In Proceedings of the Workshop on "Effective Field Theories of the Standard Model", Dobogókõ, Hungary August 22 - 26, 1991 ; U. Meissner edt., World Scientific Singapore 1992

Proton Spin and Quark Spin *

Peter Minkowski
Institute for Theoretical Physics
University of Bern
Sidlerstr. 5, CH-3012 Bern, Switzerland

December 1991

Abstract

The spin and angular momentum structure of the proton is discussed, within the framework of QCD involving the three light quark flavors u, d , s. It is argued, that unless large angular (sub)momenta - all adding up to $J_{tot} = 1/2$ - compose a considerable fraction of the proton, contrary to the simple ordering pattern seen in the spectroscopy of baryonic resonances, the asymptotic sum rule for the flavor singlet spin dependent structure function satisfies the inequality :

$$\lim_{Q^2 \to \infty} 2 \int g_1^{0\,p}(x, Q^2)\, dx = \sum_{q=u,d,s} g_q \gtrsim 0.4 \pm 0.1$$

The quantities $g_{u,d,s}$ in the equation above denote the axial current proton coupling constants for the respective quark flavor. The superscript ${}^{0\,p}$ denotes the singlet (vector)currrent correlation function of the proton, involving the spin asymmetry.

*Work supported in part by Schweizerischer Nationalfonds

We consider the spin dependent proton structure function [1] - [7]

$$g_1^{p\,jj}(x, Q^2)$$

corresponding to the (electromagnetic) current correlation function

$$W_{\mu\nu}^{jj}(q, p; h', h) =$$

$$\frac{1}{4\pi} \langle \mathcal{N}, p; h' | \int d^4 z\, e^{iqz} (j_\mu(z)\, j_\nu(0)) | \mathcal{N}, p; h \rangle$$

$$W_{\mu\nu}^{jj}(h', h) = \begin{array}{l} \left\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1^{p\,jj} + (p_\mu p_\nu)_\perp F_2^{p\,jj} \right\} \delta_{h'h} + \\ + \; i \varepsilon_{\mu\nu\sigma\tau} q^\sigma \; \bar{u}(p, h') \gamma^\tau \gamma_5 u(p, h) \frac{1}{2\nu} g_1^{p\,jj} + \cdots \end{array} \tag{1}$$

In eq. (1) standard notation for deep inelastic scattering is used :

$$f^{p\,jj} = f^{p\,jj}(x, Q^2) \quad \text{for} \quad f = F_1, F_2, g_1, \cdots$$

$$\nu = pq \quad , \quad Q^2 = -q^2 \quad , \quad x = \frac{Q^2}{2\nu} \tag{2}$$

$$(p_\mu p_\nu)_\perp = \left(p_\mu - \frac{\nu}{q^2} q_\mu \right) \left(p_\nu - \frac{\nu}{q^2} q_\nu \right)$$

whereas $h', h = \pm 1/2$ denote the helicities of the nucleons. For $|\vec{p}| \to \infty$ the (covariant) spin vector of the nucleon is given by the axial bilinear :

$$\text{for} \quad |\vec{p}| \to \infty \quad : \quad \left\{ \begin{array}{l} \bar{u}(p, h) \gamma^\tau \gamma_5 u(p, h) \approx 2 m_{\mathcal{N}} s^\tau(h) \\ s(h) \approx (2h)\, p / m_{\mathcal{N}} \end{array} \right.$$

The currents in the correlation function in eq. (1) are considered as neutral vector currents within QCD involving the three light quark flavors u, d, s. In flavor space the current j_μ is determined by a diagonal flavor matrix Q_j (Q^{em} for the electromagnetic current) :

$$j_\mu = \bar{q}_A Q^j_{AB} \gamma_\mu q \quad ; \quad A, B = u, d, s$$

$$Q^j_{AB} = Q^j_A \delta_{AB} \quad ; \quad Q^{em}_A = (2/3 , -1/3 , -1/3)$$

We are interested here only in the mean value of $g_1^{p\,jj}(x, Q^2)$ over x :

$$G_1^{p\,jj}(Q^2) = \int_0^1 dx \; g_1^{p\,jj}(x, Q^2)$$

$$G_1^{p\,jj} = G_1^{p\,em} \quad ; \quad \text{for} \quad j = j^{em} \tag{3}$$

We shall call $G_1^{p\,jj}$ in eq. (3) the spin moment or more precisely the quark spin moment of the structure function $g_1^{p\,jj}$.
Spin moments are decomposed according to the flavor matrices $(Q^j)^2$

$$(Q^j)^2 = q^{jj}_{(0)} Q^{(0)} + q^{jj}_{(8)} Q^{(8)} + q^{jj}_{(3)} Q^{(3)}$$

$$Q^{(0)} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \quad Q^{(8)} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \quad Q^{(3)} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

$$q^{jj}_{(0)} = \frac{1}{3} \left[(Q^j_u)^2 + (Q^j_d)^2 + (Q^j_s)^2 \right] \quad , \quad q^{em}_{(0)} = 2/9$$

$$q^{jj}_{(8)} = \frac{1}{6} \left[(Q^j_u)^2 + (Q^j_d)^2 \right] - \frac{1}{3} (Q^j_s)^2 \quad , \quad q^{em}_{(8)} = 1/18$$

$$q^{jj}_{(3)} = \frac{1}{2} \left[(Q^j_u)^2 - (Q^j_d)^2 \right] \quad , \quad q^{em}_{(3)} = 1/6 \tag{4}$$

as follows :

$$G_1^{p\,jj} = \frac{1}{2} \left(q_{(0)}^{jj} \; G_1^{p\,(0)} + q_{(8)}^{jj} \; G_1^{p\,(8)} + q_{(3)}^{jj} \; G_1^{p\,(3)} \right) \tag{5}$$

The basis spin moments

$$G_1^{p\,(X)} \quad \text{for} \quad X = 0 \, , \, 8 \, , \, 3$$

are in one to one correspondence with the associated axial vector current operators $j_\mu^{5\,(X)}$:

$$G_1^{p\,(X)} \quad \leftrightarrow \quad j_{\tau\,5}^{(X)} = \bar{q} \;\; Q^{(X)} \, \gamma_\tau \, \gamma_5 \, q \tag{6}$$

The current operators $j_\tau^{5\,(X)}$ in eq. (6) the precise definition of which involves precisely the spin moments $G_1^{p\,(X)}$ shall be characterized by the nucleon forward matrix elements :

$$\begin{aligned} &\langle \, \mathcal{N} \, , \, p \, ; \, h' \mid j_\tau^{5\,(X)} \, (\, 0 \,) \mid \mathcal{N} \, , \, p \, ; \, h \, \rangle \; = \; \bar{u} \, (\, p \, , \, h' \,) \, \gamma^\tau \, \gamma_5 \, u \, (\, p \, , \, h \,) \; g_{\mathcal{N}}^{(X)} \\ &g_{(\,\mathcal{N} \,=\, p\,)}^{(X)} = g^{(X)} \quad , \quad g_{(\,\mathcal{N} \,=\, n\,)}^{(0)\,,\,(8)} = g^{(0)\,,\,(8)} \quad , \quad g_{(\,\mathcal{N} \,=\, n\,)}^{(3)} = - \, g^{(3)} \\ &g^{(0)} = g_u + g_d + g_s \quad , \quad g^{(8)} = g_u + g_d - 2 \, g_s \quad , \quad g^{(3)} = g_u - g_d \end{aligned} \tag{7}$$

The relations in eq. (7) are a consequence of isospin ($SU2_{\,u\,,\,d}$) invariance. We note that a multiplicative renormalization of the axial currents $j_\tau^{5\,(X)}$ entails the identical renormalization of the (proton) matrix elements $g^{(X)}$. We denote this property by the formal relation :

$$g^{(X)} = g^{(X)} \left[\, j_\tau^{5\,(X)} \, \right] \quad \propto \quad j_\tau^{5\,(X)}$$

