Symposium 2006
für
Peter Minkowskis 'Emeritierung'

Spontaneous topics in theoretical physics

Possible signals for space-time noncommutativity in particle physics

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Introduction

Example of noncommutativity: Heisenberg algebra

$$[\hat{x}^\mu, p^\nu] = i\hbar\delta^{\mu\nu}, \quad [p^\mu, p^\nu] = 0$$

Constructing models on non-commutative space-time

* The star product: $[x^\mu \stackrel{\star}{,} x^\nu] = x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu}$.

$$(f \star g)(x) = e^{-\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}} f(x)g(y)|_{y\to x}$$

* Non-commutative coordinates:

$$x^\mu \to \hat{x}^\mu \Longrightarrow [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \quad [\theta^{\mu\nu}, \hat{x}^\rho] = 0,$$

θ - constant, antisymetric and real tensor

* Seiberg-Witten map (SW):

  In principle SW map express noncommutative functionals(parameters and functions of fields) spanned on the noncommutative space as a local functionals spanned on commutative space.

There are two essential points in which non-commutative gauge theories differ from standard gauge theories:

* The breakdown of Lorentz invariance with respect to a fixed non-zero $\theta^{\mu\nu}$ background field (which obviously fixes preferred directions)

* The appearance of new interactions and the modification of standard ones. For exapmle, triple–neutral–gauge boson, 2 fermion–2 gauge bosons, photon–neutrino interactions, etc.

Both properties have a common origin and appear in a number of phenomena

AT VERY HIGH ENERGIES AND/OR VERY SHORT DISTANCES.

SEARCH FOR THE SIGNAL OF NONCOMMUTATIVITY IN

DECAYS: 1→ 2

SM forbiden – induced by the NC space-time:

The gauge sector:

* $Z \to \gamma\gamma,\ gg$

Neutrino sector:

* $\gamma_{\text{pl}} \to \nu\bar{\nu}$

The hadron sector (neutral currents):

* $J/\psi \to \gamma\gamma$, $\Upsilon \to \gamma\gamma$

The hadron sector (flavour-changing currents):

* $K \to \pi\gamma$, $D \to (\pi, K)\gamma$, $B \to (\pi, K, D)\gamma$

SCATTERINGS: 2→ 2

SM allowed – modified by the NC space-time:

* Moller scattering: $e^-e^- \to e^-e^-$
* Bhabha scattering: $e^+e^- \to e^+e^-$
* Annihilation: $e^+e^- \to \gamma\gamma$
* Photon-photon production: $\gamma\gamma \to \gamma\gamma,\ Z\gamma,\ ZZ,\ \bar{f}f$
* Fermion pair annihilation at LHC: $\bar{f}f \to Z\gamma$

NEUTRINO DIPOLE MOMENTS: $\mathrm{d}^{\text{el}}_{\text{mag}}$, $\langle \mathrm{r}^2_\nu \rangle_{\text{NC}}$

* In the ν–mass extended SM allowed via loops – modified by the point-like NC interaction

CONSTRUCTING NCSM
VIA MOYAL-WEYL - $\star$ PRODUCT

[M. Chaichian at al. Eur. Phys. J. C**29** (2003) 413]

* Only U(N) gauge groups
* Matter content restricted to the (anti-)fundamental and adjoint rep.
* Charge quantization problem of NC Abelian GT
* Problems with UV/IR mixing; ($\frac{1}{|p\theta|^2}$ terms)
* NCQED signal in $2 \to 2$ processes: $e^+e^- \to \gamma\gamma$

[J.L. Hewett et al, Phys. Rev. **D64**, 075012 (2001)]

$$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha^2}{s}\frac{1+\cos^2\theta}{1-\cos^2\theta}[1-\sin^2\theta \ \ \sin^2\Delta_{NC}]$$

$$\Delta_{NC} = \frac{-s}{4\Lambda_{NC}^2}(c_{01}\sin\theta\cos\phi + c_{02}\sin\theta\sin\phi + c_{03}\cos\theta)$$

Experimental signatures of non-commutativity:

* Collider physics: $\Lambda_{NC} \sim \mathrm{few TeV's}$
* Low-energy non-accelerator experiments: $\Lambda_{NC} \sim 10^8$ TeV

Limits rest on the assumptions, which may have to be modified

* θ is constant across large distances with respect to the NC scale
* Unrealistic gauge groups
* Non-commutativity down to low-energy scales

* See figures in: The OPAL Collaboration: Test of non-commutative QED in the process $e^+e^- \to \gamma\gamma$; hep-ex/0303035
* $\Longrightarrow$ $\Lambda_{NC} > 141\mathrm{GeV}$ for all parameters.

CONSTRUCTING NCSM
VIA SEIBERG-WITTEN MAP

[N. Seiberg and E. Witten; String theory and non-commutative geometry, JHEP **9909**, 032 (1999)]

[X. Calmet, B. Jurčo, P. Schupp, J. Wess and M. Wohlgenannt; The standard model on non-commutative space-time, EPJ C**23** (2002) 363]

[W. Behr, N. G. Deshpande, G. Duplančić, P. Schupp, J.T. and J. Wess; The Z $\rightarrow$ gamma gamma, g g decays in the non-commutative standard model, Eur. Phys. J. C **29**, 441 (2003)]

[B. Melić, K. Passek-Kumerički, J.T., P. Schupp and M. Wohlgenannt; The Standard Model on Non-Commutative Space-Time: Electroweak Currents and Higgs Sector, EPJ C**24** (2005) 483 ibid. 499]

[F. Brandt, C.P. Martín and F. Ruiz Ruiz; Anomaly freedom in Seiberg-Witten noncommutative gauge theories JHEP **07** (2003) 068

* Based on the Seiberg-Witten mapping

* Expansion in power series in θ - new vertices

* Any gauge groups

* Arbitrary matter representation

* No charge quantization problem

* Unitarity: θ^{ij}-save, θ^{0i}-may or may not be save; carefull cannonical quantization produces always unitary theory (very hard to calculate)

* Not renormalizable at fixed order of θ

* No UV/IR mixing due to θ expansion

* The construction of covariant Yukawa couplings possible

* We construct NCSM which should be understood as an effective, anomaly free, theory

NC gauge transformation

Consider infinitesimal NC local gauge transformation $\widehat{\delta}$ of a fundamental matter field that carries a representation ρ_Ψ

$$\widehat{\delta}\widehat{\Psi} = i\rho_\Psi(\widehat{\Lambda}) \star \widehat{\Psi}$$

In Abelian case ρ_Ψ fixed by the hypercharge.

