[1614] PAULI AN PAIS

[Zürich], 25. Juli 1953
Ad „usum Delphini" only[1]

Dear $\pi\alpha\iota\varsigma$!

I worked out, what Pauli said in Leiden[2] und I found, that he is not so much more clever than $\mu\xi$, after all (a paradox!).[3]

One can now proceed further in different ways, but I feel one should *not* proceed in too conventional a way (what I wrote here is still rather conventional). I treated meson-and electromagnetic field as *external* fields, then the trouble with the coupling constants does not arise yet. The conventional way is, of course, to add with the famous „+-sign", in the Lagrangian *terms*, depending on the meson-field (and electromagnetic field) alone. I do not like this very much and I hope you will find something better.

The whole note for you is of course written in order to drive you further into the real virgin-country („Neuland"); therefore I tried to write it so, that an amplification of the group (I) is suggested. And this amplification has to be dealt together with the field-quantization.

You have, I think, quite a good chance to go on where we old's* have left the problem. By the way, my aversion against the „hot-dog-potential" (see p. IV)[4] is growing. I wonder whether one has a priori empirical arguments against the kind of coupling which here results – as I believe, in a *cogent* way, if the groupe (I) is accepted. (I still do not see, how you could ever get something else.)

All good wishes to the Jostens, too!

Yours old $\mu\xi$

[1] Das Motto *ad u. Delph.* = „Zum Gebrauch des Dauphin" geht auf die Regierungszeit Ludwig XIV zurück und bezeichnete für den noch sehr jungen Tronfolger bearbeite Ausgaben der antiken Klassiker, bei denen besonders anstößige erotische Stellen gemildert oder umschrieben worden waren. Später hat man diese Bezeichnung allgemein für eine verhüllte Aussage verwendet. Diesen Hinweis verdanke ich M. Fierz.

[2] Es handelt sich um die Ausarbeitung der Diskussionsbemerkung zum Vortrag von A. Pais während der Lorentz-Kamerlingh Onnes Konferenz in Leiden {Pauli (1953d)}. Vgl. hierzu auch die Bemerkung zum Brief [1595].

[3] Siehe die Anlage zum Brief [1614].

* Includes: Kaluza and Klein.

Anlage zum Brief [1614] [1]

Written down July 22 til 25 in order to see how it is looking

Meson-nucleon interaction and differential geometry

1. Split a 6-dimensional space in an (4 + 2)-dimensional one. Consider the 2-dimensional space as a spherical surface. Introduce on it *three* Cartesian coordinates with the restriction

$$\Omega_1^2 + \Omega_2^2 + \Omega_3^2 = 1. \tag{1}$$

The 4-dimensional space-time, denoted with x^i or briefly x, will be left *unchanged.* We consider now the linear *group*

$$\Omega'^A = C_B^A(x)\Omega^B \tag{I}$$

(fundamental for all that follows), with x-dependent coefficients. As orthogonality conditions are not elegant, I introduce coefficients $g_{AB}(x)$ possibly depending on x, *(but independent of the Ω's)* and generalize (1) to

$$g_{AB}(x)\Omega^A\Omega^B = 1; \quad A, B = 1, 2, 3 \tag{1a}$$

{although one can, of course, transform the g_{AB} always into δ_{AB} for all x by a suitable transformation of form (I)}.

Remark. In the case of a *circle* ($A, B = 1, 2$ only), my Ω^1, Ω^2 correspond to $\cos x_5$ and $\sin x_5$ of Kaluza, so that the gauge transformation $x_5' = x_5 + f(x^1 \ldots x^4)$ is indeed of the form (I). The group (I) seems to me therefore the *natural generalization of the gauge-group* in case of a two-dimensional spherical surface. (Whether or not orthogonality conditions are included in (I) is a matter of convenience. I decided to apply them *not*.)

2. The usual rules of the transformation of *covariant* vectors are (if $x' \equiv x$)

$$f_A' = \frac{\partial\Omega^B}{\partial\Omega'^A} f_B \tag{2a}$$

$$f_i' = f_i + \frac{\partial\Omega^B}{\partial xi} f_B \tag{2b}$$

($\frac{\partial\Omega^B}{\partial x^i}$ is understood for fixed Ω'). Correspondingly for a symmetrical covariant tensor (transforms like product of two vectors)

$$g_{AB}' = \frac{\partial\Omega^C}{\partial\Omega'^A}\frac{\partial\Omega^D}{\partial\Omega'^B} g_{CD}, \tag{3a}$$

$$g'_{ik} = g_{ik} + \frac{\partial\Omega^B}{\partial x^i} g_{Bk} + \frac{\partial\Omega^B}{\partial x^k} g_{Bi} + \frac{\partial\Omega^B}{\partial x^i}\frac{\partial\Omega^C}{\partial x^k} g_{BC}. \tag{3c}$$