The vector currents in the correlation function in eq. (1) as well as the traceless axial currents generating together the charges of $SU3_{\,fl} \times SU3_{\,fl}$ have identically vanishing anomalous dimensions. The spin moments $G_1^{p\,(X)}$ in eq. (6) are the product of a Wilson coefficient function and the associated axial current matrix element $g^{(X)}$ defined in eq. (7) :

$$G_1^{p\,(X)} [Q^2 / \Lambda^2] = C^{(X)} [Q^2 / \mu^2 , \kappa_\mu] \; g^{(X)} (\mu^2) + O (1 / Q^2)$$
$$g^{(X)} (\mu^2) = g^{(X)} \left[j_\tau^{5\,(X)} \Big|_\mu \right] \tag{8}$$

The redundant scale μ in eq. (8) appears in conjunction with the QCD coupling constant

$$\kappa_\mu = \frac{g_\mu^2}{16 \pi^2}$$

through the μ dependence of the short distance coefficient functions $C^{(X)}$ for a fixed value of the coupling constant. The latter satisfy a renormalization group equation :

$$- \partial_\tau C^{(X)} = \left[\gamma^{(X)} \left(j_\tau^{5\,(X)} \right) + b (\kappa) \partial_\kappa \right] C^{(X)}$$
$$\tau = \log (Q^2 / \mu^2) \quad , \quad b (\kappa) = - (\beta / g) \kappa = 9 \kappa^2 + 64 \kappa^3 + \cdots$$
$$\gamma^{(X)} = \gamma^{(X)} (\kappa) \tag{9}$$

In eq. (9) $b (\kappa)$ denotes the coupling constant rescaling function for three color triplet quark flavors, whereas

$$\gamma^{(X)} (\kappa) = \mu^2 \frac{\partial}{\partial \mu^2} \log \left[Z \left(j_\tau^{5\,(X)} \right) \right]$$

stands for the anomalous dimension of the axial current $j_\tau^{5\,(X)}$ respectively [8] :

$$\gamma^{(3)} = \gamma^{(8)} = 0$$
$$\gamma^{(0)}(\kappa) = 24\,\kappa^2 + O(\kappa^3) \tag{10}$$

Eq. (9) yields the standard solution [8] , [9] :

$$C^{(X)}[\,Q^2/\mu^2\,,\,\kappa\,] = C^{(X)}[\,1\,,\,\bar{\kappa}\,]\ \exp - \int_{\bar{\kappa}}^{\kappa} \frac{\gamma^{(X)}(\kappa')}{b(\kappa')}\, d\kappa' \tag{11}$$

$$\kappa = \kappa_\mu$$

In eq. (11) $\bar{\kappa}$ denotes the running coupling constant, determined from $\kappa = \kappa_\mu$ through the implicit equation :

$$\tau = \int_{\bar{\kappa}}^{\kappa} \frac{1}{b(\kappa')}\, d\kappa' \quad ; \quad \bar{\kappa} = \bar{\kappa}(\tau, \kappa) \quad ; \quad \bar{\kappa}(0, \kappa) = \kappa$$

$$\frac{1}{9\,\bar{\kappa}} = \log(Q^2/\Lambda^2) + \frac{64}{81} \log\log(Q^2/\Lambda^2) + O\left(\frac{\log\log(Q^2/\Lambda^2)}{\log(Q^2/\Lambda^2)}\right)$$

$$\text{for } Q^2 \to \infty \tag{12}$$

A one loop calculation [9] yields :

$$C^{(X)}[\,1\,,\,\bar{\kappa}\,] = 1 - 4\,\bar{\kappa} + O(\bar{\kappa}^2) \tag{13}$$

independent of (X) to first order in $\bar{\kappa}$.

The integrands $\gamma^{(X)}(\kappa')/b(\kappa')$ in eq. (11) are integrable at $\kappa' = 0$. This allows to express the spin moments (8) in the form, neglecting power corrections :

$$G_1^{p\,(X)}[Q^2/\Lambda^2] = C^{(X)}[1, \bar\kappa]\ E^{(X)}(\bar\kappa) \left[\frac{g^{(X)}(\mu^2)}{E^{(X)}(\kappa_\mu)} \right]$$

$$E^{(X)}(\kappa) = \exp \int_0^{\kappa} \frac{\gamma^{(X)}(\kappa')}{b(\kappa')}\, d\kappa' \quad ; \quad E^{(X)}(0) = 1 \tag{14}$$

The running coupling constant

$$\bar\kappa(Q^2/\Lambda^2) \quad ; \quad \bar\kappa(\mu^2/\Lambda^2) = \kappa_\mu$$

is only implicitely depending on the initial values of scale and coupling constant μ , κ_μ . Thus the expression in brackets on the right hand side of eq. (14) is renormalization group invariant :

$$\frac{g^{(X)}(\mu^2)}{E^{(X)}(\kappa_\mu)} = \frac{g^{(X)}(\mu'^2)}{E^{(X)}(\kappa_{\mu'})} = g^{(X)}(\infty) \quad ; \quad \kappa_{\mu'} = \bar\kappa(\mu'^2/\Lambda^2) \tag{15}$$

Since the scale dependent quantities $g^{(X)}(\mu^2)$ have no invariant meaning we shall define the axial coupling constants $g_{u,d,s}$ as referring to the axial currents normalized at infinite scale, thus resolving the ambiguity in the definition of the normalization of the (singlet) axial current and of the quantities in eq. (7) :

$$g^{(X)} = g^{(X)}(\infty) = g^{(X)}\left[j_\tau^{5\,(X)} \Big|_{\mu=\infty} \right] = \frac{g^{(X)}(\mu'^2)}{E^{(X)}(\kappa_{\mu'})} \tag{16}$$

$$j_\tau^{5\,(X)} = j_\tau^{5\,(X)} \Big|_{\mu=\infty}$$

The expressions for the spin moments in eq. (8 , 14) thus become (to first order in $\bar{\kappa}$) :

$$G_1^{p\,(X)}\,[\,Q^2/\Lambda^2\,] \;=\; C^{(X)}\,[\,1\,,\,\bar{\kappa}\,]\;\;E^{(X)}\,(\,\bar{\kappa}\,)\;\;g^{(X)}$$

$$E^{(3)\,,\,(8)} \;\equiv\; 1 \quad , \quad E^{(0)}\,(\,\bar{\kappa}\,) \;=\; 1 \,+\, \frac{8}{3}\,\bar{\kappa} \,+\, O\,(\,\bar{\kappa}^{\,2}\,)$$

$$G_1^{p\,(3)} \;\approx\; (\,1 \,-\, 4\,\bar{\kappa}\,)\,(\,g_u \,-\, g_d\,) \quad , \quad G_1^{p\,(8)} \;\approx\; (\,1 \,-\, 4\,\bar{\kappa}\,)\,(\,g_u \,+\, g_d \,-\, 2\,g_s\,)$$

$$G_1^{p\,(0)} \;\approx\; (\,1 \,-\, 4\,\bar{\kappa}\,/\,3\,)\,(\,g_u \,+\, g_d \,+\, g_s\,) \tag{17}$$

The relations in eq. (17) have been widely discussed in terms of quark and gluon spin asymmetry distributions in connection with the anomalous Ward identity of the singlet axial current [9] , [10] , [11] , [12] .