Covariant coordinates in NC theory introduced in analogy to covariant derivatives in ordinary theory

$$\widehat{x}^\mu = x^\mu + \theta^{\mu\nu}\widehat{A}_\nu$$

Locality

A $\star-$ product of ordinary functions f, g, determined by a Poisson tensor $\theta^{\mu\nu}(x)$, is local function of f, g with finite number of derivatives at each order in θ:

$$f \star g = f \cdot g + \frac{i}{2}\theta^{\mu\nu}(x)\partial_\mu f \cdot \partial_\nu g + \mathcal{O}(\theta^2)$$

Seiberg–Witten map express the non-commutative fields and parameters as local functions of the ordinary fields and parameters

$$\begin{aligned}
\widehat{\Lambda} &= \Lambda + \Lambda^\theta[V] + \Lambda^{\theta^2}[V] + \mathcal{O}(\theta^3) \\
\widehat{\psi}[\psi, V] &= \psi + \psi^\theta[\psi, V] + \psi^{\theta^2}[\psi, V] + \mathcal{O}(\theta^3) \\
\widehat{V}_\mu[V] &= V_\mu + V_\mu^\theta[V] + V_\mu^{\theta^2}[V] + \mathcal{O}(\theta^3),
\end{aligned}$$

NC field strengt $\widehat{F}_{\mu\nu} = \partial_\mu \widehat{V}_\nu - \partial_\nu \widehat{V}_\mu - i[\widehat{V}_\mu \stackrel{\star}{,} \widehat{V}_\nu]$

Gauge equivalence, and consistency conditions

Ordinary gauge transformations $\delta A_\mu = \partial_\mu \Lambda + i[\Lambda, A_\mu]$ and $\delta\Psi = i\Lambda\cdot\Psi$ induce non-commutative gauge transformations of the fields $\widehat{A}$, $\widehat{\Psi}$ with gauge parameter $\widehat{\Lambda}$

$$\delta \widehat{A}_\mu = \widehat{\delta}\widehat{A}_\mu \qquad \delta\widehat{\Psi} = \widehat{\delta}\widehat{\Psi}$$

Consistency require that any pair of non-commutative gauge parameters $\widehat{\Lambda}$, $\widehat{\Lambda'}$ satisfy

$$[\widehat{\Lambda} \stackrel{\star}{,} \widehat{\Lambda'}] + i\delta_\Lambda \widehat{\Lambda'} - i\delta_{\Lambda'}\widehat{\Lambda} = \widehat{[\Lambda, \Lambda']}.$$

Enveloping algebra-valued gauge transformation

The commutator

$$\begin{aligned}[\widehat{\Lambda} \stackrel{\star}{,} \widehat{\Lambda}'] &= \frac{1}{2}\{\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)\}[T^a, T^b] \\ &+ \frac{1}{2}[\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)]\{T^a, T^b\}\end{aligned}$$

of two Lie algebra-valued NC gauge parameters $\widehat{\Lambda} = \Lambda_a(x)T^a$ and $\widehat{\Lambda}' = \Lambda'_a(x)T^a$ does not close in the Lie algebra. For NC SU(N) & Lie algebra traceless condition incompatible with commutator. We have to consider enveloping algebra-valued NC gauge parameters

$$\widehat{\Lambda} = \Lambda^0_a(x)T^a + \Lambda^1_{ab}(x) : T^aT^b : + \Lambda^2_{abc}(x) : T^aT^bT^c : + \ldots$$

and fields. (The : : denotes ordering of the Lie algebra generators.)

NCSM ACTIONS

$$S_{\text{NCSM}} = S_{\text{fermions}} + S_{\text{gauge}} + S_{\text{Higgs}} + S_{\text{Yukawa}}$$

$$\begin{aligned} S_{\text{fermions}} &= \int d^4x \sum_{i=1}^{3} \Big(\overline{\widehat{L}}_L^{(i)} \star (i\widehat{\not{D}}\, \widehat{L}_L^{(i)}) + \overline{\widehat{Q}}_L^{(i)} \star (i\widehat{\not{D}}\, \widehat{Q}_L^{(i)}) \\ &+ \overline{\widehat{e}}_R^{(i)} \star (i\widehat{\not{D}}\, \widehat{e}_R^{(i)}) + \overline{\widehat{u}}_R^{(i)} \star (i\widehat{\not{D}}\, \widehat{u}_R^{(i)}) + \overline{\widehat{d}}_R^{(i)} \star (i\widehat{\not{D}}\, \widehat{d}_R^{(i)}) \Big) \end{aligned}$$

$$S_{\text{gauge}} = -\frac{1}{2} \int d^4x \sum_{\mathcal{R}} c_{\mathcal{R}} \mathbf{Tr} \Big(\mathcal{R}(\widehat{F}_{\mu\nu}) \star \mathcal{R}(\widehat{F}^{\mu\nu}) \Big)$$

$\mathcal{R}$ – unitary, irreducible and inequivalent representations of a gauge group. Real coefficients $c_{\mathcal{R}}$ that are subject to the constraints.

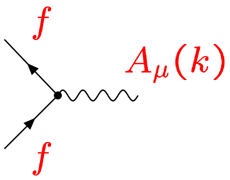
$$\frac{1}{g_{SM}^2} = \sum_{\mathcal{R}} c_{\mathcal{R}} \mathbf{Tr} \Big(\mathcal{R}(T_{SM}^a) \mathcal{R}(T_{SM}^a) \Big).$$

$$\begin{aligned} S_{\text{Higgs}} &= \int d^4x \Big(h_0^\dagger(\widehat{D}_\mu \widehat{\Phi}) \star h_0(\widehat{D}^\mu \widehat{\Phi}) - \mu^2 h_0^\dagger(\widehat{\Phi}) \star h_0(\widehat{\Phi}) \\ &- \lambda h_0^\dagger(\widehat{\Phi}) \star h_0(\widehat{\Phi}) \star h_0^\dagger(\widehat{\Phi}) \star h_0(\widehat{\Phi}) \Big) \end{aligned}$$

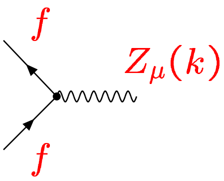
$$\begin{aligned} S_{\text{Yukawa}} &= -\int d^4x \sum_{i,j=1}^{3} \\ &\times \Big(G_e^{(ij)} \, (\overline{\widehat{L}}_L^{(i)} \star h_e(\widehat{\Phi}) \star \widehat{e}_R^{(j)}) + G_e^{\dagger\,(ij)} (\overline{\widehat{e}}_R^{(i)} \star h_e(\widehat{\Phi})^\dagger \star \widehat{L}_L^{(j)}) \\ &+ G_u^{(ij)} \, (\overline{\widehat{Q}}_L^{(i)} \star h_u(\widehat{\Phi}_c) \star \widehat{u}_R^{(j)}) + G_u^{\dagger\,(ij)} \, (\overline{\widehat{u}}_R^{(i)} \star h_u(\widehat{\Phi}_c)^\dagger \star \widehat{Q}_L^{(j)}) \\ &+ G_d^{(ij)} \, (\overline{\widehat{Q}}_L^{(i)} \star h_d(\widehat{\Phi}) \star \widehat{d}_R^{(j)}) + G_d^{\dagger\,(ij)} \, (\overline{\widehat{d}}_R^{(i)} \star h_d(\widehat{\Phi})^\dagger \star \widehat{Q}_L^{(j)}) \Big) \end{aligned}$$

