Neutrino flavors light and heavy – how heavy ?

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$\longrightarrow$

1 Minimal ν-extension of fermion families – left - and right chiral bases

$$\begin{pmatrix} u^1 & u^2 & u^3 & \nu & | & \mathcal{N} & \widehat{u}^3 & \widehat{u}^2 & \widehat{u}^1 \\ d^1 & d^2 & d^3 & e^- & | & e^+ & \widehat{d}^3 & \widehat{d}^2 & \widehat{d}^1 \end{pmatrix}^{\dot\gamma \to L}$$

(1)

$$= (f)^{\dot\gamma} \qquad \textbf{left chiral} \leftrightarrow \quad \dot\gamma = 1,2$$

$$(f)_\alpha = \varepsilon_{\alpha\gamma} \left\{ (f)^{\dot\gamma} \right\}^* = \quad \textbf{right chiral} \leftrightarrow \quad \alpha = 1,2$$

$$\begin{pmatrix} \widehat{u}^1 & \widehat{u}^2 & \widehat{u}^3 & \widehat{\nu} & | & \widehat{\mathcal{N}} & u^3 & u^2 & u^1 \\ \widehat{d}^1 & \widehat{d}^2 & \widehat{d}^3 & e^+ & | & e^- & d^3 & d^2 & d^1 \end{pmatrix}_{\alpha \to R}$$

and family $1 \to 2 \to 3$ **.**

$\rightarrow$

In eq. 1 * denotes the operation of hermitian conjugation in field variable space $f(x)$, whereas $\varepsilon_{\alpha\gamma}$ denotes the symplectic or Lorentz-invariant 2×2 matrix

(2) $$\varepsilon_{\alpha\gamma} = i\,(\sigma_2)_{\alpha\gamma} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

If the spinor indices are suppressed , the two bases shall be denoted by L , R as indicated in eq. 1 .
The symbols — represent a mirror between the left and right sides of components . This allows the mirror operation , which we exemplify acting on the right chiral basis (eq. 1) denoted $\leftrightarrow$ $\longrightarrow$

$$\leftrightarrow\!\!\!\!\!\!+ \begin{pmatrix} \widehat{u}^{\,1} & \widehat{u}^{\,2} & \widehat{u}^{\,3} & \widehat{\nu} & | & \widehat{\mathcal{N}} & u^{\,3} & u^{\,2} & u^{\,1} \\ \widehat{d}^{\,1} & \widehat{d}^{\,2} & \widehat{d}^{\,3} & e^{\,+} & | & e^{\,-} & d^{\,3} & d^{\,2} & d^{\,1} \end{pmatrix}_{\alpha \to R}$$

(3) $$= \begin{pmatrix} u^{\,1} & u^{\,2} & u^{\,3} & \widehat{\mathcal{N}} & | & \widehat{\nu} & \widehat{u}^{\,3} & \widehat{u}^{\,2} & \widehat{u}^{\,1} \\ d^{\,1} & d^{\,2} & d^{\,3} & e^{\,-} & | & e^{\,+} & \widehat{d}^{\,3} & \widehat{d}^{\,2} & \widehat{d}^{\,1} \end{pmatrix}_{\alpha \to R}$$

$$= (\leftrightarrow\!\!\!\!\!\!+ f\,)_{\alpha} \qquad \updownarrow$$

$$\begin{pmatrix} u^{\,1} & u^{\,2} & u^{\,3} & \nu & | & \mathcal{N} & \widehat{u}^{\,3} & \widehat{u}^{\,2} & \widehat{u}^{\,1} \\ d^{\,1} & d^{\,2} & d^{\,3} & e^{\,-} & | & e^{\,+} & \widehat{d}^{\,3} & \widehat{d}^{\,2} & \widehat{d}^{\,1} \end{pmatrix}^{\dot{\gamma} \to L}$$

$\longrightarrow$

1a Dirac like pairing of $(\not\leftrightarrow f)_{\alpha}$ and $(f)^{\dot\gamma}$

To be definite we can also label the components in the left chiral and right chiral bases in eqs. 1 and 3 with numbers (1-16) as shown in eq. 6 below .
Using the so defined 'flavor' components $^{r} = 1, \cdots, 16$
the mirror operation corresponds to the involution in 'flavor' number

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\ 12 & 11 & 10 & 9 & 16 & 15 & 14 & 13 & 4 & 3 & 2 & 1 & 8 & 7 & 6 & 5 \end{matrix}$$

(4)

$$= \begin{pmatrix} r \\ s \end{pmatrix} \quad \rightarrow \quad s = s (r)$$

This corresponds to

(5) $$\{ (\not\leftrightarrow f)^{s} = (f)^{r} \}^{\dot\gamma \rightarrow L} \text{ or } \alpha \rightarrow R \; ; \; s = s (r) \qquad \rightarrow$$

$$\begin{pmatrix} u^{1} & u^{2} & u^{3} & \nu & | & \mathcal{N} & \widehat{u}^{3} & \widehat{u}^{2} & \widehat{u}^{1} \\ d^{1} & d^{2} & d^{3} & e^{-} & | & e^{+} & \widehat{d}^{3} & \widehat{d}^{2} & \widehat{d}^{1} \end{pmatrix}^{\dot{\gamma} \to L}$$

$$= \begin{pmatrix} f^{1} & f^{2} & f^{3} & f^{4} & | & f^{9} & f^{10} & f^{11} & f^{12} \\ f^{5} & f^{6} & f^{7} & f^{8} & | & f^{13} & f^{14} & f^{15} & f^{16} \end{pmatrix}^{\dot{\gamma} \to L}$$

$$\dot{\gamma} \to L \ \leftrightarrow \ \alpha \to R \ : \ (f^{r})_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ (f^{r})^{\dot{\gamma}} \right\}^{*} ; \ r = 1, \cdots, 16$$

$$(\nleftrightarrow f)_{\alpha} =$$

$$= \begin{pmatrix} f^{12} & f^{11} & f^{10} & f^{9} & | & f^{4} & f^{3} & f^{2} & f^{1} \\ f^{16} & f^{15} & f^{14} & f^{13} & | & f^{8} & f^{7} & f^{6} & f^{5} \end{pmatrix}_{\alpha \to R} \qquad (6)$$

$\longrightarrow$

$$(\not\leftrightarrow f)_{\alpha \to R}$$

$$= \begin{pmatrix} u^1 & u^2 & u^3 & \widehat{\mathcal{N}} & | & \widehat{\nu} & \widehat{u}^3 & \widehat{u}^2 & \widehat{u}^1 \\ d^1 & d^2 & d^3 & e^- & | & e^+ & \widehat{d}^3 & \widehat{d}^2 & \widehat{d}^1 \end{pmatrix}_{\alpha \to R}$$

(7) $$\uplus$$

$$\begin{pmatrix} u^1 & u^2 & u^3 & \nu & | & \mathcal{N} & \widehat{u}^3 & \widehat{u}^2 & \widehat{u}^1 \\ d^1 & d^2 & d^3 & e^- & | & e^+ & \widehat{d}^3 & \widehat{d}^2 & \widehat{d}^1 \end{pmatrix}^{\dot{\gamma} \to L}$$

$$= (f)^{\dot{\gamma} \to L}$$

Merging the spinor components according to eq. 7 generates Dirac doubling $\left\{ {}_{\alpha \to R} \uplus {}^{\dot{\gamma} \to L} \right\} \to \lozenge A \; ; \; \lozenge A = 1, \cdots, 4$ $\to$

The so doubled full 16-multiplet has to be treated with special attention for the neutrino-antineutrino fields , which in the $\diamondsuit A \ \rightarrow \ L \ \& \ R$ - basis I denote by $(\ n \ , \ \widehat{n} \ )_{\ \diamondsuit A}$

$$\begin{pmatrix} u^{\ 1} & u^{\ 2} & u^{\ 3} & n & | & \widehat{n} & \widehat{u}^{\ 3} & \widehat{u}^{\ 2} & \widehat{u}^{\ 1} \\ d^{\ 1} & d^{\ 2} & d^{\ 3} & e^{\ -} & | & e^{\ +} & \widehat{d}^{\ 3} & \widehat{d}^{\ 2} & \widehat{d}^{\ 1} \end{pmatrix}_{\diamondsuit A}$$

(8)

$$n_{\ \diamondsuit A} \ = \ \begin{pmatrix} \widehat{\mathcal{N}}_{\ \alpha} \\ \nu^{\ \dot{\gamma}} \end{pmatrix} \quad , \quad \widehat{n}_{\ \diamondsuit A} \ = \ \begin{pmatrix} \widehat{\nu}_{\ \alpha} \\ \mathcal{N}^{\ \dot{\gamma}} \end{pmatrix}$$

$$\widehat{\mathcal{N}}_{\ \alpha} \ = \ \varepsilon_{\ \alpha\gamma} \ \left\{ \mathcal{N}^{\ \dot{\gamma}} \right\}^{\ *} \quad , \quad \widehat{\nu}_{\ \alpha} \ = \ \varepsilon_{\ \alpha\gamma} \ \left\{ \nu^{\ \dot{\gamma}} \right\}^{\ *}$$

The heavy neutrino states (not stable ones) are mainly absorbed and created by the fields $\mathcal{N} \ , \ \widehat{\mathcal{N}}$ **.**

$\longrightarrow$

The reversed assignments of $\mathcal{N} \leftrightarrow \widehat{\mathcal{N}}$ relative to $\nu \leftrightarrow \widehat{\nu}$ in eqs. 1 - 8 is just a matter of convention , but better suited once one of the chiral bases is selected .