The precise normalization condition for the singlet axial current given in eq. (16) is detailed, since it is this current which defines the (unique) quark spin density operator

$$\mathcal{F}_{(q)\;\;\mu\nu}{}^{\sigma} \;=\; (\,1\,/\,2\,)\,\varepsilon_{\mu\nu}{}^{\sigma\tau}\;\;j_{\tau}^{\,5\,(0)}$$

contributing to the angular momentum density (in QCD) :

$$\begin{aligned}
&\mathcal{J}_{\mu\nu}{}^{\sigma} \;=\; \mathcal{F}_{(q)\;\;\mu\nu}{}^{\sigma} \,+\, T_{\mu\nu}{}^{\sigma} \quad , \quad \partial_{\sigma}\,\mathcal{J}_{\mu\nu}{}^{\sigma} \;=\; 0 \\
&\mathcal{F}_{(q)\;\;\mu\nu}{}^{\sigma} \;=\; (\,1\,/\,2\,)\,\varepsilon_{\mu\nu}{}^{\sigma\tau}\,\sum_q\;\bar{q}\,\gamma_{\tau}\,\gamma_5\,q \\
&T_{\mu\nu}{}^{\sigma} \;=\; x_{\mu}\,\Theta_{\nu}{}^{\sigma} \,-\, x_{\nu}\,\Theta_{\mu}{}^{\sigma} \\
&\Theta_{\nu}{}^{\sigma} \;=\; \Theta_{(V)\,\nu}{}^{\sigma} \,+\, \Theta_{(q)\,\nu}{}^{\sigma} \\
&\Theta_{(V)\,\nu}{}^{\sigma} \;=\; F_{\varrho\,\nu}^{\;a}\,F^{\sigma\,\varrho\;\;a} \,+\, (\,1\,/\,4\,)\,\delta_{\nu}{}^{\sigma}\,F_{\alpha\,\beta}^{\;a}\,F^{\alpha\,\beta\;\;a} \\
&\Theta_{(q)\,\nu}{}^{\sigma} \;=\; \sum_q\,\bar{q}\,\Big[\,(\,i\,/\,2\,)\,\gamma^{\sigma}\;\overleftrightarrow{D}_{\nu}\,\Big]\,q \,-\, \delta_{\nu}{}^{\sigma}\,\mathcal{L}_q
\end{aligned} \tag{18}$$

In eq. (18) $F_{\alpha\beta}^{a}$ denotes the (eight) field strength tensors appropriate for the color group and $D_{\nu}\, q$ stands for the covariant derivative of the color triplet quark fields.
Finally the algebra of equal time commutators of spacelike components of (conserved) vector currents involves again the singlet axial current $j_{\tau}^{5\,(0)}\Big|_{\mu=\infty}$:

$$j_{\mu} = \bar{q}\; Q^{j}\, \gamma_{\mu}\, q$$

$$[\, j_{1}(t, \vec{x}),\, j_{2}(t, \vec{y})\,] = 2\, i\, \varepsilon_{1\,2\,3}\, \delta^{3}(\vec{x} - \vec{y})\; q_{(X)}^{jj}\; j^{\tau = 3,\,5\,(X)}(t, \vec{x}) \tag{19}$$

The normalization of the singlet axial current, as it appears in the quark spin density in eq. (18) is fixed by the commutation rules of the spin operators :

$$\vec{S}_{q}(t) = \frac{1}{2}\int d^{3}x\, \vec{j}^{\,5\,(0)}(t, \vec{x}) \tag{20}$$

$$\Big[\, S_{q}^{m}(t),\, S_{q}^{n}(t)\,\Big] = i\, \varepsilon_{mnr}\; S_{q}^{r}(t) \quad , \quad m, n, r = 1, 2, 3$$

The equal time commutator of vector current components in eq. (19) defines, through the right hand side, again the normalized singlet axial current as given in eq. (16). It was this relation which originally led Bjorken to derive the asymptotic sum rules [1] , [2] .

$$G_{\infty}^{\mathcal{N}\,em} = \lim_{Q^{2}\to\infty} G_{1}^{\mathcal{N}\,em}(Q^{2}/\Lambda^{2})$$

$$G_{\infty}^{p\,em} = \frac{1}{9}\, g^{(0)} + \frac{1}{36}\, g^{(8)} + \frac{1}{12}\, g^{(3)} \quad , \quad G_{\infty}^{p\,em} - G_{\infty}^{n\,em} = \frac{1}{6}\, g^{(3)} \tag{21}$$

$SU3_{fl}$ analysis of $g^{(8)}$

In the limit of exact $SU3_{fl}$ symmetry the following relations hold between the constants $g^{(3)\,,\,(8)}$ in eq. (21) and the axial current matrix elements obtained from hyperon decays :

decay	g_A / g_V	experiment	$SU3_{fl}\ fit$
$n \rightarrow p\, e^{\perp} \bar{\nu}_e$	$F + D$	1.260 ± 0.004	1.259
$\Lambda \rightarrow p\, e^{\perp} \bar{\nu}_e$	$F + D / 3$	0.718 ± 0.015	0.726
$\Xi^{\perp} \rightarrow \Lambda\, e^{\perp} \bar{\nu}_e$	$F - D / 3$	0.250 ± 0.050	0.192
$\Sigma^{\perp} \rightarrow n\, e^{\perp} \bar{\nu}_e$	$F - D$	-0.340 ± 0.017	-0.341

Table 1

The values in the rightmost column of Table 1 correspond to the following parameters, which we compare to the corresponding quantities inferred by Bourquin et al. [13], given in parentheses :

$$F = 0.46 \pm 0.025 \quad (0.458 \pm 0.016) \quad , \quad D = 0.80 \pm 0.025 \quad (0.734 \pm 0.017)$$

$$F / D = 0.58 \pm 0.06 \quad (0.62)$$

$$g^{(3)} = F + D = 1.260 \pm 0.004 \quad (1.192) \quad , \quad g^{(8)} = 3F - D = 0.58 \pm 0.10 \quad (0.64) \tag{22}$$

The error of $g^{(8)}$ given in eq. (22) does not take into account effects of $SU3_{fl}$ symmetry breaking.

Ignoring any additional error beyond the one quoted in eq. (22) for $g^{(3)} = g_A$ and $g^{(8)}$ we find for the nonsinglet (NS) linear combination occuring in the spin moment $G_{\infty}^{p\,em}$ in eq. (21) :

$$G_{\infty}^{p\,NS} = \frac{1}{36}\, g^{(8)} + \frac{1}{12}\, g^{(3)} = 0.121 \pm 0.003 \tag{23}$$

It is hard to conceive that $SU3_{fl}$ breaking effects can change the error for $G_{\infty}^{p\,NS}$ in eq. (23) appreciably, beyond 0.005 say.

The result of the EMC collaboration for the spin moment at an inferred average value of $\langle Q^2 \rangle = 10 \, GeV^2$ is [3] :

$$
\begin{aligned}
G_1^{p\,em} (\langle Q^2 \rangle / \Lambda^2) \quad & = \quad 0.126 \pm 0.010 \pm 0.015 \\
G_1^{p\,em} \approx \frac{1}{9} \, g^{(0)} + G_{\infty}^{p\,NS} \quad & = \quad 0.121 \pm 0.003 + \frac{1}{9} \, g^{(0)} \\
\rightarrow \qquad g^{(0)} \quad & = \quad 0.045 \pm 0.090 \pm 0.135
\end{aligned}
\tag{24}
$$

The remaining discussion is devoted to a theoretical estimate of the singlet axial current coupling constant $g^{(0)}$ as constrained by the angular momentum and quark spin structure of the proton [14] , [15] , [16] .

The link between quark spin and $g^{(0)}$ is provided by the relation in eq. (20). Thus we decompose the total angular momentum in the following way :

$$
\begin{aligned}
& \vec{J} = \vec{S}_q + \vec{R} \quad , \quad \vec{S}_q = \vec{S}_{123} + \vec{S}_{core} \\
& \vec{S}_{q\,,\,123\,,\,core} = \vec{S}_{q\,,\,123\,,\,core} (t)
\end{aligned}
\tag{25}
$$

In eq. (25) $\vec{S}_{123}$ denotes the total spin of three valence quarks, whereas $\vec{S}_{core}$ stands for the spin of quarks making up the core of hadrons, which we take to be $SU3_{fl}$ symmetric. Though the decomposition of angular momentum in eq. (25) is not the most general one, we impose the condition for addition of independent angular momenta :

$$
\left[\vec{S}_{123} \, , \, \vec{S}_{core} \right] = 0 \tag{26}
$$

The proton wave function thus can be decomposed into basis states with respect to the operators (at time $t = 0$ say) :

$$\vec{J}^{\,2} \rightarrow J\,(J+1) \quad , \quad J = 1/2 \qquad ; \qquad (J)_z \rightarrow m_J \quad , \quad m_J = 1/2$$

$$\vec{S}^{\,2}_{123} \rightarrow S_{123}\,(S_{123}+1) \qquad ; \qquad (S_{123})_z \rightarrow m_{123}$$

$$\left(\vec{R}\,'\right)^2 \rightarrow R'\,(R'+1) \qquad ; \qquad \left(\vec{R}\,'\right)_z \rightarrow m'$$

$$\vec{R}\,' = \vec{R} + \vec{S}_{core} \qquad ; \qquad m_J = m_{123} + m'$$

This decomposition is of the form :

$$\left| p \begin{array}{c} J \\ m_J \end{array} \right\rangle = \left| p \begin{array}{c} 1/2 \\ 1/2 \end{array} \right\rangle =$$

$$\sum \quad C \left(\begin{array}{c} 1/2 \\ 1/2 \end{array} \left| \begin{array}{cc} S_{123} & R' \\ m_{123} & m' \end{array} \right. \right) \left| \begin{array}{c} S_{123} \\ m_{123} \end{array} (\alpha) \right\rangle \left| \begin{array}{c} R' \\ m' \end{array} (\beta) \right\rangle \mathcal{A}^{\,S_{123}\;,\;R'}_{\,(\alpha)\;(\beta)}$$

$$\sum = \sum_{S_{123}\;,\;R'} \quad \sum_{m_{123}\;,\;m'\;,\;(\alpha)\;,\;(\beta)} \Big|_{\,m_{123}\,+\,m'\,=\,1/2} \tag{27}$$