And for contravariant vectors

$$f'^A = \frac{\partial\Omega'^A}{\partial\Omega^B} f^B + \left(\frac{\partial\Omega'^A}{\partial x^k}\right)_{\Omega \text{ fixed}} f^k \tag{4a}$$

$$f'^i = f^i. \tag{4b}$$

The main ,*rule of the game*' for the group (I) is to eliminate the rules (2b), (3c) and (4a) with help of a given tensor g_{AB} and g_{Ai} by defining the *underlining* of *small* indices *below* and *capital* indices *above* (for the other two possibilities the underlining is superfluous anyhow), such that the vectors (or tensors) with underlined indices transform under group (I) entirely disregarding the x-space

$$f'_{\underline{i}} = f_{\underline{i}},$$

$$f'^{\underline{A}} = \frac{\partial\Omega'^A}{\partial\Omega^B} f^{\underline{B}}. \tag{5}$$

Now define g_{AB} being given in the usual way $g^{\underline{AB}}$ by

$$g_{AC}\, g^{\underline{B}\,\underline{C}} = \delta_A^B.$$

Use $g^{\underline{A}\,\underline{B}}$ to rise indices, for instance $g_i^{\underline{A}} = g_{iB}\, g^{\underline{A}\,\underline{B}}$, and define

$$f_i = f_i - g_i^{\underline{A}} f_A,$$

$$f^{\underline{A}} = f^A + g_k^{\underline{A}} f^k. \tag{6}$$

Then the transformation law (5) actually holds. The rules for underlining *tensors* follow immediately from this, as tensors always behave like products of vectors. For the g-tensor itself follows for instance*

$$g_{\underline{i}A} \equiv 0, \qquad gik = g_{\underline{i}\,\underline{k}} + g_{iA} g_k^{\underline{A}} \tag{7a}$$

$$g^{i\underline{A}} \equiv 0, \qquad g^{AB} = g^{\underline{A}\,\underline{B}} + g^{ik} g_i^{\underline{A}} g_k^{\underline{B}} \tag{7b}$$

But

$$g^{iA} = -g^{ik} g_k^{\underline{A}} = -g^{ik} g^{\underline{A}\,\underline{B}} g_{Bk} \tag{7c}$$

$$f^i = g^{ik} f_k + g^{iA} f_A = g^{ik} f_{\underline{k}}$$

the Ω's

$$g_{Ai} = f_{AB,i}(x)\Omega^B \qquad \text{(II}$$

as g_{AB} is independent of Ω. The assumption (II) enables us to transform awa (means nullifying) *locally* the g_{Ai} by a suitable transformation (I), the secon term in the bracket of (3b) depending linear on the Ω's.

3. We define now accordingly the Dirac-matrices by

$$\frac{1}{2}(\gamma_i\gamma_k + \gamma_k\gamma_i) = g_{ik}$$

$$\frac{1}{2}(\gamma_i\gamma_A + \gamma_A\gamma_i) = g_{iA}$$

$$\frac{1}{2}(\gamma_A\gamma_B + \gamma_B\gamma_A) = g_{AB}.$$

Hence with

$$\gamma_{\underline{i}} = \gamma_i - g_i^{\underline{A}}\gamma_A = g_{\underline{i}\,\underline{k}}\gamma^k,$$

$$\gamma^{\underline{A}} = \gamma^A + g_k^{\underline{A}}\gamma^k = g^{\underline{A}\,\underline{B}}\gamma_B,$$

$$\frac{1}{2}(\gamma_{\underline{i}}\gamma_{\underline{k}} + \gamma_{\underline{k}}\gamma_{\underline{i}}) = g_{\underline{i}\,\underline{k}},$$

$$\frac{1}{2}(\gamma^i\gamma^k + \gamma^k\gamma^i) = g^{ik},$$

$$\frac{1}{2}(\gamma_{\underline{i}}\gamma_A + \gamma_A\gamma_{\underline{i}}) = 0,$$

$$\frac{1}{2}(\gamma^i\gamma^{\underline{A}} + \gamma^{\underline{A}}\gamma^i) = 0,$$

$$\frac{1}{2}(\gamma^{\underline{A}}\gamma^{\underline{B}} + \gamma^{\underline{B}}\gamma^{\underline{A}}) = g^{\underline{A}\,\underline{B}}.$$

One can identify $\gamma^{\underline{A}}$ with $\gamma^5\tau^A$, τ_A being the isotopic spin commuting wit the $\gamma_{\underline{i}}$'s. I omit here the usual discussion of Hermiticity.

The Dirac-equation becomes {see (II)}

$$\gamma^k\psi_{;k} + \gamma^A\frac{\partial\psi}{\partial\Omega^A} + M\psi = 0$$

or

$$\gamma^k\psi_{;k} + (\gamma^{\underline{A}} - \gamma^k f_{\underline{B},k}^{\underline{A}}(x)\Omega^B)\frac{\partial\psi}{\partial\Omega^A} + M\psi = 0. \qquad \text{(7}$$

away by (I) – except as a factor of the mass-term!