Lets also associate 16 base fields to the doubled basis $(f)_{\diamondsuit A}$

$$(9) \quad \left(\begin{array}{cccc|cccc} u^{1} & u^{2} & u^{3} & n & \widehat{n} & \widehat{u}^{3} & \widehat{u}^{2} & \widehat{u}^{1} \\ d^{1} & d^{2} & d^{3} & e^{-} & e^{+} & \widehat{d}^{3} & \widehat{d}^{2} & \widehat{d}^{1} \end{array} \right)_{\diamondsuit A}$$

$$= (f)_{\diamondsuit A}$$

with the numbering of components $(f^{r})_{\diamondsuit A}$ as in eqs. 4 - 6 . $\rightarrow$

Then charge conjugation (as an operation , not a symmetry) relates

$(\not\leftrightarrow f)_{\diamondsuit A} \leftrightarrow (f)_{\diamondsuit A}$

$$(C \gamma_0)_{\diamondsuit A \diamondsuit B} \{ (\not\leftrightarrow f^{r})_{\diamondsuit B} \}^{*} = (f^{r})_{\diamondsuit A} \tag{10}$$

$$r = 1, \cdots, 16$$

In eq. 10 we have *reported* the irreducible Lorentzian structure inherent in both chiral bases (L , R) **.**
This is equivalent to the $d = 1 + 3$ **chiral representation of the** γ_{μ} **matrices in the associated** $\diamondsuit A$ **basis**

$$\gamma_{\mu} = \begin{pmatrix} 0 & \sigma_{\mu} \\ \widetilde{\sigma}_{\mu} & 0 \end{pmatrix} ; \quad \begin{array}{l} \sigma_{\mu} = (\sigma_0 , \ \vec{\sigma}) \\ \widetilde{\sigma}_{\mu} = (\sigma_0 , - \vec{\sigma}) \end{array} ; \ \sigma_0 = ¶ , \ \sigma_{1,2,3} \ : \ \text{Pauli matrices}$$

$$\gamma_{5\,R} = \frac{1}{i} \gamma_0 \gamma_1 \gamma_2 \gamma_3 = \begin{pmatrix} ¶ & 0 \\ 0 & -¶ \end{pmatrix} = - \gamma_{5\,L} ; \ C = \begin{pmatrix} \varepsilon & 0 \\ 0 & -\varepsilon \end{pmatrix} = \frac{1}{i} \gamma_0 \gamma_2$$

$$\gamma_{\mu} = \eta_{\mu\nu} \gamma^{\nu} ; \ \eta_{\mu\nu} = diag (1 , -1 , -1 , -1) \ : \ \text{uncurved space-time metric} \tag{11}$$

→

Dropping for a moment the spinor indices $\diamondsuit\ A\ ,\ \cdots$ we associate with the involutory operation defined in eq. 10 a unitary transformation , denoted $\widehat{C}\ ,\ \widehat{C}^{\ 2} = ¶$ – charge conjugation

(12)
$$\begin{array}{l} \widehat{C}^{\ -1}\ f\ \widehat{C} = C\ \gamma_{\ 0}\ \{\ f\ \}^{\ *} \equiv f^{\ (C)} \\ \widehat{C}^{\ -1}\ f^{\ *}\ \widehat{C} = C\ \gamma_{\ 0}\ f \ \rightarrow \left(\ f^{\ (C)}\ \right) \equiv (\ \nleftrightarrow f\ ) \leftrightarrow \\ \left(\ \left\{\ f^{\ (C)}\ \right\}^{\ (C)}\ \right) = (\ \nleftrightarrow (\ \nleftrightarrow f\ )\ ) = (\ f\ ) \end{array}$$

On the 'side' it follows from eq. 12

(13)
$$(\ C\ \gamma_{\ 0}\ )^{\ 2} = ¶$$

$\rightarrow$

1b $\widehat{C}\,,\ \widehat{P}\,,\ \widehat{T} \rightarrow \widehat{\Theta} = \widehat{C}\,\widehat{P}\,\widehat{T}$

We continue with the discrete unitary transformation $\widehat{P}$ associated with parity and the antiunitary one $\widehat{T}$ corresponding to time reversal , neither of which is a symmetry .
Beginning with parity we associate to it the transformation , component by component $(\, f^{\,r}\,)_{\,\diamondsuit A}$ in the $\diamondsuit A$ basis , suppressing the spinor index (as in eq. 12)

$$\widehat{P}^{\,-1}\,(\,f\,(\,x\,)\,)\,\widehat{P} = (\,i\,\gamma_{\,0}\,f\,(\,Px\,)\,) = \left(\,f^{\,(P)}\,(\,x\,)\,\right)$$

$$\widehat{T}^{\,-1}\,(\,f\,(\,x\,)\,)\,\widehat{T} = (\,C\,\gamma_{\,5}\,f\,(\,Tx\,)\,) = \left(\,f^{\,(T)}\,(\,x\,)\,\right)$$

$$\gamma_{\,5} = +\,\gamma_{\,5\,R}$$ + to be definite

$$x = (\,x^{\,0}\,,\,\vec{x}\,)\ ;\ Px = (\,x^{\,0}\,,\,-\,\vec{x}\,)\ ;\ Tx = (\,-\,x^{\,0}\,,\,\vec{x}\,)$$

(14)

$\longrightarrow$

The mirror relation in eq. 12 is algebraic , so we must check the consistency of the $\widehat{P}$, $\widehat{T}$ **operators as defined in eq. 14 . We do this in appendix 1 , from where we report (from eqs. 71 - 73)**

$$\widehat{C}^{\,2} \sim ¶ \,,\; \widehat{P}^{\,2} \sim -¶ \,,\; \widehat{T}^{\,2} \sim -¶$$

(15)

$$\widehat{C}\,\widehat{P} \sim \widehat{P}\,\widehat{C} \,,\; \widehat{C}\,\widehat{T} \sim \widehat{T}\,\widehat{C} \,,\; \widehat{P}\,\widehat{T} \sim -\widehat{T}\,\widehat{P}$$

Next we turn to the antiunitary *symmetry* $\widehat{\Theta} = \widehat{C}\,\widehat{P}\,\widehat{T}$

$$\widehat{\Theta}^{\,-1} \left(f \left(x \right) \right) \widehat{\Theta} = \left(C \gamma_0 \left(\tfrac{1}{i} \right)^* \gamma_0 C \gamma_5 f^{\,*} \left(-x \right) \right)$$

$$= \left(\tfrac{1}{i} \gamma_5 f^{\,*} \left(-x \right) \right)$$

$$\widehat{\Theta}^{\,2} = \widehat{C}\,\widehat{P}\,\widehat{T}\,\widehat{C}\,\widehat{P}\,\widehat{T} \sim \widehat{P}\,\widehat{T}\,\widehat{P}\,\widehat{T} \sim -\widehat{P}\,\widehat{T}^{\,2}\,\widehat{P} \sim -¶$$

(16)

→

We project back the action of $\widehat{\Theta}$ [1'] [2] to the left and right chiral bases respectively (eq. 7) , which is equivalent to replacing $\gamma_5 = \gamma_{5\,R}$ in eq. 16 by - 1 for the left chiral - and by + 1 for the right chiral components

$$\widehat{\Theta}^{-1} \left(f^{\dot{\gamma}} (x) \right) \widehat{\Theta} = \left(i \left\{ f^{\dot{\gamma}} \right\}^{*} (-x) \right)$$

(17) $$\widehat{\Theta}^{-1} \left(f_{\alpha} (x) \right) \widehat{\Theta} = \left(- i \left\{ f_{\alpha} \right\}^{*} (-x) \right)$$

$$f_{\alpha} (y) = \varepsilon_{\alpha \gamma} \left\{ f^{\dot{\gamma}} \right\}^{*} (y)$$

The 3 relations in eq. 17 are not independent and checked for consistency in appendix 1 , from where we also recover the relation

(18) $\widehat{\Theta}^{2} \sim - ¶ \rightarrow (- 1)^{\# f} =$ **fermion number parity**

$\longrightarrow$

A few remarks as to $\widehat{\Theta}$ [3] are given here :

1) $\widehat{\Theta}$ combines two *intertwined* operations

$$f^{\dot{\gamma}}(x) \rightarrow \{f^{\dot{\gamma}}\}^*(x) = g^{\gamma}(x) \quad ; \quad g^{\gamma} f'_{\gamma} \ : \ \text{Lorentz (pseudo)scalar}$$

(19)

$$x \rightarrow -x = PT\,x \quad : \quad \text{space-time inversion}$$

2) Space-time inversion is *not* a local transformation , and only unambiguously defined in uncurved space-time .