FEYNMAN RULES: NCSM

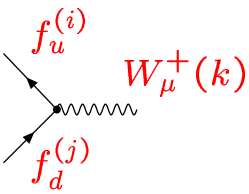
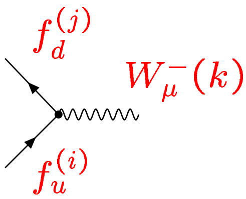
* Electroweak currents and Higgs sector

- 

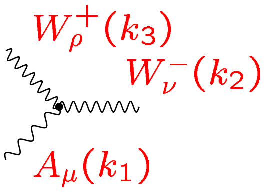
$$
\begin{aligned}
& i\,e\,Q_f\,\left[\gamma_\mu - \frac{i}{2}\,k^\nu\left(\theta_{\mu\nu\rho}\,p^\rho_{\rm in} - \theta_{\mu\nu}\,m_f\right)\right] \\
& \quad = \quad i\,e\,Q_f\,\gamma_\mu \\
& \qquad\quad +\frac{1}{2}\,e\,Q_f\,\left[(p_{\rm out}\theta p_{\rm in})\gamma_\mu - (p_{\rm out}\theta)_\mu(\not{p}_{\rm in} - m_f) - (\not{p}_{\rm out} - m_f)(\theta p_{\rm in})_\mu\right]\,, \\
(\theta k)_\mu & \quad = \quad \theta_{\mu\nu}k^\nu = -(k\theta)_\mu \qquad \text{(def.)}
\end{aligned}
$$

- 

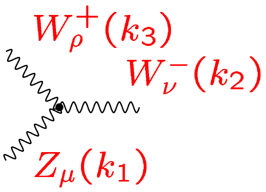
$$
\begin{aligned}
& \frac{i\,e}{\sin 2\theta_W}\left\{\left(\gamma_\mu - \frac{i}{2}\,k^\nu\theta_{\mu\nu\rho}\,p^\rho_{\rm in}\right)\left(c_{V,f} - c_{A,f}\,\gamma_5\right)\right. \\
& \quad \left. -\frac{i}{2}\theta_{\mu\nu}\,m_f\left[p^\nu_{\rm in}\left(c_{V,f} - c_{A,f}\,\gamma_5\right) - p^\nu_{\rm out}\left(c_{V,f} + c_{A,f}\,\gamma_5\right)\right]\right\}\,,
\end{aligned}
$$

-  

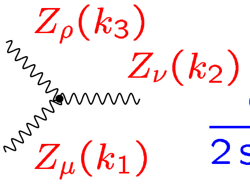
$$
\begin{aligned}
& \frac{i\,e}{2\sqrt{2}\sin\theta_W}\binom{V_f^{(ij)}}{V*_f^{(ij)}}\left\{\left[\gamma_\mu - \frac{i}{2}\,\theta_{\mu\nu\rho}\,k^\nu\,p^\rho_{\rm in}\right](1-\gamma_5)\right. \\
& \quad \left. -\frac{i}{2}\,\theta_{\mu\nu}\left[\binom{m_{f_u^{(i)}}}{m_{f_d^{(j)}}}\,p^\nu_{\rm in}(1-\gamma_5) - \binom{m_{f_d^{(j)}}}{m_{f_u^{(i)}}}\,p^\nu_{\rm out}(1+\gamma_5)\right]\right\},
\end{aligned}
$$

- 

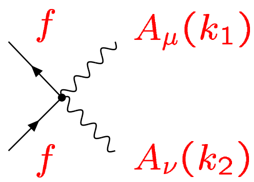
$$ie\left\{g^{\mu\nu}(k_1-k_2)^\rho+g^{\nu\rho}(k_2-k_3)^\mu+g^{\rho\mu}(k_3-k_1)^\nu\right.$$
$$\left.+\frac{i}{2}M_W^2\left[\theta^{\mu\nu}k_1^\rho+\theta^{\mu\rho}k_1^\nu+g^{\mu\nu}(\theta k_1)^\rho-g^{\nu\rho}(\theta k_1)^\mu+g^{\rho\mu}(\theta k_1)^\nu\right]\right\},$$

- 

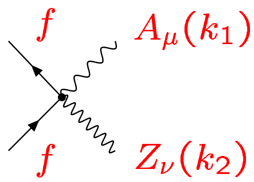
$$ie\cot\theta_W\left\{g^{\mu\nu}(k_1-k_2)^\rho+g^{\nu\rho}(k_2-k_3)^\mu+g^{\rho\mu}(k_3-k_1)^\nu\right.$$
$$+\frac{i}{2}M_W^2\left[\theta^{\mu\nu}k_1^\rho+\theta^{\mu\rho}k_1^\nu+g^{\mu\nu}(\theta k_1)^\rho-g^{\nu\rho}(\theta k_1)^\mu+g^{\rho\mu}(\theta k_1)^\nu\right]$$
$$-\frac{i}{4}M_Z^2\left[\theta^{\mu\nu}(k_1-k_2)^\rho+\theta^{\nu\rho}(k_2-k_3)^\mu+\theta^{\rho\mu}(k_3-k_1)^\nu\right.$$
$$\left.\left.-2g^{\mu\nu}(\theta k_3)^\rho-2g^{\nu\rho}(\theta k_1)^\mu-2g^{\rho\mu}(\theta k_2)^\nu\right]\right\}.$$

- 

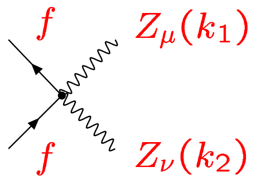
$$\frac{e\,M_Z^2}{2\sin 2\theta_W}\left[\theta^{\mu\nu}(k_1-k_2)^\rho+\theta^{\nu\rho}(k_2-k_3)^\mu+\theta^{\rho\mu}(k_3-k_1)^\nu\right.$$
$$\left.-2g^{\mu\nu}(\theta k_3)^\rho-2g^{\nu\rho}(\theta k_1)^\mu-2g^{\rho\mu}(\theta k_2)^\nu\right].$$

- 

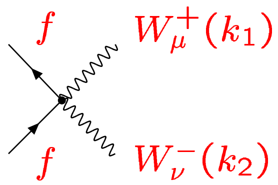
$$\frac{-1}{2}\, e^2\, Q_f^2 \theta_{\mu\nu\rho}\, (k_1^\rho - k_2^\rho)\,,$$

- 

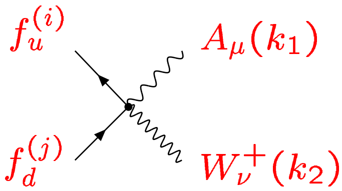
$$\frac{-\,e^2\, Q_f}{2\sin 2\theta_W} \times \left[\theta_{\mu\nu\rho}\,(k_1^\rho - k_2^\rho)(c_{V,f} - c_{A,f}\gamma_5) - 2\theta_{\mu\nu}\, m_f\, c_{A,f}\gamma_5\right],$$