Remarks

1. It is

$$\psi_{;k}\frac{\partial \psi}{\partial x^k} + \Lambda_k \psi$$

with a suitable Λ_k. In the coordinate system where $g_{AB} = \delta_{AB}$ one has, however, $\Lambda_k = 0$.

2. The homogeneous character of the Ω's {see (1a), above} can be taken care of by adding a suitable chosen $\lambda g_{AB}\Omega^B$ to every $\frac{\partial}{\partial \Omega^A}$. One can fix λ by claiming $\Omega^A \frac{\partial \psi}{\partial \Omega^A} = 0$ (λ arbitrary – possibly Ω-depending).

The homogeneous coordinates are very convenient for practical computations as the spherical harmonics are *polynoms* in the Ω's.

3. For the case of the circle ($A, B = 1, 2$ only) one can try to write the Dirac-equations in an electromagnetic field in this way. Or (Kaluza-Klein). But then only very particular solutions of (7) are interpreted (and therefore I could not accept it). By developing ψ in a Fourier series on the circle, one obtains a quantum number n (which is a *component* of your „orbital-momentum quantum number" of the Ω-space, which in your case gives rise to $S, P, D \ldots$ terms). Then only the lowest value of n ($n = 0$ or 1) could be interpreted in the Kaluza-Klein-fashion.

4. The constant term M_ψ may be superfluous (?). – I am looking forward to get your paper. – Would you like to assume a field $F(x)\psi$ *instead of* $M\psi$? Or $F_B(x)\Omega^B\psi$?

But now comes my *main result.* What are the necessary and sufficient conditions, that g_{Ai} {means $f_{AB,i}(x)$} can be transformed away in the whole x-space? Answer: there exists a *tensor*

$$F^{\underline{A}}{}_{\underline{i}\,\underline{k}} = f^{\underline{A}}{}_{B\underline{i}\,\underline{k}}\Omega^B \equiv \frac{\partial g^{\underline{A}}_{k}}{\partial x^i} - \frac{\partial g^{\underline{A}}{}_{i}}{\partial x^k} + \frac{\partial g^{\underline{A}}{}_{i}}{\partial \Omega^B} g^{\underline{B}}{}_{k} - \frac{\partial g^{\underline{A}}{}_{k}}{\partial \Omega^B} g^{\underline{B}}{}_{i} \tag{8}$$

or also

$$f^{\underline{A}}{}_{B\underline{i}\,\underline{k}}(x) = \frac{\partial f^{\underline{A}}{}_{B,k}}{\partial x^i} - \frac{\partial f^{\underline{A}}{}_{Bi}}{\partial x^k} + f^{\underline{A}}{}_{C,i} f^{\underline{C}}{}_{B,k} - f^{\underline{A}}{}_{Ck} f^{\underline{C}}{}_{Bi}. \tag{8a}$$

This is the *true* physical field, the analogon of the *field-strength*. If and only if they vanish, the field $f^A{}_{B,i}$ can be made identically zero for all x's, by a transformation (I).

I have checked the tensor-character of (8) in different ways.

[1] Das folgende Manuskript befand sich nicht bei der Pauli-Korrespondenz, die Pais auf Grund des Rundschreibens zur Einsammlung der Briefe an Franca Pauli gesandt hatte. Es wurde erst

nachträglich von ihm zusammen mit folgendem Begleitschreiben vom 1. Mai 1963 an Franca Pauli geschickt: „A few days ago, while I was sorting out some papers, I came across the enclosed pages written by the late Professor Pauli. As I have not send them to you earlier for photocopying, I do so now. I look forward to the return of these pages at your convenience. – It may be helpful to add that these notes must have been written in the second part of 1953. Reference to the work contained therein is found in a paper by P. Gulmanelli entitled *Su una teoria dello spin isotopico* which was published in Milan.“ Aus dem Brieftext und dem Hinweis auf eine Textstelle geht eindeutig die Zusammengehörigkeit von Brief und Manuskript hervor. – Das Manuskript ist auch in der Zusammenstellung der wichtigsten Arbeiten zur Entwicklung der Eichtheorien von Lochlainn O'Raifeartaigh [1997, S. 171–175] enthalten.

* For our purpose (disregarding of gravitation) it is sufficient to identify $g_{\underline{ik}}$ and g^{ik} with the diag $\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$ of special relativity; similar with γ^i and $\gamma_{\underline{i}}$.

** Potential der Dyson-Betheschen „Wurstfabrik“. {Vgl. hierzu Dysons Bemerkung über die Anwendung der Tamm-Dancoff-Methode in seinem Schreiben [1937].}