3) Hence 2) implies that upon inclusion of gravity the proof that $\widehat{\Theta}$ is an (antunitary) symmetry cannot be inferred from locality and Lorentz invariance pertinent to uncurved space-time .

4) A *subtle* question arises : is there in the general case an extension of $\widehat{\Theta}$ which continues to be an antihermitian *genuine symmetry* ?

$\longrightarrow$

2 Projecting out leptons and antileptons

We go backwards along the developed bases , starting with $\Diamond A$ (eqs. 8 and 9)

$$\begin{pmatrix} u^{1} & u^{2} & u^{3} & n & | & \widehat{n} & \widehat{u}^{3} & \widehat{u}^{2} & \widehat{u}^{1} \\ d^{1} & d^{2} & d^{3} & e^{-} & | & e^{+} & \widehat{d}^{3} & \widehat{d}^{2} & \widehat{d}^{1} \end{pmatrix}_{\Diamond A}$$

(20) $$= \left(f \right)_{\Diamond A}$$

$$\searrow \swarrow$$

$$\begin{pmatrix} n & | & \widehat{n} \\ e^{-} & | & e^{+} \end{pmatrix}_{\Diamond A} = \left(\ell \;\; | \;\; \widehat{\ell} \right)_{\Diamond A} \quad \longrightarrow$$

The mirror reflection defined in eq. 6 is naturally projected on the lepton fields , as defined in eq. 20 , where we have introduced the shorthand as displayed in eq. 22 below , wherein we extend charge conjugation to the chiral components of charged leptons $e^{\pm}$

$$\left(e^{+} \right)_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ \left(e^{-} \right)^{\dot{\gamma}} \right\}^{*}$$

$$\left(e^{-} \right)_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ \left(e^{+} \right)^{\dot{\gamma}} \right\}^{*}$$

(21)

$$\left(e^{+} \right)^{\dot{\gamma}} = \widetilde{\varepsilon}^{\dot{\gamma}\dot{\alpha}} \left\{ \left(e^{-} \right)_{\alpha} \right\}^{*}$$

$$\left(e^{-} \right)^{\dot{\gamma}} = \widetilde{\varepsilon}^{\dot{\gamma}\dot{\alpha}} \left\{ \left(e^{+} \right)_{\alpha} \right\}^{*}$$

$$\widetilde{\varepsilon}^{\dot{\gamma}\dot{\alpha}} \equiv \widetilde{\varepsilon}^{\gamma\alpha} = \varepsilon_{\alpha\gamma} \equiv \varepsilon_{\dot{\alpha}\dot{\gamma}}$$

in matrix form , ^{t} = transposed : $\widetilde{\varepsilon} = \varepsilon^{-1} = \varepsilon^{t} = - \varepsilon$ $\longrightarrow$

$$\ell_{\diamondsuit A} = \begin{pmatrix} n \\ e^{-} \end{pmatrix}_{\diamondsuit A} \quad ; \quad \widehat{\ell}_{\diamondsuit A} = (C\gamma_0)_{\diamondsuit A \diamondsuit B} \{ \ell_{\diamondsuit B} \}^{*}$$

and $L \leftrightarrow \widehat{L}$

$$\widehat{\ell}_{\diamondsuit A} = \begin{pmatrix} \widehat{n} \\ e^{+} \end{pmatrix}_{\diamondsuit A}$$

$$n_{\diamondsuit A} = \begin{pmatrix} \widehat{\mathcal{N}}_{\alpha} \\ \nu^{\dot{\gamma}} \end{pmatrix} \quad , \quad \widehat{n}_{\diamondsuit A} = \begin{pmatrix} \widehat{\nu}_{\alpha} \\ \mathcal{N}^{\dot{\gamma}} \end{pmatrix}$$

$$e^{-}_{\diamondsuit A} = \begin{pmatrix} (e^{-})_{\alpha} \\ (e^{-})^{\dot{\gamma}} \end{pmatrix} \quad , \quad e^{+}_{\diamondsuit A} = \begin{pmatrix} (e^{+})_{\alpha} \\ (e^{+})^{\dot{\gamma}} \end{pmatrix}$$

(22)

$\longrightarrow$

The complete projections on the L - and R - chiral bases (eq. 6) are done in appendix A2 , reporting eq. 78 as eq. 23 below

$$\left(\ell \mid \widehat{\ell} \right)^{\dot{\gamma} \rightarrow L} = \begin{pmatrix} \nu & \mid & \mathcal{N} \\ e^{-} & \mid & e^{+} \end{pmatrix}^{\dot{\gamma} \rightarrow L}$$

(23)
$$\left(\widehat{\ell} \mid \ell \right)_{\alpha \rightarrow R} = \begin{pmatrix} \widehat{\mathcal{N}} & \mid & \widehat{\nu} \\ e^{-} & \mid & e^{+} \end{pmatrix}_{\alpha \rightarrow R}$$

$$\widehat{\mathcal{N}}_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ \mathcal{N}^{\dot{\gamma}} \right\}^{*} , \quad \left(e^{-} \right)_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ \left(e^{+} \right)^{\dot{\gamma}} \right\}^{*}$$

$$\widehat{\nu}_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ \nu^{\dot{\gamma}} \right\}^{*} , \quad \left(e^{+} \right)_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ \left(e^{-} \right)^{\dot{\gamma}} \right\}^{*}$$

and $\longleftrightarrow$ **h.c.**

$\longrightarrow$

Two remarks shall conclude this section

1) When transfroming between L - and R - chiral bases it is by no means essential to associate *upper* dotted spinor indices $\dot{\gamma} \rightarrow L$ to *lower* undotted ones $\alpha \rightarrow R$ since individually indices can be raised and lowered by the symplectic invaraint matrices (eq. 21)

(24) $$(\varepsilon = i \sigma_2 , \widetilde{\varepsilon}) \; ; \; \widetilde{\varepsilon} = \varepsilon^{-1} = \varepsilon^{t} = - \varepsilon$$

but it is essential to keep one each of the dotted and undotted type , related by *local* hermition conjugation .

2) The mirror association $\leftrightarrow$ is a feature specifi c to chiral bases pertaining respectively to the 16 and $\overline{16}$ representations of SO10 , and constitute the (a) *minimal* neutrino mass induced extension of fermion families . In possible extended fermion representations , e.g. involving the 27 and $\overline{27}$ representations of the exceptional group E6 , the number of fermions in one of the chiral bases can well be odd .

3 Mass and mass from mixing

$\longrightarrow$

Charged fermions are not like neutrinos [1]

We shall consider - 'pour fixer les idees' - 3 fermion families in the (left-) chiral basis,

forming a substrate for the local gauge group

SL (2,C) [or SO (1,3)] x SO10

$$\left(\begin{matrix} u^1 & u^2 & u^3 \\ d^1 & d^2 & d^3 \end{matrix} \quad \begin{matrix} \nu & | & N \\ l^- & | & l^+ \end{matrix} \quad \begin{matrix} \hat{u}^1 & \hat{u}^2 & \hat{u}^3 \\ \hat{d}^1 & \hat{d}^2 & \hat{d}^3 \end{matrix} \right)_F^{\mathring{\gamma}}$$

$$f_F^{\mathring{\gamma}} \qquad \mathring{\gamma} = 1, 2 \qquad F = e, \mu, \tau$$

→ Fig. 1

[1] Ettore Majorana, ' Teoria simmetrica dell elettrone e positrone ' , Nuovo Cimento 14 (1937) 171.

Figure 1:

Key questions → why 3 ? why SO10 ?

From ref. [4] PM , Venice , 22. February 2005 .