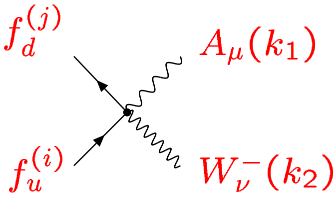
- 

$$\frac{-\,e^2}{2\sin^2 2\theta_W}\,\theta_{\mu\nu\rho}\,(k_1^\rho - k_2^\rho)\,(c_{V,f} - c_{A,f}\gamma_5)^2\,,$$

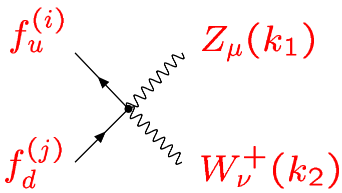
- 

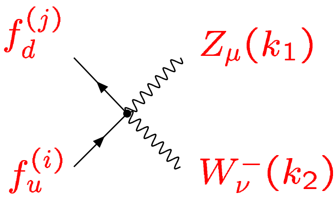
$$\frac{-\,e^2}{8\sin^2\theta_W}\left[\theta_{\mu\nu\rho}\,(p_{\text{in}}^\rho + k_1^\rho)(1-\gamma_5) + 2\theta_{\mu\nu} m_f\right],$$

- 



$$\frac{-e^2}{4\sqrt{2}\sin\theta_W}\left\{\theta_{\mu\nu\rho}\left[\binom{Q_{f_u^{(i)}}}{Q_{f_d^{(j)}}}(p_{\text{in}}^\rho+k_1^\rho)-\binom{Q_{f_d^{(j)}}}{Q_{f_u^{(i)}}}(p_{\text{in}}^\rho+k_2^\rho)\right](1-\gamma_5)\right.$$
$$\left.+\,\theta_{\mu\nu}\left[\binom{m_{f_u^{(i)}}Q_{f_d^{(j)}}}{m_{f_d^{(j)}}Q_{f_u^{(i)}}}(1-\gamma_5)-\binom{m_{f_d^{(j)}}Q_{f_u^{(i)}}}{m_{f_u^{(i)}}Q_{f_d^{(j)}}}(1+\gamma_5)\right]\right\}\binom{V_f^{(ij)}}{V_f^{*(ij)}},$$

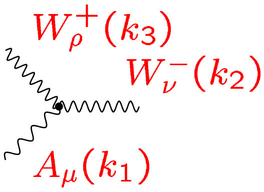
- 



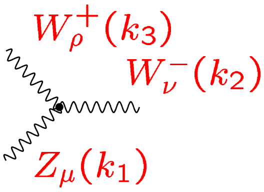
$$\frac{-\,e^2}{4\sqrt{2}\sin\theta_W\sin 2\theta_W}\binom{V_f^{(ij)}}{V_f^{*(ij)}}$$
$$\left\{\theta_{\mu\nu\rho}\left[\begin{pmatrix}c_{V,f_u^{(i)}}+c_{A,f_u^{(i)}}\\ c_{V,f_d^{(j)}}+c_{A,f_d^{(j)}}\end{pmatrix}(p_{\text{in}}^\rho+k_1^\rho)-\begin{pmatrix}c_{V,f_d^{(j)}}+c_{A,f_d^{(j)}}\\ c_{V,f_u^{(i)}}+c_{A,f_u^{(i)}}\end{pmatrix}(p_{\text{in}}^\rho+k_2^\rho)\right](1-\gamma_5)\right.$$
$$+\,\theta_{\mu\nu}\left[\begin{pmatrix}m_{f_u^{(i)}}\left[c_{V,f_d^{(j)}}+3c_{A,f_d^{(j)}}\right]\\ m_{f_d^{(j)}}\left[c_{V,f_u^{(i)}}+3c_{A,f_u^{(i)}}\right]\end{pmatrix}(1-\gamma_5)\right.$$
$$\left.\left.-\begin{pmatrix}m_{f_d^{(j)}}\left[c_{V,f_u^{(i)}}+3c_{A,f_u^{(i)}}\right]\\ m_{f_u^{(i)}}\left[c_{V,f_d^{(j)}}+3c_{A,f_d^{(j)}}\right]\end{pmatrix}(1+\gamma_5)\right]\right\}.$$

Similarly: $ffWWZ$, $ffWW\gamma$ and $ff\gamma WZ$.

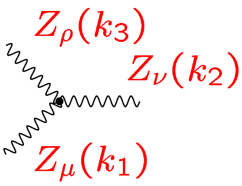
In this section: f $=$ quarks and leptons

- 

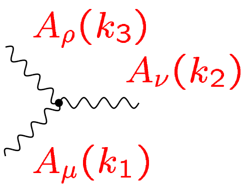
$$(\mathsf{WWA})_{\mathsf{mNCSM}} + 2\,e\,\sin 2\theta_W \mathsf{K}_{WW\gamma}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,,$$

- 

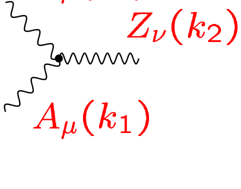
$$(\mathsf{WWZ})_{\mathsf{mNCSM}} + 2\,e\,\sin 2\theta_W \mathsf{K}_{WWZ}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,,$$

- 

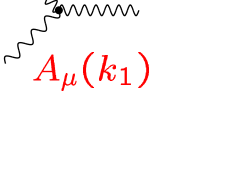
$$(\mathsf{ZZZ})_{\mathsf{mNCSM}} + 2\,e\,\sin 2\theta_W \mathsf{K}_{ZZZ}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,,$$

- 

$$2\,e\,\sin 2\theta_W \mathsf{K}_{\gamma\gamma\gamma}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,,$$

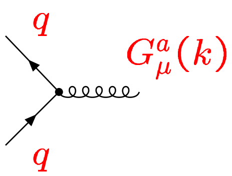
- 

$$-2\,e\,\sin 2\theta_W \mathsf{K}_{Z\gamma\gamma}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,,$$

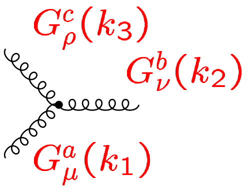
- 

$$-2\,e\,\sin 2\theta_W \mathsf{K}_{ZZ\gamma}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,.$$

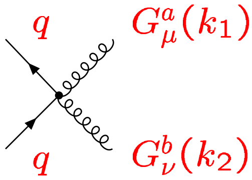
$*$ Strong interactions included

- 

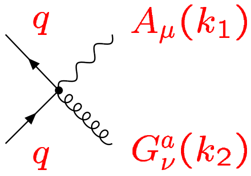
$$i\, g_s \left[\gamma_\mu - \frac{i}{2}\, k^\nu \left(\theta_{\mu\nu\rho}\, p^\rho_{\text{in}} - \theta_{\mu\nu}\, m_q\right)\right]\, T^a_S$$
$$\begin{aligned} = \quad & i\, g_s\, \gamma_\mu\, T^a_S \\ & + \frac{1}{2}\, g_s\, \left[(p_{\text{out}}\theta p_{\text{in}})\gamma_\mu - (p_{\text{out}}\theta)_\mu(\not{p}_{\text{in}} - m_q) - (\not{p}_{\text{out}} - m_q)(\theta p_{\text{in}})_\mu\right]\, T^a_S\,, \end{aligned}$$