In eq. (27) $C\,(\cdots)$ denotes the Clebsch Gordan coefficients. The labels (α) , (β) distinguish different states with given quantum numbers S_{123} , m_{123} and R' , m' respectively.

$$\mathcal{A}^{\,S_{123}\;,\;R'}_{\,(\alpha)\;(\beta)}$$

denotes the amplitude for the particular state with quantum numbers S_{123} , R' and labels (α) , (β) respectively. (α) runs over a finite set :

$$(\alpha) = \left\{ \begin{array}{ccc} m & \text{for} & S_{123} = 3/2 \\ s\;,\;m\;,\;a & \text{for} & S_{123} = 1/2 \end{array} \right. \tag{28}$$

The symbols s , m , a in eq. (28) refer to the symmetry class of the combined spin - flavor group of the three valence quarks : s = symmetric , m = mixed , a = antisymmetric.

The label (β) in eq. (28) represents orbital wave functions, which depend on the positions of valence and core quarks as well as gluons.

Angular momentum quantum numbers S_{123} , R' extend over the combinations :

$$(S_{123} , R') = \left\{ \begin{array}{l} \left(1/2 \;\middle|\; \begin{array}{c} 0 \\ 1 \end{array} \right) \\ \\ \left(3/2 \;\middle|\; \begin{array}{c} 2 \\ 1 \end{array} \right) \end{array} \right. \tag{29}$$

We continue to assume exact $SU3_{fl}$ invariance for valence as well as core parts of the wave functions in eq. (27). Thus the following association holds :

$$\vec{S}_{123} = \left(\vec{S}_u + \vec{S}_d \right)_{val}$$

$$\begin{array}{lllll} g_u - g_s & = & 2F & = & \left\langle p \begin{array}{c} 1/2 \\ 1/2 \end{array} \middle| \; 2 \, (S_u)_{z \, val} \; \middle| p \begin{array}{c} 1/2 \\ 1/2 \end{array} \right\rangle \\ g_d - g_s & = & F - D & = & \left\langle p \begin{array}{c} 1/2 \\ 1/2 \end{array} \middle| \; 2 \, (S_d)_{z \, val} \; \middle| p \begin{array}{c} 1/2 \\ 1/2 \end{array} \right\rangle \end{array} \tag{30}$$

From eqs. (27 , 30) it follows :

$$g_q - g_s = \sum \; C \begin{pmatrix} 1/2 & \Big| & \widetilde{S}_{123} & R' \\ 1/2 & \Big| & m_{123} & m' \end{pmatrix} \; C \begin{pmatrix} 1/2 & \Big| & S_{123} & R' \\ 1/2 & \Big| & m_{123} & m' \end{pmatrix}$$

$$\left\langle \begin{matrix} \widetilde{S}_{123} \\ m_{123} \end{matrix} \; (\widetilde{\alpha}) \; \middle| \; 2 \, (S_q)_{z \, val} \; \middle| \; \begin{matrix} S_{123} \\ m_{123} \end{matrix} \; (\alpha) \right\rangle$$

$$\left(\mathcal{A}^{\widetilde{S}_{123} \, , \, R'}_{(\widetilde{\alpha}) \, (\beta)} \right)^* \left(\mathcal{A}^{S_{123} \, , \, R'}_{(\alpha) \, (\beta)} \right)$$

$$q = u \, , \, d \quad ; \quad \sum = \sum_{\widetilde{S}_{123} \, | \, S_{123} \; ; \; (\widetilde{\alpha}) \, | \, (\alpha) \; ; \; m_{123} \, , \, m' \, , \, R' \, , \, (\beta)} \tag{31}$$

The sum in eq. (31) involves the nondiagonal matrix elements

$$\left\langle \begin{matrix} \widetilde{S}_{123} \\ m_{123} \end{matrix} \; (\widetilde{\alpha}) \; \middle| \; 2 \, (S_q)_{z \, val} \; \middle| \; \begin{matrix} S_{123} \\ m_{123} \end{matrix} \; (\alpha) \right\rangle$$

$$\text{for} \;\; \widetilde{S}_{123} \neq S_{123} \quad ; \quad (\widetilde{\alpha}) = (\alpha) \quad ; \quad \{ \, (\widetilde{\alpha}) \neq (\alpha) \, \}$$

The above matrix element is multiplied by the density matrix :

$$\varrho \begin{pmatrix} \widetilde{S}_{123} & \Big| & S_{123} & \\ (\widetilde{\alpha}) & \Big| & (\alpha) & ; \; R' \end{pmatrix} = \sum_{(\beta)} \left(\mathcal{A}^{\widetilde{S}_{123} \, , \, R'}_{(\widetilde{\alpha}) \, (\beta)} \right)^* \left(\mathcal{A}^{S_{123} \, , \, R'}_{(\alpha) \, (\beta)} \right) \tag{32}$$

The symmetry class (α) through the Fermi statistics is coupled with the symmetry class of the orbital valence quark variables. The sum over (β) in eq. (32) includes the orbital overlap of the associated wave functions. The latter vanishes for wave functions belonging to different orbital symmetry classes. Thus the density matrix ϱ in eq. (32) is diagonal with respect to the variables $(\widetilde{\alpha})$, (α) :

$$\varrho \left(\begin{array}{c} \widetilde{S}_{123} \\ (\widetilde{\alpha}) \end{array} \middle| \begin{array}{c} S_{123} \\ (\alpha) \end{array} ; \; R' \right) = \delta_{(\widetilde{\alpha})\,(\alpha)} \; \varrho \left(\widetilde{S}_{123} \; \middle| \; S_{123} \; ; \; (\alpha) \; , \; R' \right) \tag{33}$$

Thus eq. (31) becomes :

$$g_{q} - g_{s} = \sum \left\{ \begin{array}{l} \left\langle \begin{array}{c} \widetilde{S}_{123} \\ m_{123} \end{array} (\alpha) \middle| \; 2 \, (\, S_{q} \,)_{z \; val} \; \middle| \begin{array}{c} S_{123} \\ m_{123} \end{array} (\alpha) \right\rangle \\ C \left(\begin{array}{c} 1/2 \\ 1/2 \end{array} \middle| \begin{array}{cc} \widetilde{S}_{123} & R' \\ m_{123} & m' \end{array} \right) \quad C \left(\begin{array}{c} 1/2 \\ 1/2 \end{array} \middle| \begin{array}{cc} S_{123} & R' \\ m_{123} & m' \end{array} \right) \\ \varrho \left(\widetilde{S}_{123} \; \middle| \; S_{123} \; ; \; (\alpha) \; , \; R' \right) \end{array} \right.$$

$$\varrho \left(\widetilde{S}_{123} \; \middle| \; S_{123} \; ; \; (\alpha) \; , \; R' \right) = \sum_{(\beta)} \left(\mathcal{A}^{\widetilde{S}_{123} \, , \, R'}_{(\alpha) \; (\beta)} \right)^{*} \left(\mathcal{A}^{S_{123} \, , \, R'}_{(\alpha) \; (\beta)} \right)$$

$$\sum = \sum_{\widetilde{S}_{123} \, , \, S_{123} \, , \, m_{123} \, , \, R', \, (\alpha)} \quad ; \quad m' = 1/2 - m_{123} \tag{34}$$