→

Lets call the above extension of the standard model the 'minimal nu-extended SM' . [a]

$$\begin{pmatrix} \bullet & \bullet & \bullet & \nu & | & \mathcal{N} & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \ell & | & \widehat{\ell} & \bullet & \bullet & \bullet \end{pmatrix}^{\dot{\gamma}}_{F = e\ ,\ \mu\ ,\ \tau}$$

$$\downarrow \tag{25}$$

$$\begin{pmatrix} \nu & \mathcal{N} \\ \ell & \widehat{\ell} \end{pmatrix}^{\dot{\gamma}}_{F = e\ ,\ \mu\ ,\ \tau}$$

$\longrightarrow$

[a] [5] Harald Fritzsch and Peter Minkowski, "Unified interactions of leptons and hadrons", Annals Phys.93 (1975) 193 and Howard Georgi, "The state of the art - gauge theories", AIP Conf.Proc.23 (1975) 575.

The right-chiral base fields are then associated to 1 for 1

(26)

$$(f^{\,r})_{F\,\alpha} = \varepsilon_{\alpha\gamma} \left[(f^{\,r})_{F}^{\dot{\gamma}} \right]^{*}$$

$$(\varepsilon = i\,\sigma_{2})_{\alpha\gamma} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The matrix ε is the symplectic ($Sp\,(1)$) unit, as implicit in Ettore Majorana's original paper [1] .
The local gauge theory is based on the gauge (sub-) group

(27) $$SL\,(2\,,\,C\,) \times SU3_{\,c} \times SU2_{\,L} \times U1_{\,\mathcal{Y}}$$

... why ? why 'tilt to the left' ?

→

3a Yukawa interactions and mass terms

The doublet(s) of scalars are related to the 'tilt to the left' .

(28) $$\begin{pmatrix} \nu & \mathcal{N} \\ \ell & \widehat{\ell} \end{pmatrix}_F \leftrightarrow \begin{pmatrix} \varphi^{\,0} & \Phi^{\,+} \\ \varphi^{\,-} & \Phi^{\,0} \end{pmatrix} = z$$

The green entries in eq. 28 denote singlets under $SU2_{\,L}$.

The quantity z is associated with the quaternionic or octonionic structure inherent to the $(\,2\,,\;2\,)$ representation of $SU2_{\,L} \otimes SU2_{\,R}$ (beyond the electroweak gauge group) $[\,6\,]$ [a] . $\longrightarrow$

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[a] e.g. $[\,6\,]$ F. Gürsey and C.H. Tze , "On the role of division- , Jordan- and related algebras in particle physics", Singapore, World Scientifi c (1996) 461. $\leftrightarrow$ related to the number 3 .

The Yukawa couplings are of the form (notwithstanding the quaternionic or octonionic structure of scalar doublets)

$$\mathcal{H}_{Y} = \left[\begin{array}{c} (\varphi^{0})^{*}, (\varphi^{-})^{*} \\ \Phi^{0}, -\Phi^{+} \end{array} \right] \lambda_{F' F} \times$$

(29)

$$\times \left\{ \varepsilon_{\dot{\gamma}\dot{\delta}} \mathcal{N}^{\dot{\delta}}_{F'} \left[\begin{array}{c} \nu^{\dot{\gamma}} \\ \ell^{\dot{\gamma}} \end{array} \right]_{F} \right\} + h.c.$$

$$\mathcal{N}_{\dot{\gamma} F'} = \varepsilon_{\dot{\gamma}\dot{\delta}} \mathcal{N}^{\dot{\delta}}_{F'} \; ; \; \varepsilon_{\dot{\gamma}\dot{\delta}} = \overline{\varepsilon_{\gamma\delta}} = \varepsilon_{\gamma\delta}$$

The only allowed Yukawa couplings by $SU2_{L} \otimes U1_{\mathcal{Y}}$ invariance are those in eq. 29 , with arbitrary complex couplings $\lambda_{F' F}$. →

Spontaneous breaking of $SU2_L \otimes U1_{\mathcal{Y}}$ through the vacuum expected value(s)

$$\langle \Omega \mid \begin{pmatrix} \varphi^0 & \Phi^+ \\ \varphi^- & \Phi^0 \end{pmatrix} (x) \mid \Omega \rangle =$$

(30)

$$= \langle z(x) \rangle = \begin{pmatrix} v_{ch} \, (v^u_{ch}) & 0 \\ 0 & v_{ch} \, (v^d_{ch}) \end{pmatrix}$$

$$v_{ch} = \frac{1}{\sqrt{2}} \left(\sqrt{2} \, G_F \right)^{-1/2} = 174.1 \text{ GeV}$$

independent of the space-time point x [a] →

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[a] The implied parallelizable nature of $\langle z(x) \rangle$ is by far not trivial and relates in a wider context including $SU2_L -$ triplet scalar representations to potential (nonabelian) monopoles and dyons.

induces a neutrino mass term through the Yukawa couplings $\lambda_{F' F}$ in eq. 29

$$ {}_{F'}\mathcal{N}\, \nu_{F} = \mathcal{N}_{\dot\gamma\, F'}\, \nu^{\dot\gamma}_{F} = \nu_{\dot\gamma\, F}\, \mathcal{N}^{\dot\gamma}_{F'} $$

(31) $$ \mu_{F' F} = v_{ch}\, \lambda_{F' F} $$

$$ \rightarrow\ \mathcal{H}_{\mu} = {}_{F'}\mathcal{N}\, \mu_{F' F}\, \nu_{F} + h.c. = \nu^{T}\, \mu^{T}\, \mathcal{N} + h.c. $$

The matrix μ defined in eq. 31 is an arbitrary complex 3×3 matrix, analogous to the similarly induced mass matrices of charged leptons and quarks. In the setting of primary SO10 breakdown , a general (not symmetric) Yukawa coupling $\lambda_{F' F}$ implies the existence in the scalar sector of at least two irreducible representations $(10) \oplus (120)$ [a]. $\longrightarrow$

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[a] **key question $\longrightarrow$ a 'drift' towards unnatural complexity ? It becomes even worse including the heavy neutrino mass terms : 256 (complex) scalars.**

3b 'Mass from mixing' in vacuo or 'seesaw'

neutrinos oscillate like neutral Kaons (yes , but how ?) - Bruno Pontecorvo [a]

The special feature, pertinent to (electrically neutral) neutrinos is, that the $\nu-$ extending degrees of freedom $\mathcal{N}$ are singlets under the whole SM gauge group $G_{SM} = SU3_{c} \otimes SU2_{L} \otimes U1_{\mathcal{Y}}$, in fact remain singlets under the larger gauge group $SU5 \supset G_{SM}$. This allows an arbitrary (Majorana-) mass term, involving the bilinears formed from two $\mathcal{N}$-s.

In the present setup (minimal ν-extended SM) the full neutrino mass term is thus of the form $\longrightarrow$

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[a] [7] Bruno Pontecorvo, "Mesonium and antimesonium", JETP (USSR) 33 (1957) 549, english translation Soviet Physics, JETP 6 (1958) 429. Here let me continue the 'flow of thought' embedding neutrino masses in SO10.

$$\mathcal{H}_{\mathcal{M}} = \frac{1}{2}\,[\,\nu\,\mathcal{N}\,]\,\mathcal{M}\begin{bmatrix}\nu\\ \mathcal{N}\end{bmatrix} + h.c.$$

(32)

$$\mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} \; ; \; \mathcal{M} = \mathcal{M}^T \rightarrow M = M^T$$

Again within primary SO10 breakdown the full $\mathcal{M}$ extends the scalar sector to the representations $(10) \oplus (120) \oplus (126)$ [a] $\longrightarrow$

---

[a] **It is from here where – to the best of my knowledge – the discussion of the** *necessarily nonvanishing nature* **and of the magnitude of the light neutrino masses (re-) started in 1974 [8] , [9] .**
The structure in eq. 32 is reserved for the minimal $SU2_L \times U1_{\mathcal{Y}}$ case 'tilted to the left' [10] .