- 

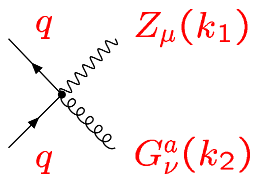
$$\begin{aligned} & g_s\, f^{abc}\, [g_{\mu\nu}(k_1 - k_2)_\rho + g_{\nu\rho}(k_2 - k_3)_\mu + g_{\rho\mu}(k_3 - k_1)_\nu] \\ & + \frac{1}{2}\, g_s\, d^{abc}\, \Theta_3((\mu, k_1), (\nu, k_2), (\rho, k_3))\,. \end{aligned}$$

- 

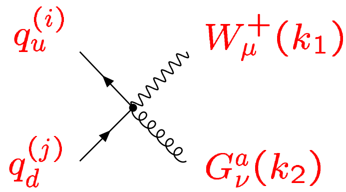
$$-\frac{g_s^2}{2}\, \left\{\theta_{\mu\nu\rho}\ (k^\rho_1 - k^\rho_2)\, T^a_S T^b_S + i\left[\theta_{\mu\nu\rho}\ (p^\rho_{\text{in}} + k^\rho_2) - \theta_{\mu\nu} m_q\right]\ f^{abc} T^c_S\right\}\,,$$

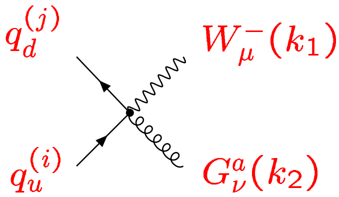
- 

$$-\frac{1}{2}\, g_s\, e\, Q_q\, \theta_{\mu\nu\rho}\, (k^\rho_1 - k^\rho_2)\, T^a_S,$$

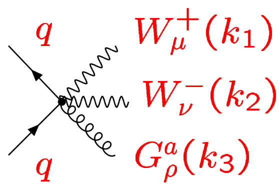
- 

$$\frac{-\,e\,g_s}{2\sin 2\theta_W}\left[\theta_{\mu\nu\rho}\,(k_1^\rho - k_2^\rho)(c_{V,q} - c_{A,q}\gamma_5) + 2\theta_{\mu\nu}\,m_q\,c_{A,q}\gamma_5\right] T^a_S\,,$$

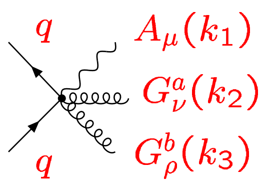
- 



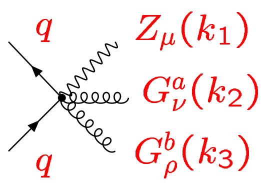
$$\frac{-e\,g_s}{4\sqrt{2}\sin\theta_W}\begin{pmatrix}V_{ij}\\ V^*_{ij}\end{pmatrix}\left\{\theta_{\mu\nu\rho}(k_1^\rho - k_2^\rho)\;(1-\gamma_5)\right.$$
$$\left. -\;\theta_{\mu\nu}\left[\begin{pmatrix}m_{u^{(i)}}\\ m_{d^{(j)}}\end{pmatrix}(1-\gamma_5) - \begin{pmatrix}m_{d^{(j)}}\\ m_{u^{(i)}}\end{pmatrix}(1+\gamma_5)\right]\right\} T^a_S$$

- 

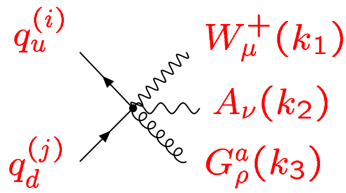
$$-\frac{e^2\,g_s}{8\sin^2\theta_W}\theta_{\mu\nu\rho}\,(1-\gamma_5)\,T^a_S\,,$$

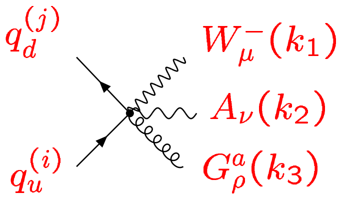
- 

$$-\frac{i}{2}e\,g_s^2\,Q_q\theta_{\mu\nu\rho}\,f^{abc}T^c_S\,,$$

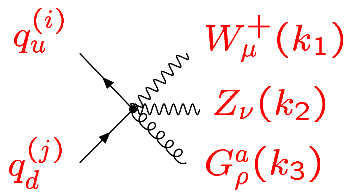
- 

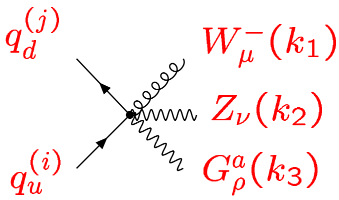
$$-\frac{i\,e\,g_s^2}{2\sin 2\theta_W}\theta_{\mu\nu\rho}\,(c_{V,q} - c_{A,q}\gamma_5)\,f^{abc}T^c_S\,,$$

- 

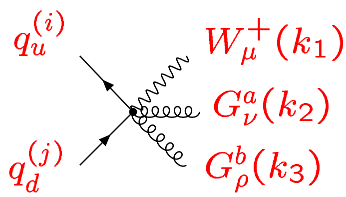


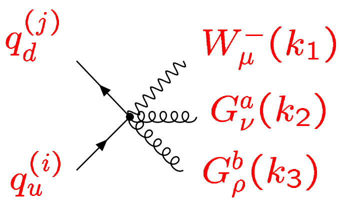
$$\frac{e^2\, g_s}{4\sqrt{2}\sin\theta_W} \begin{pmatrix} V_{ij} \\ -V_{ij}^* \end{pmatrix} \theta_{\mu\nu\rho}\,(1-\gamma_5)\; T_S^a\,,$$

- 

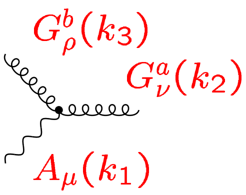


$$\frac{e^2\, g_s\,\cos\theta_W}{4\sqrt{2}\sin^2\theta_W} \begin{pmatrix} V_{ij} \\ -V_{ij}^* \end{pmatrix} \theta_{\mu\nu\rho}\,(1-\gamma_5)\, T_S^a\,,$$

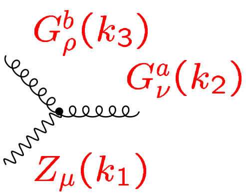
- 



$$\frac{-i\, e\, g_s^2}{4\sqrt{2}\sin\theta_W} \begin{pmatrix} V_{ij} \\ V_{ij}^* \end{pmatrix} \theta_{\mu\nu\rho}\,(1-\gamma_5)\; f^{abc}\, T_S^c\,.$$

- 

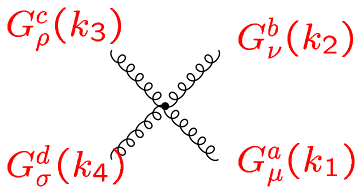
$$-2\,e\,\sin 2\theta_W \mathsf{K}_{\gamma gg}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,\delta^{ab}\,,$$