From eq. (29) we see, that the interference term involves the quantum numbers :

$$\left(\begin{array}{c} \widetilde{S}_{123} = 3/2 \\ m_{123} \end{array} (\alpha) \middle| (\alpha) \begin{array}{c} S_{123} = 1/2 \\ m_{123} \end{array} \right) \quad ; \quad \begin{array}{l} m_{123} = \pm 1/2 \; , \; \alpha = \text{mixed} \\ R' = 1 \end{array}$$

We note here the Clebsch Gordan coefficients entering eq. (34) in this case :

$$C\left(\begin{array}{c|cc} 1/2 & 3/2 & 1 \\ 1/2 & +1/2 & 0 \end{array}\right) = -\frac{1}{\sqrt{3}} \quad \Bigg| \quad C\left(\begin{array}{c|cc} 1/2 & 1/2 & 1 \\ 1/2 & +1/2 & 0 \end{array}\right) = +\frac{1}{\sqrt{3}}$$

$$C\left(\begin{array}{c|cc} 1/2 & 3/2 & 1 \\ 1/2 & -1/2 & 1 \end{array}\right) = +\frac{1}{\sqrt{6}} \quad \Bigg| \quad C\left(\begin{array}{c|cc} 1/2 & 1/2 & 1 \\ 1/2 & -1/2 & 1 \end{array}\right) = -\frac{2}{\sqrt{6}} \tag{35}$$

The matrix elements in eq. (34) for the case of mixed symmetry are :

$$\tau_q = \left\langle \begin{array}{l} \widetilde{S}_{123} = 3/2 \\ m_{123} = \pm 1/2 \end{array} (\alpha) \right| 2 \left(S_q \right)_{z\ val} \left| \begin{array}{l} S_{123} = 1/2 \\ m_{123} = \pm 1/2 \end{array} (\alpha) \right\rangle \Bigg|_{(\alpha) = m}$$

$$\tau_q = (2/3)\, \sigma_q \quad ; \quad \sigma_q = \begin{cases} +1 & \text{for q = u} \\ -1 & \text{for q = d} \end{cases} \tag{36}$$

The nondiagonal matrix elements τ_q in eq. (36) are independent of the sign of the magnetic quantum number m_{123} .

The nondiagonal contribution to the nonsinglet axial coupling constants $g_q - g_s$ thus are obtained collecting the appropriate factors in eqs. (34 , 35 , 36) :

$$g_q - g_s \,|_I = -\frac{8}{9}\,\sigma_q\, I \quad ; \quad \sigma_q = \begin{cases} +1 & \text{for q = u} \\ -1 & \text{for q = d} \end{cases}$$

$$I = c\,|\,\varrho_I\,| \quad ; \quad c = Re\left(\frac{\varrho_I}{|\,\varrho_I\,|}\right) \quad ; \quad -1 \le c \le 1$$

$$\varrho_I = \varrho\left(\ \tilde{S}_{123} = 3/2 \ \middle|\ S_{123} = 1/2 \ ; \ (\alpha) = m \ , \ R' = 1\ \right) \tag{37}$$

We introduce here the probalbilities corresponding to the diagonal elements of the density matrix ϱ :

$$p\,(\,S_{123}\,,\,(\alpha)\,,\,R'\,) = \varrho\left(\ S_{123}\ \middle|\ S_{123}\ ;\ (\alpha)\ ,\ R'\ \right)$$

$$p\,[\,S_{123}\,,\,(\alpha)\,,\,R'\,] = P\,[\,S_{123}\,]\ r_{(S_{123})}\,[\,(\alpha)\,,\,R'\,]$$

$$S_{123} = \begin{cases} 1/2 : R' = \begin{cases} \underline{0} & ; \ (\alpha) = \underline{s}\,,\,m\,,\,a \\ \underline{1} & ; \ (\alpha) = s\,,\,m\,,\,\underline{a} \end{cases} \\ 3/2 : R' = \begin{cases} \underline{2} \\ 1 \end{cases} \ ; \ (\alpha) = \underline{m} \end{cases} \tag{38}$$

In eq. (38) $P\,[\,S_{123}\,]\ ;\ S_{123} = 1/2\,,\,3/2$ shall denote the probability for the total valence quark spin to take on the value $1/2$ and $3/2$ respectively, whereas the quantities $r_{(S_{123})}\,[\,(\alpha)\,,\,R'\,]$ stand for the relative probabilities, given the value of S_{123} , of the quantum numbers (α) , R' :

$$P[1/2] + P[3/2] = 1$$

$$\sum_{(\alpha),R'} \left\{ r_{(S_{123})} [(\alpha), R'] \right\} = 1 \quad ; \quad \text{for } S_{123} = 1/2, 3/2$$

The configurations , potentially dominating for a given value of S_{123} within the proton, are underlined in eq. (38) .

The triangular inequalities for density matrix elements imply for the interference term in eq. (37) :

$$|I| \leq \left(P[1/2] \, P[3/2] \, r_{(1/2)}[m,1] \, r_{(3/2)}[m,1] \right)^{1/2} \tag{39}$$

We turn to the diagonal elements of the valence quark spin operators in eq. (34) :

$$\left\langle \begin{matrix} S_{123} \\ m_{123} \end{matrix} (\alpha) \middle| 2(S_q)_{z\,val} \middle| \begin{matrix} S_{123} \\ m_{123} \end{matrix} (\alpha) \right\rangle$$

$$= D_q[S_{123}, (\alpha)] \left\langle \begin{matrix} S_{123} \\ m_{123} \end{matrix} \middle| 2(S_{123})_z \middle| \begin{matrix} S_{123} \\ m_{123} \end{matrix} \right\rangle \tag{40}$$

$$q = u, d \quad ; \quad D_u[S_{123}, (\alpha)] + D_d[S_{123}, (\alpha)] = 1$$

The spin projection constants $D_q[S_{123}, (\alpha)]$ in eq. (40) are given in Table 2 below.

The probabilities of the configurations defined in eqs. (38 , 39) are numbered in the following way :

$$\begin{aligned}
\underline{w}_1 &= P[1/2]\, r_{(1/2)}\, [\underline{s}, \underline{0}] \quad ; \quad w_2 = P[1/2]\, r_{(1/2)}\, [s, 1] \\
w_3 &= P[1/2]\, r_{(1/2)}\, [m, 0] \quad ; \quad w_4 = P[1/2]\, r_{(1/2)}\, [m, 1] \\
w_5 &= P[1/2]\, r_{(1/2)}\, [a, 0] \quad ; \quad \underline{w}_6 = P[1/2]\, r_{(1/2)}\, [\underline{a}, \underline{1}] \\
\underline{w}_7 &= P[3/2]\, r_{(3/2)}\, [\underline{m}, \underline{2}] \quad ; \quad w_8 = P[3/2]\, r_{(3/2)}\, [m, 1]
\end{aligned} \tag{41}$$

The matrix elements of the total valence quark spin, which contain only diagonal terms, are determined by the Lande factors , which are given together with the probabilities of the various configurations in Table 2 :

$$\left\langle p \begin{array}{c} 1/2 \\ 1/2 \end{array} \right| \; 2\,(S_{123})_z \; \left| p \begin{array}{c} 1/2 \\ 1/2 \end{array} \right\rangle_{S_{123},\, R'}$$

$$= g_L(J, S_{123}, R')\,|_{J=1/2} \left\langle p \begin{array}{c} 1/2 \\ 1/2 \end{array} \right| \; 2\,(J)_z \; \left| p \begin{array}{c} 1/2 \\ 1/2 \end{array} \right\rangle$$

$$g_L(J, J_1, J_2) = \frac{J(J+1) + J_1(J_1+1) - J_2(J_2+1)}{2J(J+1)} \tag{42}$$

(α)	S_{123}	R'	w	g_L	D_u	D_d
$\underline{s}$	$1/2$	$\underline{0}$	$\underline{w}_1$	1	$4/3$	$-1/3$
		1	w_2	$-1/3$		
m	$1/2$	0	w_3	1	$2/3$	$1/3$
		1	w_4	$-1/3$		
$\underline{m}$	$3/2$	$\underline{2}$	$\underline{w}_7$	-1		
		1	w_8	$5/3$		
$\underline{a}$	$1/2$	0	w_5	1	0	1
		$\underline{1}$	$\underline{w}_6$	$-1/3$		