Especially the 0 entry needs explanation. It is an exclusive property of the minimal ν-extension assumed here.
Since the 'active' flavors ν_F all carry $I_{3\,w} = \frac{1}{2}$ terms of the form

(33) $$\frac{1}{2}\ _{F'}\nu\,\chi_{F'F}\,\nu_F = \frac{1}{2}\,\nu^T\chi\,\nu\ ;\ \chi = \chi^T$$

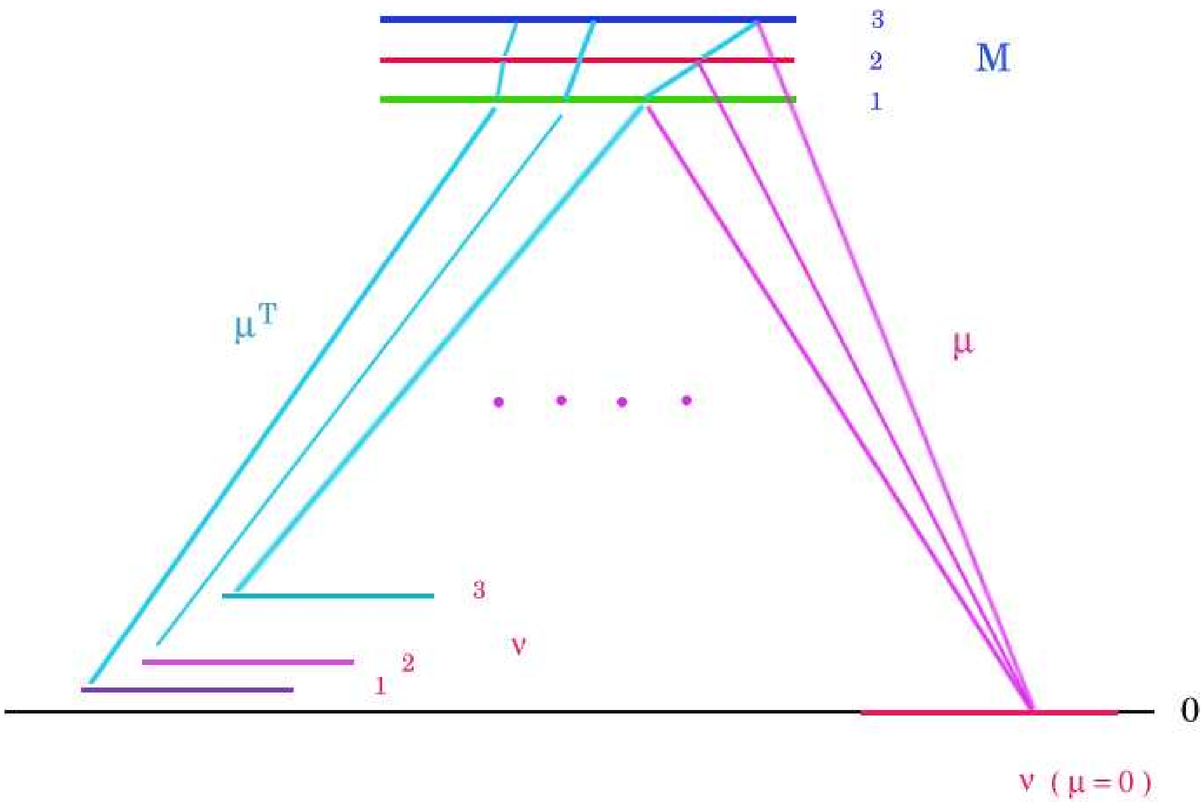
cannot arise as Lagrangean masses, except induced by an I_w-triplet of scalars, developing a vacuum expected value independent from the doublet(s) .

'seesaw' $\swarrow\!\!\nearrow$

The relative 'size' of μ and M shall define the 'mass from mixing' situation and segregates 3 heavy neutrino flavors from the 3 light ones :

(34) $$\swarrow \quad ||\,\mu\,|| \ll ||\,M\,|| \quad \nearrow$$

$\longrightarrow$



Mass from mixing

Figure 2:

Key questions → which is the scale of M ? $O(10^{10})$ GeV → is there any evidence for this scale today ? hardly ! → and what about susy ?

→

3c Diagonalization of $\mathcal{M}$

We shall use the generic expansion parameter $\vartheta = ||\mu|| / ||M|| \ll 1$ – and determine a unitary 6×6 matrix U with the property [11]

$$\mathcal{M} = U \mathcal{M}_{diag} U^T \rightarrow \mathcal{M}_{diag} =$$

$$\mathcal{M}_{diag} (m_1 , m_2 , m_3 ; M_1 , M_2 , M_3)$$

(35)

$$0 \leq m_1 \leq \cdots \leq M_3 , \ m_3 \ll M_1$$

and $U = T U_0 \ ; \ T^{-1} \mathcal{M} T^{-1\,T} = \mathcal{M}_{bl.diag.} \rightarrow$

$$= \begin{pmatrix} \mathcal{M}_1 & 0 \\ 0 & \mathcal{M}_2 \end{pmatrix} = U_0 \mathcal{M}_{diag} U_0^T$$

→

The matrix T in eq. 35 describes the mixing of light and heavy flavors, determined from a 3×3 submatrix t .

$$T =$$

(36) $$\begin{pmatrix} \left(1 + t\,t^{\dagger}\right)^{-1/2} & \left(1 + t\,t^{\dagger}\right)^{-1/2} t \\ -t^{\dagger} \left(1 + t\,t^{\dagger}\right)^{-1/2} & \left(1 + t^{\dagger} t\right)^{-1/2} \end{pmatrix}$$

The upper left 3×3 block of T (eq. 36) $\left(1 + t\,t^{\dagger}\right)^{-1/2}$ causes the (3×3) mixing matrix governing oscillations of light (anti)neutrino's to deviate from unitarity , i.e. it becomes subunitary, but by a tiny amount since as we will discuss below

(37) $$\| t \|^{2} = \sum_{kl=1}^{3} | t_{kl} |^{2} = O\left(10^{-21}\right)$$

$\longrightarrow$

The matrix t in eq. 36 is reduced to diagonal form through two unitary 3×3 matrices u and w [a]

$$t = u \left(\tan a_{diag} \right) w^{-1}$$

(38)
$$a_{diag} = a_{diag} \left(a_1 , a_2 , a_3 \right)$$

$$0 \leq a_k \leq \pi / 2$$

$$a_k \ll \pi / 2 \text{ for } \vartheta = || \mu || / || M || \ll 1$$

t is determined from the quadratic equation

(39)
$$t = \mu^T M^{-1} - t \mu \bar{t} M^{-1}$$

which can be solved recursively $\longrightarrow$

---

[a] **In eq. 38 a_{diag} defines the three (real) heavy-light mixing angles $a_{1,2,3}$, which without loss of generality can be chosen in the first quadrant, but which are small for $\vartheta = || \mu || / || M || \ll 1$**

setting

(40)
$$t_{n+1} = \mu^T M^{-1} - t_n \mu \bar{t}_n M^{-1}$$
$$t_0 = 0 \, , \; t_1 = \mu^T M^{-1} \, ,$$
$$t_2 = t_1 - \mu^T M^{-1} \mu \mu^\dagger \overline{M}^{-1} M^{-1}$$
$$\ldots$$
$$\lim_{n \to \infty} t_n = t$$

In order to control convergence we introduce ← the specific norms [a]

(41)
$$|| \mu ||^2 = tr \, \mu \mu^\dagger$$
$$|| M ||^{-2} = tr \, M^{-1} M^{-1 \dagger}$$
$$\vartheta = || \mu || / || M || \ll 1$$

→

---

[a] The sequence defined in eq. 40 is convergent for $\vartheta < 1$.

u , w **in eq. 38 contain all 9 CP violating phases, pertaining to T .**
The above was intended to 'explain' why the (un)observed light neutrino masses are so much smaller than charged fermion ones.
key question → does it ? wait . $t = u \, (\, \tan \, a_{diag} \,) \, w^{-1}$ **defined in eq. (38) and its determining equation, repeated below**

$$t = \mu^{T} M^{-1} - t \, \mu \, \bar{t} \, M^{-1}$$

ensure block diagonal form of $\mathcal{M}_{bl.diag.}$ **.**

$$\mathcal{M}_{bl.diag.} = T^{-1} \mathcal{M} \, T^{-1 \, T}$$

(42)

$$\mathcal{M}_{bl.diag.} = \begin{pmatrix} \mathcal{M}_1 & 0 \\ 0 & \mathcal{M}_2 \end{pmatrix}$$

→

$(\mathcal{M}_1, \mathcal{M}_2)$ **forming** $\mathcal{M}_{bl.diag.}$ **defined in eq. 42 become**

$$\mathcal{M}_1 = (1 + t\,t^\dagger)^{-1/2} \times$$
$$\times \left[-t\,\mu - \mu^T t^T + t\,M\,t^T \right] \times$$
$$\times (1 + t\,t^\dagger)^{-1/2\,T}$$

(43)

$$\mathcal{M}_2 = (1 + t^\dagger t)^{-1/2} \times$$
$$\times \left[\mu\,\bar{t} + t^\dagger \mu^T + M \right] \times$$
$$\times (1 + t^\dagger t)^{-1/2\,T}$$

Comparing $\mathcal{M}_1$ **with** $t\,\mathcal{M}_2\,t^T$ **we find** $\rightarrow$

the relation [a] , $[\ 12\ ]$

(44) $$\mathcal{M}_{\ 1} \ = \ - \, t \, \mathcal{M}_{\ 2} \, t^{\ T}$$

It follows from the assumptions detailed in footnote a , that $Det \, t \ \neq \ 0$ and hence the heavy-light mixing angles $a_{\ 1,2,3} \ > \ 0$ defined in eq. 38 are strictly bigger than 0.
The lowest approximation, $t \ \rightarrow \ t_{\ 1}$ and and $\mathcal{M}_{\ 2} \ \rightarrow \ M$, yields the first nontrivial approximation of the light neutrino mass matrix in 'second order mixing'

(45) $$\mathcal{M}_{\ 1} \sim \mathcal{M}_{\ 1}^{\ (2)} \ = \ - \, \mu^{\ T} \, M^{\ -1} \, \mu$$

→

[a] In the scenario adopted here, we further assume $Det \, M \ \neq \ 0$ and $Det \, \mu \ \neq \ 0$. This leaves no room for light 'sterile' neutrinos, which would imply a nonminimal $\nu-$extension of the standard model. This would be mandatory, if the results of the LSND collaboration are correct. $[\ 12\ ]$ G.B. Mills for the LSND Collaboration , 'Results on neutrinos from LSND' , published in *Stanford 1998, Gravity from the Hubble length to the Planck length* 467-475, see the MiniBooNE Experiment [13] .