- 

$$-2\,e\,\sin 2\theta_W \mathsf{K}_{Zgg}\,\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3))\,\delta^{ab}\,.$$

In this section: q = quarks

•



$$
\begin{aligned}
& i g_s^2 \left\{ f^{abx} f^{cdx} \left(g^{\mu\sigma} g^{\nu\rho} - g^{\mu\rho} g^{\nu\sigma} \right) + f^{acx} f^{bdx} \left(g^{\mu\sigma} g^{\nu\rho} - g^{\mu\nu} g^{\rho\sigma} \right) \right. \\
& \qquad \left. + f^{adx} f^{bcx} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\nu} g^{\rho\sigma} \right) \right\} \\
& + \frac{i}{2} g_s^2 \left\{ f^{abx} d^{cdx} \Theta_4((\mu,k_1),(\nu,k_2),(\rho,k_3),(\sigma,k_4)) \right. \\
& \qquad + [(\mu,k_1,a) \leftrightarrow (\rho,k_3,c)] + [(\mu,k_1,a) \leftrightarrow (\sigma,k_4,d)] \\
& \qquad + [(\nu,k_2,b) \leftrightarrow (\rho,k_3,c)] + [(\nu,k_2,b) \leftrightarrow (\sigma,k_4,d)] \\
& \qquad \left. + [(\mu,k_1,a) \leftrightarrow (\rho,k_3,c), (\nu,k_2,b) \leftrightarrow (\sigma,k_4,d)] \right\} .
\end{aligned}
$$

The three and four gauge boson vertex function in momentum space:

$$
\begin{aligned}
\Theta_3((\mu,k_1),(\nu,k_2),(\rho,k_3)) \quad = & \\
& - (k_1 \theta k_2) \left[(k_1 - k_2)^\rho g^{\mu\nu} + (k_2 - k_3)^\mu g^{\nu\rho} + (k_3 - k_1)^\nu g^{\rho\mu} \right] \\
& - \theta^{\mu\nu} \left[k_1^\rho (k_2 k_3) - k_2^\rho (k_1 k_3) \right] \\
& - \theta^{\nu\rho} \left[k_2^\mu (k_3 k_1) - k_3^\mu (k_2 k_1) \right] \\
& - \theta^{\rho\mu} \left[k_3^\nu (k_1 k_2) - k_1^\nu (k_3 k_2) \right] \\
& + (\theta k_2)^\mu \left[g^{\nu\rho} k_3^2 - k_3^\nu k_3^\rho \right] + (\theta k_3)^\mu \left[g^{\nu\rho} k_2^2 - k_2^\nu k_2^\rho \right] \\
& + (\theta k_3)^\nu \left[g^{\mu\rho} k_1^2 - k_1^\mu k_1^\rho \right] + (\theta k_1)^\nu \left[g^{\mu\rho} k_3^2 - k_3^\mu k_3^\rho \right] \\
& + (\theta k_1)^\rho \left[g^{\mu\nu} k_2^2 - k_2^\mu k_2^\nu \right] + (\theta k_2)^\rho \left[g^{\mu\nu} k_1^2 - k_1^\mu k_1^\nu \right] ,
\end{aligned}
$$

$$
\begin{aligned}
\Theta_4((\mu,k_1),(\nu,k_2),(\rho,k_3),(\sigma,k_4)) \quad = & \\
& (k_3 \theta k_4) \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) \\
& + \theta^{\mu\nu} [k_3^\sigma k_4^\rho - g^{\rho\sigma} (k_3 k_4)] + \theta^{\rho\sigma} (k_3^\mu k_4^\nu - k_3^\nu k_4^\mu) \\
& - \theta^{\mu\rho} [k_3^\sigma k_4^\nu - g^{\nu\sigma} (k_3 k_4)] - \theta^{\mu\sigma} [k_3^\nu k_4^\rho - g^{\nu\rho} (k_3 k_4)] \\
& + \theta^{\nu\rho} [k_3^\sigma k_4^\mu - g^{\mu\sigma} (k_3 k_4)] + \theta^{\nu\sigma} [k_3^\mu k_4^\rho - g^{\mu\rho} (k_3 k_4)] \\
& + (\theta k_3)^\mu \left(k_4^\nu g^{\rho\sigma} - k_4^\rho g^{\nu\sigma} \right) + (\theta k_4)^\mu \left(k_3^\nu g^{\rho\sigma} - k_3^\sigma g^{\nu\rho} \right) \\
& - (\theta k_3)^\nu \left(k_4^\mu g^{\rho\sigma} - k_4^\rho g^{\mu\sigma} \right) - (\theta k_4)^\nu \left(k_3^\mu g^{\rho\sigma} - k_3^\sigma g^{\mu\rho} \right) \\
& + (\theta k_3)^\rho \left(k_4^\mu g^{\nu\sigma} - k_4^\nu g^{\mu\sigma} \right) - (\theta k_4)^\rho \left(k_3^\mu g^{\nu\sigma} - k_3^\nu g^{\mu\rho} \right) \\
& - (\theta k_3)^\sigma \left(k_4^\mu g^{\nu\rho} - k_4^\nu g^{\mu\rho} \right) + (\theta k_4)^\sigma \left(k_3^\mu g^{\nu\rho} - k_3^\nu g^{\mu\rho} \right) .
\end{aligned}
$$

FORBIDDEN DECAYS

GAUGE SECTOR: $Z \to \gamma\gamma\ ,\ gg$

[W. Behr, N.G. Deshpande, G. Duplančić, P. Schupp, J.T. and J. Wess, EPJ C**29** 441(2003)],

From $\mathcal{L}_{Z\gamma\gamma} \Rightarrow$ the gauge-invariant amplitude $\mathcal{M}_{Z\to\gamma\gamma}$

$$\sum_{\text{spins}} |\mathcal{M}_{Z\to\gamma\gamma}|^2 = -\theta^2 + \frac{8}{M_Z^2}(p\theta^2 p) - \frac{16}{M_Z^4}(k\theta k')^2 .$$

Z-boson at rest

$$\Gamma_{Z\to\gamma\gamma} = \frac{\alpha}{12} M_Z^5 \sin^2 2\theta_W \mathsf{K}_{Z\gamma\gamma}^2 \left[\frac{7}{3}(\vec{E}_\theta)^2 + (\vec{B}_\theta)^2\right]$$

$$\vec{E}_\theta = (\theta^{01}, \theta^{02}, \theta^{03}),\ \ \vec{B}_\theta = (\theta^{23}, \theta^{13}, \theta^{12})$$

$$\begin{aligned} \theta^2 &= (\theta^2)^\mu_\mu = \theta_{\mu\nu}\theta^{\nu\mu} = \frac{2}{\Lambda_{\mathrm{NC}}^4}\left(\vec{E}_\theta^2 - \vec{B}_\theta^2\right) \\ &\equiv \frac{2}{\Lambda_{\mathrm{NC}}^4}\left(\sum_{i=1}^{3}(c^{0i})^2 - \sum_{i,j=1\,;\,i<j}^{3}(c^{ij})^2\right) \end{aligned}$$

