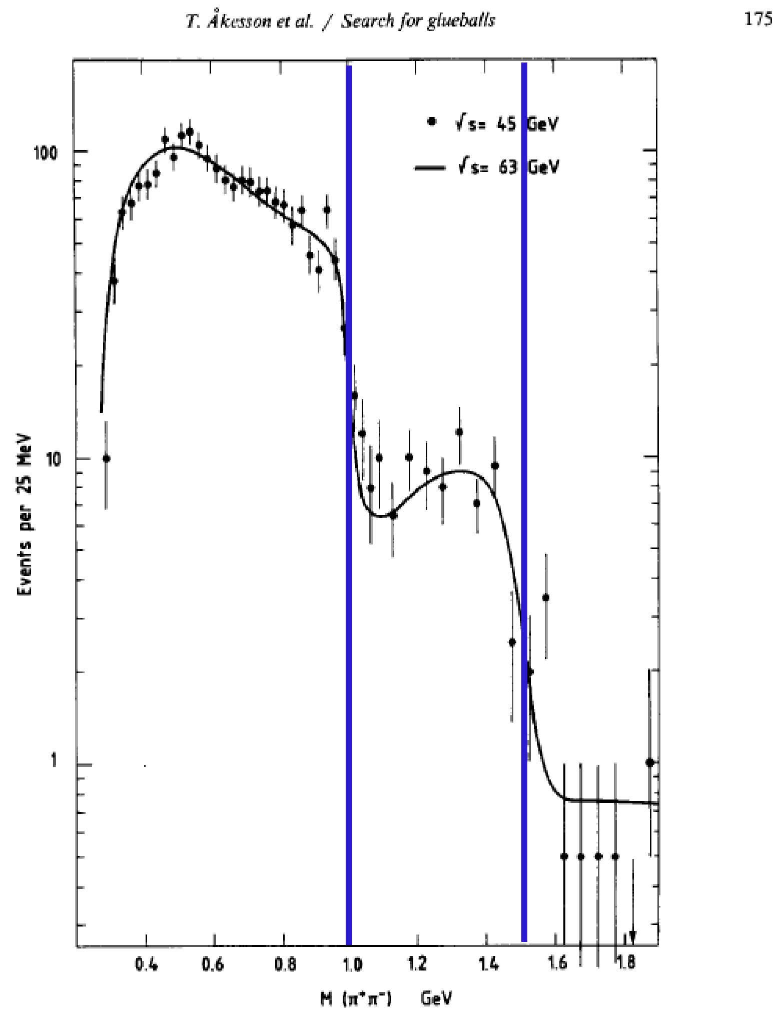


Fig 10 : The imaginary part , $\Im\, T00 \;=\; (\; W \;/\; K\;)\; \Im\, t00$ **in Fig 7 ,**

of the $\pi\pi$ **,** $I = 0$ ***ideal*** **elastic s-wave from threshold to** $\sim\; 1.526$ **GeV**



p p ---> p (pi+ pi-) p central production

Fig 11 : Central production of pion pairs at the ISR $(W_{pp} = 63$ GeV$)$ within the invariant mass range $0.5 < M_{\pi\pi} < 1.9$ GeV from the Axial Field Spectrometer collaboration [18-1986]

I quote from ref. [18-1986] :

”We conclude by commenting that our exclusive data show interesting behaviour in the $\pi^+\pi^-$ - D-wave, but the production is dominated by the S-wave. Although the S*(980) reveals itself in a most striking manner, there is no evidence* for any new 0^{++} states. The lack of a low-mass scalar glueball candidate poses a problem for some conventional models of the glueball spectrum.” [a]

---

[a] The results of the AFS collaboration was a guideline to our common work with Wolfgang Ochs [19-1999] .