Table 2

From eqs. (37) - (42) and Table 2 we infer the following expressions for the nonsinglet axial coupling constants $g_u - g_s$, $g_d - g_s$ or equivalently $g_A = g_u - g_d$, $g_d - g_s$:

$$g_A = \left[\begin{array}{l} \frac{5}{3}\, \underline{w}_1 - \frac{5}{9}\, w_2 + \frac{1}{3}\, w_3 - \frac{1}{9}\, w_4 \\ \\ - \frac{1}{3}\, \underline{w}_7 + \frac{5}{9}\, w_8 - w_5 + \frac{1}{3}\, \underline{w}_6 \\ \\ - \frac{16}{9}\, I \end{array} \right]$$

$$g_d - g_s = \left[\begin{array}{l} - \frac{1}{3}\, \underline{w}_1 + \frac{1}{9}\, w_2 + \frac{1}{3}\, w_3 - \frac{1}{9}\, w_4 \\ \\ - \frac{1}{3}\, \underline{w}_7 + \frac{5}{9}\, w_8 + w_5 - \frac{1}{3}\, \underline{w}_6 \\ \\ + \frac{8}{9}\, I \end{array} \right] \qquad (43)$$

$$I = \tilde{c}\, \sqrt{w_4\, w_8} \quad ; \quad -1 \leq \tilde{c} \leq 1$$

$$\sum_{i=1}^{8} w_i = 1$$

At first sight it may appear impossible to learn anything from expressing the two measurable axial coupling constants g_A, $g_d - g_s$ (in the limit of $SU3_{fl}$ symmetry) in terms of eight unknown parameters :

$$w_2 , w_3 , \cdots , w_8 , \tilde{c}$$

The expressions in eq. (43) entail in particular an inequality for $g_d - g_s$ (among related inequalities like e. g. $-1 \leq g_A \leq 5/3$) :

$$g_d - g_s = \left[-\frac{1}{3} + \frac{4}{9} w_2 + \frac{2}{3} w_3 + \frac{4}{9} w_5 + \frac{2}{9} \left(w_4 + 4 \tilde{c} \sqrt{w_4 w_8} + 4 w_8 \right) \right]$$

$$\geq -\frac{1}{3} \; ; \; \left(= -\frac{1}{3} \leftrightarrow w_2 = w_3 = w_4 = w_5 = w_8 = 0 \right) \tag{44}$$

Comparing $g_d - g_s = F - D$ with the corresponding quantity in Table 1 :

$$g_A / g_V (\Sigma^{\perp} \rightarrow n e^{\perp} \bar{\nu}_e) = F - D = -0.340 \pm 0.017 \tag{45}$$

it follows, that indeed $g_d - g_s$ is very near to the lower limit $-1/3$.

We recall here the approximations which underly the present discussion of the NS axial nucleon coupling constants g_A , $g_d - g_s$:

i) $SU3_{fl}$ symmetry allows to determine $g_{d\,val}$ from strange baryon decays (Table 1 , eqs. (22 , 45)) .

ii) Nontrivial color correlations between the valence quark and the core wave functions are neglected.

The quark spin in the core of the proton

As a consequence of eqs. (44 , 45) the following discussion shall be restricted to the extremal case :

$$g_d - g_s = -\frac{1}{3} \quad \leftrightarrow \quad w_2 = w_3 = w_4 = w_5 = w_8 = 0$$

In the above limit eq. (43) yields the relation :

$$\begin{aligned} \frac{4}{3}\,\underline{w}_6 + 2\,\underline{w}_7 &= \frac{5}{3} - g_A \approx 0.41 \\ g^{(8)} = 3F - D &= g_A - 2/3 \approx 0.59 \\ &\leftrightarrow 0.58 \pm 0.10 \quad \text{from the } SU3_{fl} \text{ fit} \end{aligned} \tag{46}$$

The remaining dominant contributions are given in Table 3 below, together with the Lande factors relative to the residual angular momentum R' , $g'_L = 1 - g_L$ and the spectroscopic $[\, SU6 \, , \, R' \,]$ assignments [17] :

(α)	S_{123}	R'	w	g'_L	$[\, SU6 \, , \, R' \,]$
$\underline{s}$	1 / 2	$\underline{0}$	$\underline{w}_1$	0	[56 , 0]
$\underline{m}$	3 / 2	$\underline{2}$	$\underline{w}_7$	2	[70 , 2]
$\underline{a}$	1 / 2	$\underline{1}$	$\underline{w}_6$	4 / 3	[20 , 1]

Table 3

We recall the angular momentum pieces defined in eqs. (25 - 27) :

$$\vec{J} \,=\, \vec{S}_{q} + \vec{R} \quad , \quad \vec{S}_{q} \,=\, \vec{S}_{123} + \vec{S}_{core} \quad , \quad \vec{R}\,' \,=\, \vec{R} + \vec{S}_{core}$$

Since the different values of R' belong to different symmetry classes as indicated in Table 3, there are no nondiagonal contributions to the matrix elements of $\vec{S}_{core}$ since the nondiagonal elements of the associated density matrix vanish :

$$\varrho \left(\tilde{R}\,' \quad (\tilde{\alpha}) \;\middle|\; R\,' \quad (\alpha) \quad ; \; S_{123} \right) \;=\; \sum_{(\beta)} \left(\mathcal{A}^{\,S_{123}\,,\,\tilde{R}\,'}_{(\tilde{\alpha})\,(\beta)} \right)^{*} \left(\mathcal{A}^{\,S_{123}\,,\,R\,'}_{(\alpha)\,(\beta)} \right)$$

$$= \quad 0 \quad \text{for} \quad \tilde{R}\,'\,,\,(\tilde{\alpha}) \,\neq\, R\,'\,,\,(\alpha) \tag{47}$$

The diagonal matrix elements of $\vec{S}_{core}$ are obtained from those of $\vec{R}\,'$:

$$\left\langle \begin{matrix} R\,' \\ m\,' \end{matrix} \;(\alpha)\;,\, S_{c}\,,\, R \;\middle|\; 2\,(\,S_{core}\,)_{z} \;\middle|\; \begin{matrix} R\,' \\ m\,' \end{matrix} \;(\alpha)\;,\, S_{c}\,,\, R \right\rangle$$

$$= g_{L}\,(\,R\,'\,,\,S_{c}\,,\,R\,)\,\left\langle \begin{matrix} R\,' \\ m\,' \end{matrix} \;(\alpha)\;,\, S_{c}\,,\, R \;\middle|\; 2\,(\,R\,'\,)_{z} \;\middle|\; \begin{matrix} R\,' \\ m\,' \end{matrix} \;(\alpha)\;,\, S_{c}\,,\, R \right\rangle$$

$$\left| \begin{matrix} R\,' \\ m\,' \end{matrix} \;(\alpha)\;,\, S_{c}\,,\, R \right\rangle$$

$$= \sum \quad C \begin{pmatrix} R\,' & \Big| & S_{c} & R \\ m\,' & \Big| & m_{c} & m_{R} \end{pmatrix} \left| \begin{matrix} S_{c} \\ m_{c} \end{matrix} \;(\alpha)\;,\,(\beta_{1}) \right\rangle \left| \begin{matrix} R \\ m_{R} \end{matrix} \;(\alpha)\;,\,(\beta_{2}) \right\rangle \times$$

$$\times\, \mathcal{A}^{\,S_{123}\,,\,R\,'\,,\,S_{c}\,,\,R}_{(\alpha)\,(\beta 1)\,(\beta 2)} \tag{48}$$