Remaining dagonalization of $\mathcal{M}_{bl.diag.}$

We go back to eq. 35 $U = T\,U_0$ **:**

U_0 **diagonalizes the** 3×3 **blocks** $\mathcal{M}_1, \mathcal{M}_2$ [a]

$$T^{-1}\,\mathcal{M}\,T^{-1\,T} = \mathcal{M}_{bl.diag.}\ ;\ \mathcal{M}_{bl.diag.} = \begin{pmatrix} \mathcal{M}_1 & 0 \\ 0 & \mathcal{M}_2 \end{pmatrix}$$

$$U_0 = \begin{pmatrix} u_0 & 0 \\ 0 & v_0 \end{pmatrix} \sim U_0\,I\,;\ I = I_{diag}(\pm 1, \cdots, \pm 1)$$

$$\begin{matrix} \mathcal{M}_1 = u_0\, m_{diag}(m_1, m_2, m_3)\, u_0^T \\ \mathcal{M}_2 = v_0\, M_{diag}(M_1, M_2, M_3)\, v_0^T \end{matrix}\ ;\ \mathcal{M}_1 = -t\,\mathcal{M}_2\, t^T$$

(46)

$\longrightarrow$

---

[a] U_0 is determined modulo diagonal (orthogonal , 6×6) matrices $I = I_{diag}$ as shown in eq. 46 , representing the discrete abelian group $(Z_2)^{\otimes 6}$.

T is constructed as a sequence (eq. 40) , convergent for $\vartheta = || \mu || / || M || < 1$, as shown above , and thus unique. As a consequence of eqs. 44 , 46 – t beeing determined (within T) – $\mathcal{M}_1$ and $\mathcal{M}_2$ and hence also u_0 and v_0 are not independent of each other. We shall keep as independent variables $\mathcal{M}_1$ and t (or equivalently T) .

4 Generic mixing and mass estimates

Here we introduce the arithmetic mean measure for 3×3 matrices A , not to be confused with the norms $|| \, . \, ||$ defined in eq. 41

(47) $$| A | = | Det \, A |^{1/3}$$

Eq. 44 then implies

→

$$|\mathcal{M}_1| / |\mathcal{M}_2| = |t|^2$$

(48) $$|\mathcal{M}_1| = |m_{diag}| = (m_1 m_2 m_3)^{1/3}$$

$$|\mathcal{M}_2| = |M_{diag}| = (M_1 M_2 M_3)^{1/3}$$

We consider the arithmetic mean of the light and heavy neutrino masses and the coorresponding 'would be' masses if μ and μ^T would be the only parts of the full 6×6 mass matrix $\mathcal{M}$

$$\overline{m} = (m_1 m_2 m_3)^{1/3}$$

(49) $$\overline{M} = (M_1 M_2 M_3)^{1/3}$$

$$\mu = u_\mu \mu_{diag}(\mu_1, \mu_2, \mu_3) v_\mu^{-1}$$

$$\overline{\mu} = (\mu_1, \mu_2, \mu_3)^{1/3}$$

→

Then beyond eq. 48 there is one more (exact) relation [a]

$$\widehat{t} = (\tan a_1 \tan a_2 \tan a_3)^{1/3} = | t |$$

$$| \mu |^2 = | \mathcal{M}_1 | | \mathcal{M}_2 | \rightarrow$$

$$\overline{m} / \overline{\mu} = \widehat{t} \, , \; \overline{m} / \overline{M} = \widehat{t}^{\,2} \quad (50)$$

or equivalently

$$\overline{m} = \widehat{t} \, \overline{\mu} \quad \swarrow\!\!\nearrow \quad \overline{M} = \widehat{t}^{\,-1} \, \overline{\mu}$$

seesaw (of type I)

$\longrightarrow$

---

[a] **for MSSM inspired seesaw of type II realizations see e.g. [14] .**

The estimates below are based on the assumption that the scalar doublets (2) are part of a complex 10-representation of SO10 with Yukawa couplings of the form

(51)
$$\mathcal{H}_Y = \lambda_{F'F} \begin{pmatrix} 16 & 16 & 10 \\ B & A & D \end{pmatrix}_{F'} f_B \, f_{A\,F} + h.c.$$

$$\rightarrow \lambda_{F'F} = \lambda_{FF'}$$

It follows that at the unification scale we have [a] $\longrightarrow$

[a] **In order to obtain a general (not a symmetric) heavy-light mass matrix μ a combination of SO10 representations $(120) \oplus (10)$ is needed, which however would 'destroy' the mass relation in eq. 51 .**

key question $\longrightarrow$ is this relevant ? estimate shall be estimate .

(52) $$\mu = \mu^{T} = \mu_{u}$$

We shall use the relation at a scale near 100 GeV

(53) $$\mu \sim \frac{1}{3} (\mu_{u})$$

The factor $\frac{1}{3}$ accounts for the color rescaling reducing the (colored) up-quark mass matrix from the unification scale down to 100 GeV . It follows using the definitions in eq. 49 and the quark masses $m_{u} \sim 5.25$ MeV , $m_{c} \sim 1.25$ GeV and $m_{t} \sim 180$ GeV

(54)
$$\overline{\mu}_{u} = (m_{u} m_{c} m_{t})^{1/3} \sim 1 \text{ GeV} \longrightarrow$$
$$\overline{\mu} \sim \frac{1}{3} \text{ GeV}$$

$\longrightarrow$

Further lets approximate the mass square differences obtained from the combined neutrino oscillation measurements by

$$\Delta m^2_{12} \sim 10^{-4} \textbf{ eV}^2$$
(55)
$$\Delta m^2_{23} \sim 2.5\, 10^{-2} \textbf{ eV}^2 ,$$

Finally 'pour fixer les idées' I set the lowest light neutrino mass ~ 1 meV and assume hierarchical (123) light masses. This implies

$$m_1 \sim 1 \textbf{ meV} , \; m_2 \sim 10 \textbf{ meV}$$
(56)
$$m_3 \sim 50 \textbf{ meV} \rightarrow \overline{m} \sim 8 \textbf{ meV}$$

$\rightarrow$

It follows from eq. 50

$$\widehat{t} = \overline{m} \, / \, \overline{\mu} \sim 2.4 \; 10^{-11}$$

(57) $$\overline{M} = \overline{\mu} \, / \, \widehat{t} \sim 1.4 \; 10^{10} \text{ GeV}$$

$$\widehat{t}^{\,2} \sim 5.8 \; 10^{-22}$$

Light neutrino masses are indeed small. [a]

5 Why is the large mass - scale so large ? – tentative thoughts $\longrightarrow$

---

[a] **puzzling questions $\longrightarrow$ is susy bringing down in 'small steps' the B-L protecting mass scale $\overline{M} = \sim 1.4 \, . \, 10^{7}$ TeV to 1 TeV ? – Or is $\overline{M} = \sim 1.4 \, . \, 10^{7}$ TeV in view of seesaw type II too small ? $\mu \rightarrow e \, \gamma$ at a rate of ?**

– tentative thoughts

1) Exact symmetries are all (?) linked to unbroken gauge fields .
The most difficult such form the substrate of *gravity* , with base quanta of spin 2 , which yet expose a characteristic high (mass-) scale

(58) $m_{Pl} = (G_N)^{-1/2} (\hbar, c)^{1/2} \sim 1.22 . 10^{19}$ **GeV** $/ c^2$

Next in line of unbroken gauge fields are *charge-like gauge bosons* base quanta of spin 1 with gauge group