Z-boson at rest and polarized along the 3-axis

$$\begin{aligned} \Gamma_{Z^3\to\gamma\gamma} &= \frac{\alpha}{4} M_Z^5 \sin^2 2\theta_W \mathsf{K}_{Z\gamma\gamma}^2 \\ &\times \left[\frac{2}{5}\left((\theta^{01})^2 + (\theta^{02})^2\right) + \frac{23}{15}(\theta^{03})^2 + (\theta^{12})^2\right] \end{aligned}$$

The same Lorentz structure of $\mathcal{L}_{Z\gamma\gamma}$ and $\mathcal{L}_{Zgg}$:

$$\frac{\Gamma_{Z\to gg}}{\Gamma_{Z\to\gamma\gamma}} = \frac{\Gamma_{Z^3\to gg}}{\Gamma_{Z^3\to\gamma\gamma}} = 8\frac{\mathsf{K}_{Zgg}^2}{\mathsf{K}_{Z\gamma\gamma}^2}.$$

Experimental situation

Decay mode: $Z \to \gamma\gamma$

$$BR = \frac{\Gamma(Z \to \gamma\gamma)}{\Gamma_{tot}(Z)} \begin{cases} < 5.2 \times 10^{-5} & \text{L3} & 1995 \\ < 5.5 \times 10^{-5} & \text{DELPHI} & 1994 \\ < 1.4 \times 10^{-4} & \text{OPAL} & 1991 \end{cases}$$

$e^+e^- \to \gamma\gamma$ near Z resonance is an ideal process to test QED. The present statistic enables comparison of data with the QED up to $\mathcal{O}(\alpha^3)$.

Deviation of the experimentaly measured cross sections from the QED prediction
$\longrightarrow$ evidence for $Z \to \gamma\gamma$ (SM forbidden) and $Z \to \pi^0\gamma \,/\, \eta\gamma$.

Decay mode: $Z \to gg$

could be observed through $Z \to 2\,\text{jets}$ processes.

Taking into account discrepancy between the experimentally observed hadronic width for Z boson and the theoretical SM estimate, we estimate the upper bound for any new hadronic mode to be of the order 10^{-3} GeV.

$$\Gamma_{Z \to gg} < 10^{-3}\ \text{GeV}$$

The forbidden decay $Z \to \gamma\gamma$ and the real decays $Z \to \pi^0\gamma\,/\,\eta\gamma$ would have the same experimental signature as the SM forbidden process

$$e^+e^- \to Z^* \to \gamma\gamma$$

Rare decays at high energies, the two photons from π^0 or η decays are very close seen in EM calorimeter as a single high energy photon:

$$e^+e^- \to Z^* \to (\pi^0, \eta)\gamma \to (\gamma\gamma)\gamma$$

The measurement of the total cross section as a function of center of mass energy ($\sqrt{s}$) can be used to set limits on (L3 COLLABORATION – 1995):

$$\begin{aligned} BR(Z \to \pi^0\gamma) &< 5.2 \times 10^{-5} \\ BR(Z \to \eta\gamma) &< 7.6 \times 10^{-5} \\ BR(Z \to \gamma\gamma) &< 5.2 \times 10^{-5} \end{aligned}$$

Theoretical estimates $Br(Z \to \pi^0\gamma\,/\,\eta\gamma) \sim 10^{-10}$.

(Arnellos at al. Nucl.Phys.B 196 (1982) 378)

Discusion: GAUGE SECTOR

The range of the scale of non-commutativity:
$1 \geq \Lambda_{\mathrm{NC}}/\mathrm{TeV} \geq 0.25$

– For central value $|\mathrm{K}_{Z\gamma\gamma}| \simeq 0.1$ we have

$$4 \times 10^{-8} \lesssim BR^{\mathrm{nmNCSM}}(Z \to \gamma\gamma) \lesssim 10^{-5}$$

– For pure electroweak part of nmNCSM (extracted from pentahedron), any pair of electroweak couplings never vanish simultaneously – Figures from:

[G. Duplančić, P. Schupp and J.T.; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]

– From L3 Collboration (1995) experiment:

$$BR(Z \to \gamma\gamma) \; < \; 5.2 \times 10^{-5}$$

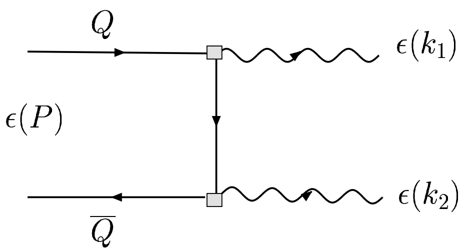
and for the central value $|\mathrm{K}_{Z\gamma\gamma}| \simeq 0.1$ we found the following bound on the scale of noncommutativity

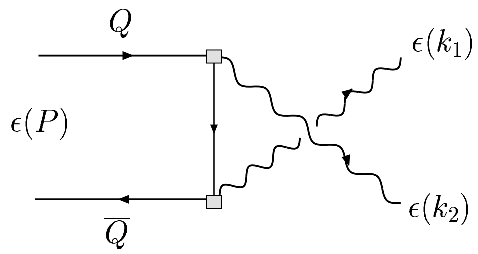
$$\Lambda_{\mathrm{NC}} \; > \; 162 \text{ GeV}$$

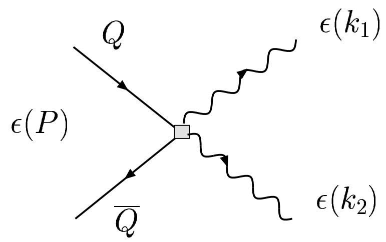
HADRON SECTOR

* NEUTRAL CURRENT DECAYS: $\overline{Q}Q_{1^{--}}(J/\psi,\,\Upsilon)\to\gamma\gamma$

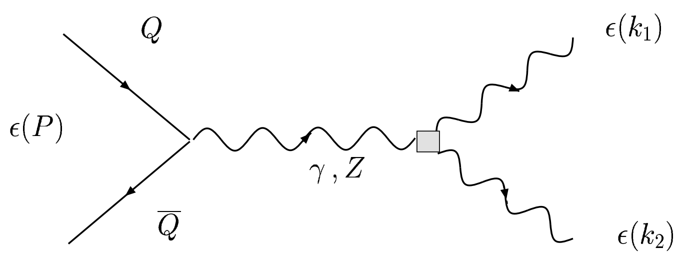
[B. Melic, K. Passek-Kumericki and J.T.; Quarkonia decays into two photons induced by the space-time non-commutativity, Phys. Rev. D **72** (2005) 054004]