Thus the matrix element of the core spin in the proton generates the following contributions to the strange axial current coupling constant :

$$3\, g_s \;=\; \left\langle p \begin{array}{c} 1/2 \\ 1/2 \end{array} \right| \; 2\,(S_{core})_z \; \left| p \begin{array}{c} 1/2 \\ 1/2 \end{array} \right\rangle$$

$$3\, g_s \;=\; \left\{ \begin{array}{l} \underline{w}_6 \, g_L^{\prime\,(6)} \sum_{S_c\,,\,R} g_L (R' = 1 \,,\, S_c \,,\, R)\, ww^{(6)} (S_c \,,\, R) \\ + \;\; \underline{w}_7 \, g_L^{\prime\,(7)} \sum_{S_c\,,\,R} g_L (R' = 2 \,,\, S_c \,,\, R)\, ww^{(7)} (S_c \,,\, R) \end{array} \right\}$$

$$g_L^{\prime\,(6)} = g_L (1/2 \,,\, 1 \,,\, 1/2) = 4/3 \quad ; \quad g_L^{\prime\,(7)} = g_L (1/2 \,,\, 2 \,,\, 3/2) = 2 \tag{49}$$

In eq. (49) the quantities $ww^{(j)} (S_c \,,\, R)$, $j = 6 \,,\, 7$ denote the relative probabilities given the respective configurations denoted by the label $(j) = (6) \,,\, (7)$ in Table 3, of all subconfigurations with quantum numbers $S_c \,, R$ respectively :

$$\sum_{S_c\,,\,R} ww^{(j)} (S_c \,,\, R) = 1 \quad ; \quad \text{for } (j) = (6) \,,\, (7)$$

Addition of angular momenta implies the selection rules :

$$R = S_c + \Delta \quad ; \quad \Delta = \left\{ \begin{array}{ll} 1 \,,\, 0 \,,\, -1 & \text{for } (j) = (6) \\ 2 \,,\, 1 \,,\, 0 \,,\, -1 \,,\, -2 & \text{for } (j) = (7) \end{array} \right. \tag{50}$$

The factors multiplying the relative probabilities $ww^{(j)}$, $(j) = (6) \,,\, (7)$ shall be characterized by the quantities Δ in eq. (50) , rewriting eq. (49) in the form :

$$-\,3\, g_s = \frac{2}{3} \left(\underline{w}_6 \, \left\langle g_\Delta^{(6)} \right\rangle + \underline{w}_7 \, \left\langle g_\Delta^{(7)} \right\rangle \right)$$

$$\left\langle g_\Delta^{(j)} \right\rangle = \sum_{S_c\,,\,\Delta} g_\Delta^{(j)} (S_c \,,\, \Delta)\; ww^{(j)} (S_c \,,\, \Delta) \qquad \text{for } \; (j) = (6) \,,\, (7) \tag{51}$$

The quantities $g_{\Delta}^{(j)} (S_c , \Delta) , ww^{(j)} (S_c , \Delta) , (j) = (6) , (7)$ in eq. (51) are :

$$g_{\Delta}^{(6)} (S_c , \Delta) = - 2 g_L (R' = 1 , S_c , R = S_c + \Delta)$$

$$g_{\Delta}^{(7)} (S_c , \Delta) = - 3 g_L (R' = 2 , S_c , R = S_c + \Delta)$$

$$ww^{(j)} (S_c , \Delta) = ww^{(j)} (S_c , R = S_c + \Delta) \qquad \text{for} \quad (j) = (6) , (7)$$

The quantities $g_{\Delta}^{(6) , (7)}$ take on the following values :

Δ	$g_{\Delta}^{(6)}$	$g_{\Delta}^{(7)}$	
2	–	S_c	
1	S_c		$(S_c - 2) / 2$
0	$- 1$	$- 3 / 2$	
$- 1$	$- S_c - 1$		$- (S_c + 3) / 2$
$- 2$	–	$- S_c - 1$	

(52)

Eqs. (51 , 51) allow , in all generality , arbitrary values for the core spin :

$$- \infty \leq 3 g_s \leq + \infty$$

Yet the range of values in eq. (52) implies the inequality :

$$- 3 \; g_{\,s} \; \leq \; \frac{2}{3} \; \Big(\underline{w}_{\,6} \;\; \langle \, S_{\,c} \, \rangle_{\,(6)} \; + \; \underline{w}_{\,7} \;\; \langle \, S_{\,c} \, \rangle_{\,(7)} \Big)$$

$$\left\langle \begin{array}{c} S_{\,c} \\ \\ R \end{array} \right\rangle_{(j)} \; = \; \sum_{\,S_{\,c} \, , \, \Delta} \; \left(\begin{array}{c} S_{\,c} \\ \\ S_{\,c} \, + \, \Delta \end{array} \right) \; ww^{\,(j)} \, (\, S_{\,c} \, , \; \Delta \,) \qquad \text{for} \quad (j) \; = \; (6) \; , \; (7) \tag{53}$$

or equivalently

$$- 3 \; g_{\,s} \; \leq \; \frac{2}{3} \; (\, \underline{w}_{\,6} \; + \; \underline{w}_{\,7} \,) \;\; \langle \, S_{\,c} \, \rangle_{\,(6,7)}$$

$$\left\langle \begin{array}{c} S_{\,c} \\ \\ R \end{array} \right\rangle_{(6,7)} \; = \; \frac{\underline{w}_{\,6}}{\underline{w}_{\,6} \; + \; \underline{w}_{\,7}} \; \left\langle \begin{array}{c} S_{\,c} \\ \\ R \end{array} \right\rangle_{(6)} \; + \; \frac{\underline{w}_{\,7}}{\underline{w}_{\,6} \; + \; \underline{w}_{\,7}} \; \left\langle \begin{array}{c} S_{\,c} \\ \\ R \end{array} \right\rangle_{(7)} \tag{54}$$

In eqs. (53 , 54) the equality only holds if the configurations

$$\underline{w}_{\,6} \quad : \quad R \; = \; S_{\,c} \; + \; 1 \quad \leftrightarrow \quad \langle \, R \, \rangle_{\,(6)} \; = \langle \, S_{\,c} \, \rangle_{\,(6)} \; + \; 1$$

$$\underline{w}_{\,7} \quad : \quad R \; = \; S_{\,c} \; + \; 2 \quad \leftrightarrow \quad \langle \, R \, \rangle_{\,(7)} \; = \langle \, S_{\,c} \, \rangle_{\,(7)} \; + \; 2$$

dominate over the remaining configurations.

Eq. (46) implies

$$A / 3 = 0.136 \leq \frac{2}{3} (\underline{w}_6 + \underline{w}_7) \leq 0.205 = A / 2$$

$$\uparrow \left(\frac{4}{3} \underline{w}_6 + 2 \underline{w}_7 = A = \frac{5}{3} - g_A \approx 0.41 \right) \tag{55}$$

Combining eqs. (54) and (55) we obtain :

$$- 3 g_s \leq \frac{A}{2} \langle S_c \rangle_{(6,7)} \quad ; \quad A = \frac{5}{3} - g_A \approx 0.41 \tag{56}$$

Finally eqs. (46 , 46 , 46) yield the relations :

$$g^{(0)} = g^{(8)} + 3 g_s = 1 - A - \frac{2}{3} \left(\underline{w}_6 \left\langle g_\Delta^{(6)} \right\rangle + \underline{w}_7 \left\langle g_\Delta^{(7)} \right\rangle \right)$$

$$\geq 1 - A \left(1 + \frac{1}{2} \langle S_c \rangle_{(6,7)} \right) \tag{57}$$

From eqs. (52 - 57) we infer two inequalities for the mean value of the core quark spin S_c and the remaining angular momentum in the core R respectively :

$$\left\{ \begin{array}{ll} \langle S_c \rangle_{(6,7)} & - 2 \\ & \geq F (1 - g^{(0)}) \\ \langle R \rangle_{(6,7)} & - 1 \end{array} \right\} \quad ; \quad F = \frac{2}{5/3 - g_A} = 4.92 \pm 0.05 \tag{58}$$

We shall set $F = 5$ in the following. This just simplifies the inequalities in eq. (58) without significantly changing any numerical conclusion relative to the exact range of values. Thus eq. (58) becomes :

$$\langle S_c \rangle_{(6,7)} \geq 3 - 5\, g^{(0)} \quad ; \quad g^{(0)} \geq \left(3 - \langle S_c \rangle_{(6,7)} \right) / 5$$

$$\langle R \rangle_{(6,7)} \geq 4 - 5\, g^{(0)} \quad ; \quad g^{(0)} \geq \left(4 - \langle R \rangle_{(6,7)} \right) / 5 \tag{59}$$

From eq. (59) we infer, that a vanishing value of $g^{(0)}$ implies large values for the angular momenta in the core of the proton :

$$g^{(0)} = 0 \quad \rightarrow \quad \langle S_c \rangle_{(6,7)} \geq 3 \quad , \quad \langle R \rangle_{(6,7)} \geq 4 \tag{60}$$

I shall call the large and compensating values of the core angular momenta $\vec{S}_c$, $\vec{R}$ (eq. (60)) the double helix structure of the nucleon.