(59) $$G_3 = SU3_c \times U1_{e.m.}$$

The substrate of fermions , carrying the charges of G_3 contains the three families , with neutrino flavors *omitted* $\longrightarrow$

in the left - chiral basis (eq. 1) for one family

(60) $$\begin{pmatrix} u^{1} & u^{2} & u^{3} & \nu & | & \mathcal{N} & \widehat{u}^{3} & \widehat{u}^{2} & \widehat{u}^{1} \\ d^{1} & d^{2} & d^{3} & e^{-} & | & e^{+} & \widehat{d}^{3} & \widehat{d}^{2} & \widehat{d}^{1} \end{pmatrix}^{\dot{\gamma} \to L}$$

$$= (f)^{\dot{\gamma}}$$

The scales of fermion masses, while *parameters* within the conserved gauge interactions of G_3 arise exclusively through spontaneous breaking of the SM interactions with gauge group

(61) $$G_{SM} = SU3_c \times SU2_L \times U1_{\mathcal{Y}}$$
$$(Q_{e.m.} / e = I_{3\,w} + \mathcal{Y} \; ; \; I_{3\,w} = I_{3\,L})_f$$

$\rightarrow$

through Yukawa couplings to one (2) doublets of scalars as defined in eq.28

(62) $$\begin{pmatrix} \nu & \mathcal{N} \\ \ell & \widehat{\ell} \end{pmatrix}_F \leftrightarrow \begin{pmatrix} \varphi^{\,0} & \Phi^{\,+} \\ \varphi^{\,-} & \Phi^{\,0} \end{pmatrix} = z$$

in straightforward generalizations of eq. 29

(63) $$\mathcal{H}_{\,Y} = \begin{bmatrix} (\,\varphi^{\,0}\,)^{\,*}\,,\,(\,\varphi^{\,-}\,)^{\,*} \\ \Phi^{\,0}\,,\,-\,\Phi^{\,+} \end{bmatrix} \lambda_{\,F'\,F} \times$$
$$\times \left\{ \varepsilon_{\dot\gamma\dot\delta}\,\mathcal{N}^{\,\dot\delta}_{\,F'} \begin{bmatrix} \nu^{\,\dot\gamma} \\ \ell^{\,\dot\gamma} \end{bmatrix}_{F} \right\} + h.c.$$

$\rightarrow$

This way a *plethora* of base fermion masses is generated [a]

(64)
$$m_{u\,*} = 5.25 \text{ MeV} \quad m_{c\,m_c} \sim 1.3 \text{ GeV} \quad m_t \sim 170 \text{ GeV}$$
$$m_{d\,*} = 8.75 \text{ MeV} \quad m_{s\,*} = 175 \text{ MeV} \quad m_{b\,m_b} \sim 4.2 \text{ GeV}$$
$$m_e = 0.511 \text{ MeV} \quad m_\mu = 105.7 \text{ MeV} \quad m_\tau = 1.777 \text{ GeV}$$

The mass values in eq. 64 represent a downfeed from the broken gauge sector of the SM into *parameter space* of the $SU3_c \times U1_{e.m.}$ unbroken gauge sector .

While the so induced mass scales overlap with the electroweak (breaking) scale(s) , the central scale of strong interactions (QCD) is *not generated the same way* : it can be represented by the nucleon mass in the limit $m_{u,d} \rightarrow 0$ or the inverse slope of Regge trajectories

(65)
$$M_N\,(m_{u,d} \rightarrow 0) \sim 0.9 \text{ GeV} \;;\; 1/\alpha' \sim 1.02 \text{ GeV}$$

→

[a] For the quark masses in the $\overline{\text{MS}}$ scheme – say – a reference scale μ must be specified : $*$ stands here for $\mu \sim 1$ GeV [15] , [16] .

2) Hence we are led to include – through the intertwined nature of *scales* – as next step G_{SM} alongside G_3 , where a typical scale characterizing electroweak gauge breaking can be taken as

(66) $$v_{ch} = \frac{1}{\sqrt{2}} \left(\sqrt{2}\, G_F \right)^{-1/2} = 174.1 \text{ GeV}$$

We face the appearance of 3 spin 1/2 fermion families , 90 (96) degrees of freedom excluding (including) heavy neutrino flavors . In this connection the nature of global , ungauged charge-like symmetries : exactly $\leftrightarrow$ approximately conserved

$$B\,,\; L_F\,,\; L = \sum_F L_F\,,\; B - L$$

becomes an urgent question .

$\longrightarrow$

Individual B and L_F conservations are broken by anomalies in the SM , whereas $B - L$ without minimal neutrino mass extension to $\mathcal{N}_F$ flavors – $B - L\,(15)$ is broken by a graviational anomaly

$$d^4 x \sqrt{|g|}\, D^{\mu} j_{\mu}^{B - L\,(15)} = 3\, \widehat{A}_1 (X)$$

$$\widehat{A}_1 (X) = - \frac{1}{24}\, tr X^2$$

(67)
$$(X)^a{}_b = \frac{1}{2\pi} \frac{1}{2}\, d x^{\mu} \wedge d x^{\nu} \left(R^a{}_b \right)_{\mu\nu}$$

$\left(R^a{}_b \right)_{\mu\nu}$: **Riemann curvature tensor**

mixed components : $a \; b \longrightarrow$ **tangent space**

$\mu\,\nu \longrightarrow$ **covariant space**

$$D^{\mu} j_{\mu}^{B - L\,(16)} = 0$$

In eq. 67 $\widehat{A} (X \to \lambda) = \frac{1}{2} \lambda / \sinh (\frac{1}{2} \lambda)$ $\rightarrow$

denotes the Atiyah - Hirzebruch character or $\widehat{A} -$ genus [17] with its integral over a compact , euclidean signatured closed manifold M_4 , capable of carrying on SO4 - spin structure , becomes the index of the associated *elliptic* Dirac equation

(68) $$\int \widehat{A} (X_E) = n_R - n_L = \text{integer}$$

In eq. 68 $n_{R,L}$ denote the numbers of right - and left - chiral solutions of the Dirac equation on M_4 . The index $_E \rightarrow X_E$ shall indicate the euclidean transposed curvature 2 - form , opposite to physical uncurved space time [18] .
For the latter case the first relation in eq. 67 yields the integrated form – in the limit of infinitely heavy $\mathcal{N}_F$ – $\rightarrow$

$$\Delta_{R-L}\, n_{\nu} \; = \; \int d^{4}x \, \sqrt{|\,g\,|} \; D^{\mu} \, j_{\mu}^{\;B\,-\,L\,(15)} \; = \; 3 \; \Delta \, n \, (\, \widehat{A} \,)$$

$3 \; =$ **number of families = odd** $\; ; \quad m_{\nu_F} \; \rightarrow \; 0$

(69)

In eq. 69 $\Delta_{R-L}\, n_{\nu}$ **denotes the difference of right - chiral** $(\, \widehat{\nu} \,)$ **and left - chiral** $(\, \nu \,)$ **flavors between times** $t \; \rightarrow \; \pm \, \infty$ **.**

Here a subtlety arises *precisely* **because the number of families on the level of** G_{SM} **is odd , and the light neutrino flavors are not 'Dirac - doubled' , which according to eq. 69 could potentially lead to a change in fermion number beeing odd , which violates the rotation by** $2\,\pi$ **symmetry , equivalent to** $\widehat{\Theta}^{\,2}$ **,** *unless*

(70) $\quad \Delta \, n \, (\, \widehat{A} \,) \; = \;$ **even** $\quad (\, \surd$ **for** $dim \; = \; 4$ **mod** $8 \,)$

$\rightarrow$

This leads to the following collection of questions *undecidable* (or *unanswerable*) within the SM and with this on the level of scales characteristic of electroweak gauge breaking

questions	*potential* answers at unknown scale
3 colors ?	**octonion structure**
3 families ?	$\longrightarrow$ **exceptional groups**
quantized elementary electric charges ?	1 / **replica-product of** } **charge-like *simple* gauge group(s)**
origin of the 'tilt to the left' ?	**no anwer but : what means left ?**
B , B - L violation driven by $\overline{M} \sim 10^{\,10}$ **GeV ?**	**large scale protects at low scale**
spins other than $0 \, , \, \frac{1}{2} \, , \, 1 \, , \, 2$	**supersymmetry , supergravity**
$d \, = \, 1 \, + \, 3$ **dimensions and gravity ?**	**target space** $\leftrightarrow$ **base space**

$\longrightarrow$

3) **Apparent paradox of unification of forces and stability of *large , primary* breakdown scales of order $\overline{M} \sim 10^{10}$ GeV $- - m_{Pl} \sim 10^{19}$ GeV**