best wishes Murray

Fig 80 : thank you

Supplementary slides

a3-10

Fig 11 : Alpine folding

fig-5

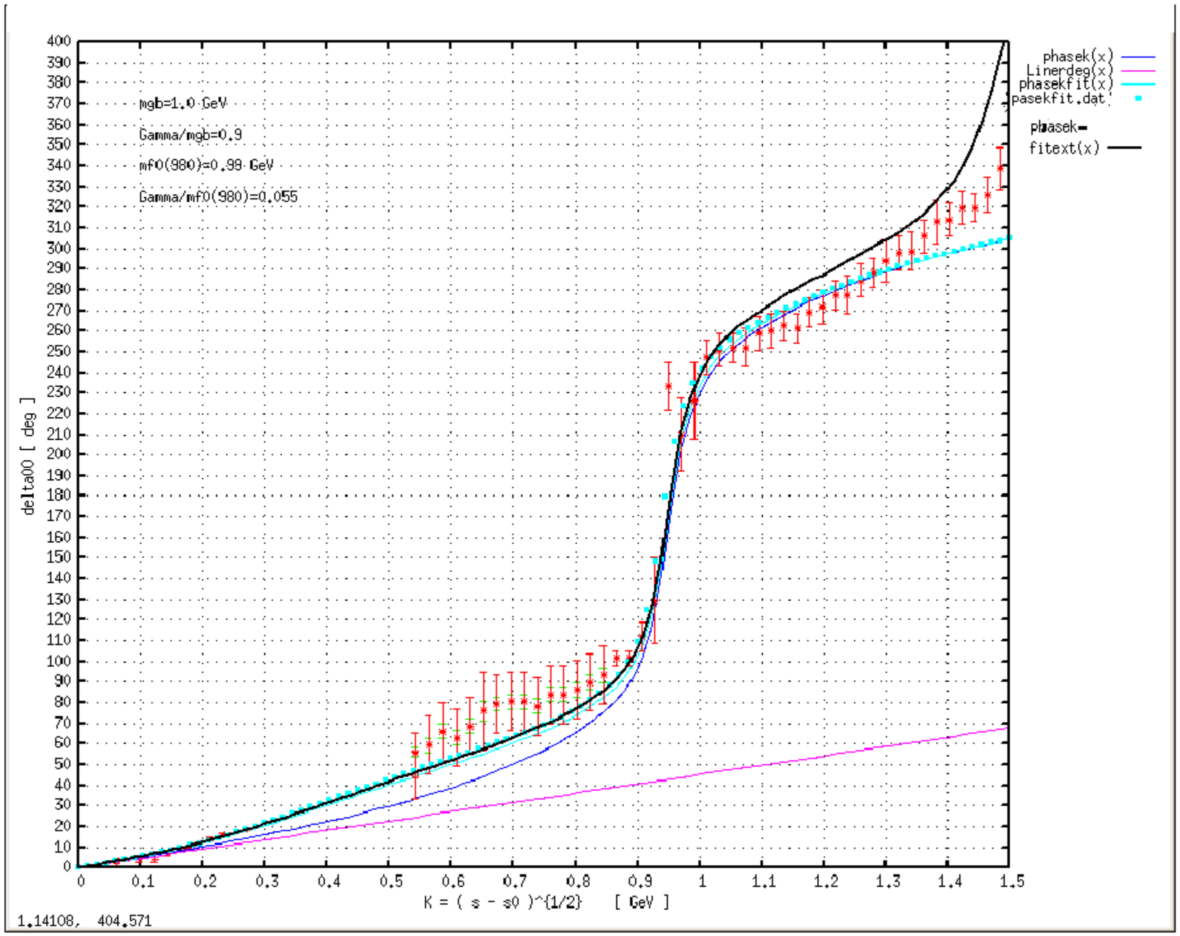


Fig a-5 : $\pi\pi$; $I = 0$ **phase shifts from [17-2008]** $\longleftrightarrow$

fig-5a

To figure 5-a **:**

This is an extension of other figures to include three resonances f0(980) , gb and f0(1500).

— **: minimal meromorphic phase from the superposition of gb and f0(980)**
with mass $mf0 \; = \; 0.99$ **GeV and ratio** $\Gamma_{\,f0} \, / \, mf0 \; = \; 0.055$ **.**

— **: background phase added with mass and width parameters**
as for — and $K_{\,1} \; = \; 0.62$ **GeV**

▪ ▪ ▪ **with different mass for f0(980)** $mf0 \; = \; 0.98$ **GeV ,** $\Gamma_{\,f0} \, / \, mf0 \; = \; 0.055$ **.**

— **: minimal meromorphic phase from the superposition of**
gb , f0(980) *and* **f0(1500) with mass and width parameters**
$mf0(1500) \; = \; 1.51$ **GeV ,** $\Gamma_{\,f0(1500)} \, / \, mf0(1500) \; = \; 0.07$ **,**
and background parameters $\eta^{\,3}_{\,bg} \; = \; 1$
to keep qualitative features of f0(1500) only $\left(K_{\,1} = 0.62 \right.$ **GeV** $,\; B = 4.2$ **GeV** $\left. ^{-2} \right)$**.**

The rise of the s-wave phase towards the end of the
high fringe region was remarked in ref. [f3-2008] .
It formed the entry point of the discussion in ref. [f2-2008] .

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