The clear mass separation of nucleon isobars with respect to total angular momentum does not show any evidence of the above double helix structure. A value of $\langle S_c \rangle_{(6,7)}$ as large as 3 cannot be excluded from first principles at present. Nevertheless I estimate

$$\langle S_c \rangle_{(6,7)} \leq 1 \quad \rightarrow \quad g^{(0)} \geq 0.4 \; (\pm\, 0.1) \tag{61}$$

The error in eq. (61) reflects the error from the $SU3_{fl}$ fit (eqs. (46 , 22) and Table 1) .

Conclusions

The spin sum rule in polarized deep inelastic lepton proton scattering relates the asymptotic moment of the structure function $g_1^{ep} (x , Q^2)$ to the weighted sum of axial current coupling constants of the proton (eqs. (3 , 7 , 21)) :

$$G_{\infty}^{p\,em} = \lim_{Q^2 \rightarrow \infty} \int_0^1 dx \; g_1^{p\,em} (x , Q^2) = \frac{1}{9}\, g^{(0)} + \frac{1}{36}\, g^{(8)} + \frac{1}{12}\, g^{(3)}$$

$$g^{(0)} = g_u + g_d + g_s \quad , \quad g^{(8)} = g_u + g_d - 2\, g_s \quad , \quad g^{(3)} = g_u - g_d$$

In the limit of $SU3_{fl}$ symmetry the nucleon configurations are decomposed into two parts, a valence quark and a core component, each separately color singlet, forming direct product wave functions.
Then the coupling constant $g_d - g_s$ satisfies the inequality (eq. (44)) :

$$g_d - g_s \geq - \frac{1}{3}$$

The nonsinglet axial current coupling constants of the proton whence analyzed in an $SU3_{fl}$ invariant way [7] lead to the (approximate) values (eq. (22)) :

$$g^{(8)} = = 0.58 \pm 0.10 \quad , \quad g_d - g_s = -0.340 \pm 0.017$$

The nonsinglet coupling constants serve as input to the present analysis of the singlet coupling constant $g^{(0)}$, based on the approximation (eq. (46)) :

$$g_d - g_s = - \frac{1}{3} \quad \rightarrow \quad g^{(8)} = g_A - 2 / 3 \approx 0.59 \quad (= 0.58 \pm 0.10)$$

For the above limiting value of $g_d - g_s$ the configurations, as classified by

1. the symmetry class of valence quark flavor times spin : $(\alpha) = s , m , a$
2. the valence quark spin S_{123}
3. the remaining angular momentum $\vec{R}\,' = \vec{J} - \vec{S}_{123}$

are reduced to three (Table 3 , eqs. (25 - 27)) .

They are denoted by the associated probabilities $\underline{w}_1 , \underline{w}_6 , \underline{w}_7$
and the respective values for $((\alpha) , S_{123} , R\,')$

$$\underline{w}_1 : (s , 1 / 2 , 0) \quad , \quad \underline{w}_6 : (a , 1 / 2 , 1) \quad , \quad \underline{w}_7 : (m , 3 / 2 , 2)$$

$\underline{w}_6$, $\underline{w}_7$ determine the deviation of g_A from $5/3$ (eq. (46)) :

$$\frac{4}{3} \underline{w}_6 + 2 \underline{w}_7 = \frac{5}{3} - g_A \approx 0.41$$

The remaining angular momentum

$$\vec{R}' = \vec{J} - \vec{S}_{123} = \vec{S}_c + \vec{R}$$

is decomposed into the core part of quark and antiquark spins S_c and the remainder R , representing the sum of orbital angular momenta due to valence quarks, quarks, antiquarks and gluons in the core and the spin due to gluons.

The distribution of the core spin S_c over the configurations (6 , 7) , characterized by $\underline{w}_6$, $\underline{w}_7$ determines the singlet axial currrent coupling constant (eq. (49))

$$g^{(0)} = g^{(8)} + 3 g_s$$

As main result of this paper we derive inequalities between the singlet coupling constant and the mean values of S_c and R over the subconfigurations (6) and (7) (eq. (58)) .

In the approximation

$$F = \frac{2}{5/3 - g_A} = 4.92 \pm 0.05 \rightarrow 5$$

the inequalities become (eq. (59)) :

$$\langle S_c \rangle_{(6,7)} \geq 3 - 5 g^{(0)} \quad ; \quad g^{(0)} \geq \left(3 - \langle S_c \rangle_{(6,7)} \right) / 5$$

$$\langle R \rangle_{(6,7)} \geq 4 - 5 g^{(0)} \quad ; \quad g^{(0)} \geq \left(4 - \langle R \rangle_{(6,7)} \right) / 5$$

$g^0 = 0$ implies large values for the angular momenta in the core of the proton :

$$g^{(0)} = 0 \quad \rightarrow \quad \langle S_c \rangle_{(6,7)} \geq 3 \quad , \quad \langle R \rangle_{(6,7)} \geq 4$$

Such a double helix configuration, dominating the nucleon core wave functions, cannot be excluded, yet we give a lower bound for $g^{(0)}$ based on the estimate :

$$\langle S_c \rangle_{(6,7)} \leq 1 \quad \rightarrow \quad g^{(0)} \geq 0.4 \ (\pm 0.1)$$

References

[1] J. D. Bjorken , Phys. Rev. **148** (1966) 1467 ,
Phys. Rev. **D1** (1971) 1376

[2] J. Ellis and R. L. Jaffe , Phys. Rev **D9** (1974) 1444 ,
Erratum Phys. Rev **D10** (1974) 1669

[3] J. Ashman et al. (EMC Collaboration) , Phys. Lett. **206B** (1988) 364
Nucl. Phys. **B328** (1989) 1

[4] (SLAC results) G. Baum et al. , Phys. Rev. Lett. **51** (1983) 1135
see also : V. W. Hughes et al. , Phys. Lett. **212B** (1988) 511

[5] E. Reya , in Frontiers in Particle Physics, Proceedings of the
XII Warsaw Symposium on Particle Physics ,
Z. Ajduk, S. Pokorski and A. K. Wróblewski edts. ,
World Scientific Publishing Company, Singapore, 1990
University of Dortmund preprint ,
DO-89/15 (1989)

[6] G. Altarelli , CERN preprint CERN/TH-5675/90 (1990)

[7] A. V. Manohar , University of California San Diego preprint ,
UCSD/PTH 90-28 (1990)

[8] J. Kodeira , Nucl. Phys. **B165** (1980) 129 ,
Phys. Rev. **D20** (1979) 627 ,
Nucl. Phys. **B159** (1979) 99

[9] G. Altarelli and G. G. Ross , Phys. Lett. **212B** (1988) 391

[10] A. V. Efremov and O. V. Teryayev , Dubna preprint , E2-88-287 (1988)

[11] R. D. Carlitz , J. C. Collins and A. H. Muller , Phys. Lett. **214B** (1988) 229

[12] R. L. Jaffe , Phys. Lett. **193B** (1987) 101

[13] M. Bourquin et al. , Z. Phys. **C21** (1983) 27

[14] H. Fritzsch , Max Planck Institute Munich report ,
MPI-PAE/PTh 18/89

[15] G. Veneziano , Mod. Phys. Lett. **A4** (1989) 1605

[16] G. M. Shore and G. Veneziano , CERN preprint , CERN-TH-5689/90 (1990)

[17] A. J. G. Hey and R. L. Kelly , Phys. Lett. **96C** (1983) 71