In a *concluding remark* (not a *conclusion*) let me point out that the example of the large scale inherent in heavy neutrino masses is not only responsible for the small masses of observed neutrino flavors , but at the same time the stability of $\overline{M} \sim 10^{10}$ GeV – a key feature unexplained so far – serves as *protecting scale* for the *approximative* conservation of leptonic (as well as baryonic) numbers at electroweak scales much below $\overline{M}$.
As a consequence even a unified charge-like gauge group $SO10 \, , \, E6 \cdots$ acts first through a primary breakdown of local gauge invariance at scales $M_{unif} \sim 10^{16}$ GeV , which produces essential deviations from unified symmetries .

asymmetry is the sister of symmetry

- - - - - - -

A1 Appendix 1 $\left(\widehat{C} \, , \, \widehat{P} \, , \, \widehat{T} \right)$

Combining eqs. 12 and 14 it follows , for $\widehat{C}$, $\widehat{P}$

$$\left(\{ \, f^{\,(C)} \, \}^{\,(P)} \right) \, = \, (- \, i \, C \, f^{\,*} \, (\, P x \,) \,) \, = \, \left(i \, \gamma_{\,0} \, f^{\,(C)} \, (\, P x \,) \right) \, = \, \left(\{ \, \nleftrightarrow \, f \, \}^{\,(P)} \right) \; (\surd)$$

$$\widehat{C} \, \widehat{P} \, \sim \, \widehat{P} \, \widehat{C} \; ; \; \widehat{P}^{\,2} \, \sim \, - \, ¶$$

(71)

The symbol $\sim$ in eq. 71 shall indicate that the operators are restricted to their action on the fermion families $(\, f \,)_{\,\diamond A}$ and their (re)projectable chiral L and R bases .

For $\widehat{C}$, $\widehat{T}$ we have

$$\begin{aligned} \left(\{ \, f^{\,(C)} \, \}^{\,(T)} \right) \, = \, (\, C \, \gamma_{\,0} \, C \, \gamma_{\,5} \, f^{\,*} \, (\, T x \,) \,) \quad &= \quad (\, C \, \gamma_{\,5} \, C \, \gamma_{\,0} \, f^{\,*} \, (\, T x \,) \,) \\ &= \quad \left(\{ \, \nleftrightarrow \, f \, \}^{\,(T)} \right) \end{aligned} \quad (\surd)$$

(72)

$$\widehat{C} \, \widehat{T} \, \sim \, \widehat{T} \, \widehat{C} \; ; \; \widehat{T}^{\,2} \, \sim \, - \, ¶$$

We note without explicit proof

(73)

$$\widehat{T} \, \widehat{P} \, \sim \, - \, \widehat{P} \, \widehat{T}$$

We repeat eq. 17 as eq. 74 below

$$\widehat{\Theta}^{-1} \left(f^{\dot{\gamma}} (x) \right) \widehat{\Theta} = \left(i \, \{ f^{\dot{\gamma}} \}^{*} (-x) \right)$$

(74) $$\widehat{\Theta}^{-1} (f_{\alpha} (x)) \widehat{\Theta} = (- i \, \{ f_{\alpha} \}^{*} (-x))$$

$$f_{\alpha} (y) = \varepsilon_{\alpha\gamma} \{ f^{\dot{\gamma}} \}^{*} (y)$$

From the last relation in eq. 74 we obtain

$$\begin{aligned} \widehat{\Theta}^{-1} f_{\alpha} (y) \widehat{\Theta} = \varepsilon_{\alpha\gamma} \left\{ \widehat{\Theta}^{-1} f^{\dot{\gamma}} \widehat{\Theta} \right\}^{*} (y) \; &= \; - i \, \varepsilon_{\alpha\gamma} f^{\dot{\gamma}} (-y) \\ &= \; (- i) \left\{ \varepsilon_{\alpha\gamma} \left(f^{\dot{\gamma}} \right)^{*} (-y) \right\}^{*} \\ &= \; (- i) \{ f_{\alpha} \}^{*} (-y) \; (\surd) \end{aligned}$$

$$\widehat{\Theta}^{2} \sim - ¶$$

(75)

A2 Appendix 2 : chiral projection onto chiral bases $^{\dot{\gamma} \rightarrow L}$ and $_{\alpha \rightarrow R}$

We report from eq. 6

$$\begin{pmatrix} u^{\,1} & u^{\,2} & u^{\,3} & \nu & | & \mathcal{N} & \widehat{u}^{\,3} & \widehat{u}^{\,2} & \widehat{u}^{\,1} \\ d^{\,1} & d^{\,2} & d^{\,3} & e^{\,-} & | & e^{\,+} & \widehat{d}^{\,3} & \widehat{d}^{\,2} & \widehat{d}^{\,1} \end{pmatrix}^{\dot{\gamma} \rightarrow L}$$

(76)

$$= \begin{pmatrix} f^{\,1} & f^{\,2} & f^{\,3} & f^{\,4} & | & f^{\,9} & f^{\,10} & f^{\,11} & f^{\,12} \\ f^{\,5} & f^{\,6} & f^{\,7} & f^{\,8} & | & f^{\,13} & f^{\,14} & f^{\,15} & f^{\,16} \end{pmatrix}^{\dot{\gamma} \rightarrow L}$$

$$\searrow \swarrow$$

$$\begin{pmatrix} f^{\,4} & | & f^{\,9} \\ f^{\,8} & | & f^{\,13} \end{pmatrix}^{\dot{\gamma} \rightarrow L} = \begin{pmatrix} \ell & | & \widehat{\ell} \end{pmatrix}^{\dot{\gamma} \rightarrow L} \quad \longrightarrow$$

$$\dot{\gamma} \to L \ \leftrightarrow \ \alpha \to R \ : \ (\, f^{\,r}\,)_{\alpha} \ = \ \varepsilon_{\alpha\gamma} \left\{ (\, f^{\,r}\,)^{\dot{\gamma}} \right\}^{*} ; \ r = 1, \cdots, 16$$

$$(\, \not\leftrightarrow \ f \,)_{\alpha} \ =$$

(77)

$$= \begin{pmatrix} f^{\,12} & f^{\,11} & f^{\,10} & f^{\,9} & | & f^{\,4} & f^{\,3} & f^{\,2} & f^{\,1} \\ f^{\,16} & f^{\,15} & f^{\,14} & f^{\,13} & | & f^{\,8} & f^{\,7} & f^{\,6} & f^{\,5} \end{pmatrix}_{\alpha \to R}$$

$$= \begin{pmatrix} u^{\,1} & u^{\,2} & u^{\,3} & \widehat{\mathcal{N}} & | & \widehat{\nu} & \widehat{u}^{\,3} & \widehat{u}^{\,2} & \widehat{u}^{\,1} \\ d^{\,1} & d^{\,2} & d^{\,3} & e^{\,-} & | & e^{\,+} & \widehat{d}^{\,3} & \widehat{d}^{\,2} & \widehat{d}^{\,1} \end{pmatrix}_{\alpha \to R}$$

$$\searrow \ \swarrow$$

$$\begin{pmatrix} f^{\,9} & | & f^{\,4} \\ f^{\,13} & | & f^{\,8} \end{pmatrix}_{\alpha \to R} \ = \ \begin{pmatrix} \widehat{\ell} & | & \ell \end{pmatrix}_{\alpha \to R} \qquad \rightarrow$$

Collecting both chiral projections on (anti-)lepton flavors in eqs. 76 and 77 we obtain

$$\left(\begin{array}{ccc} \ell & | & \widehat{\ell} \end{array} \right)^{\dot{\gamma} \to L} = \left(\begin{array}{ccc} \nu & | & \mathcal{N} \\ e^{-} & | & e^{+} \end{array} \right)^{\dot{\gamma} \to L}$$

(78)
$$\left(\begin{array}{ccc} \widehat{\ell} & | & \ell \end{array} \right)_{\alpha \to R} = \left(\begin{array}{ccc} \widehat{\mathcal{N}} & | & \widehat{\nu} \\ e^{-} & | & e^{+} \end{array} \right)_{\alpha \to R}$$

$$\widehat{\mathcal{N}}_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ \mathcal{N}^{\dot{\gamma}} \right\}^{*} \ , \ \left(e^{-} \right)_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ \left(e^{+} \right)^{\dot{\gamma}} \right\}^{*}$$

$$\widehat{\nu}_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ \nu^{\dot{\gamma}} \right\}^{*} \ , \ \left(e^{+} \right)_{\alpha} = \varepsilon_{\alpha\gamma} \left\{ \left(e^{-} \right)^{\dot{\gamma}} \right\}^{*}$$

and $\leftrightarrow$ **h.c.**

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There is an error in the definition of the $\widehat{A}$ genus in ref. [18] , which is corrected in eq. 67 here .