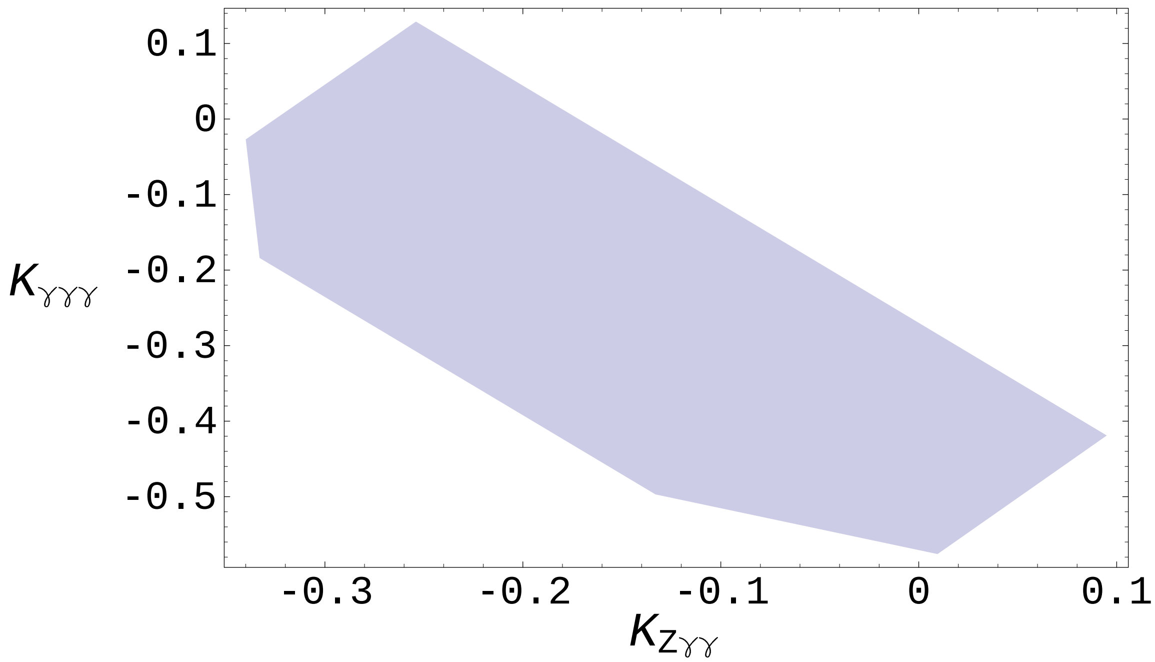




$$\mathcal{M}_{\mathrm{mNCSM}} = i\,\pi\,4\sqrt{3\,M}\,\alpha\,e_Q^2|\Psi_{\overline{Q}Q}(0)|\,\epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P)$$
$$\times\Bigg\{-(k_1-k_2)^\rho\left[\theta^{\mu\nu}-2g^{\mu\nu}\frac{(k_1\theta k_2)}{M^2}\right]$$
$$+2g^{\mu\rho}\left[(k_1\theta)^\nu-2k_1^\nu\frac{(k_1\theta k_2)}{M^2}\right]+2g^{\nu\rho}\left[(k_2\theta)^\mu+2k_2^\mu\frac{(k_1\theta k_2)}{M^2}\right]\Bigg\}$$



$$\mathcal{M}_{\mathrm{nmNCSM}} = -\,i\,\pi\frac{16\sqrt{3\,M}}{M^2}\alpha|\Psi_{\overline{Q}Q}(0)|\,\epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P)$$
$$\times\Theta_3((\mu,k_1),(\nu,k_2),(\rho,P))\left[e_Q\sin 2\theta_W K_{\gamma\gamma\gamma}+\left(\frac{M}{M_Z}\right)^2 c_V^Q K_{Z\gamma\gamma}\right]$$



The allowed region for the $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants.

Hadronization: collinear quarks; annihilation and the WFO:

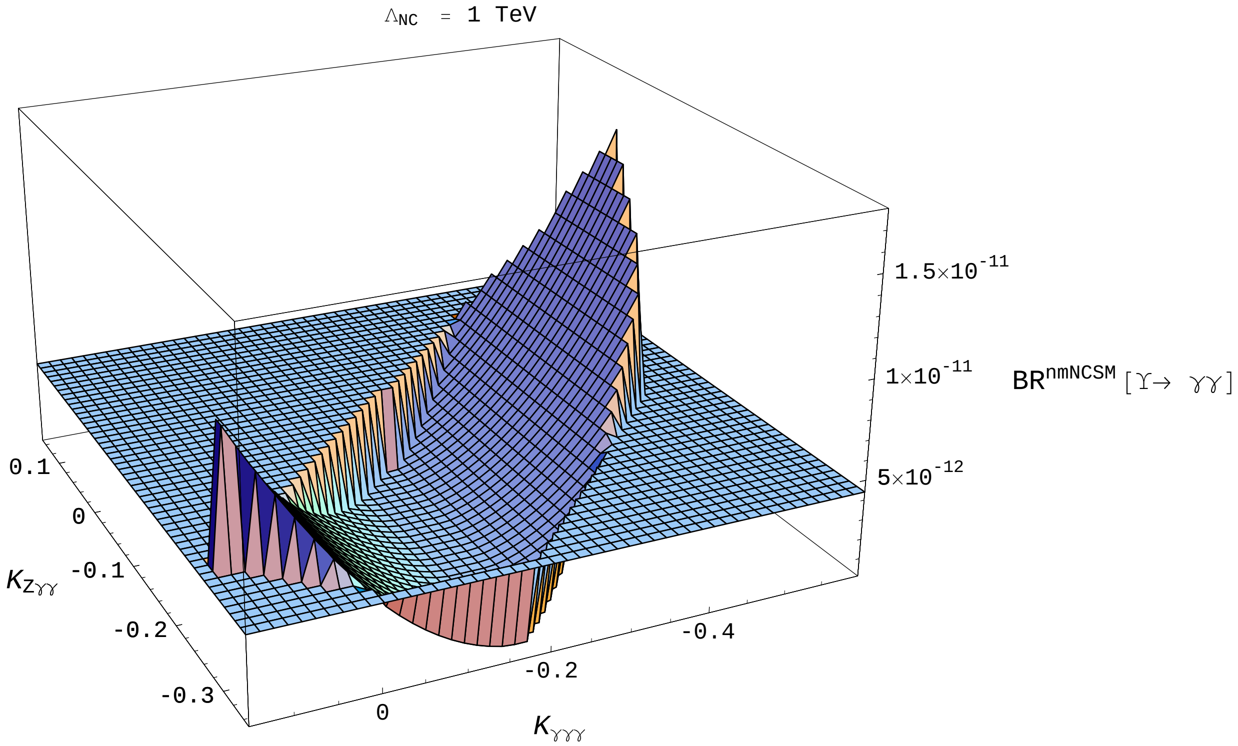
$$\langle 0|q_i^\alpha \overline{q}_j^\beta|\overline{Q}Q_{1^{--}}(P)\rangle = -\frac{|\Psi_{\overline{Q}Q}(0)|}{\sqrt{12M}}[(\not{P}+M)\not{\epsilon}]^{\alpha\beta}\ \delta_{ij},$$

$$|\Psi_{\overline{Q}Q}(0)|^2 = \frac{\Gamma(\overline{Q}Q_{1^{--}}\to \ell^+\ell^-)M^2}{16\pi\alpha^2 e_Q^2}$$

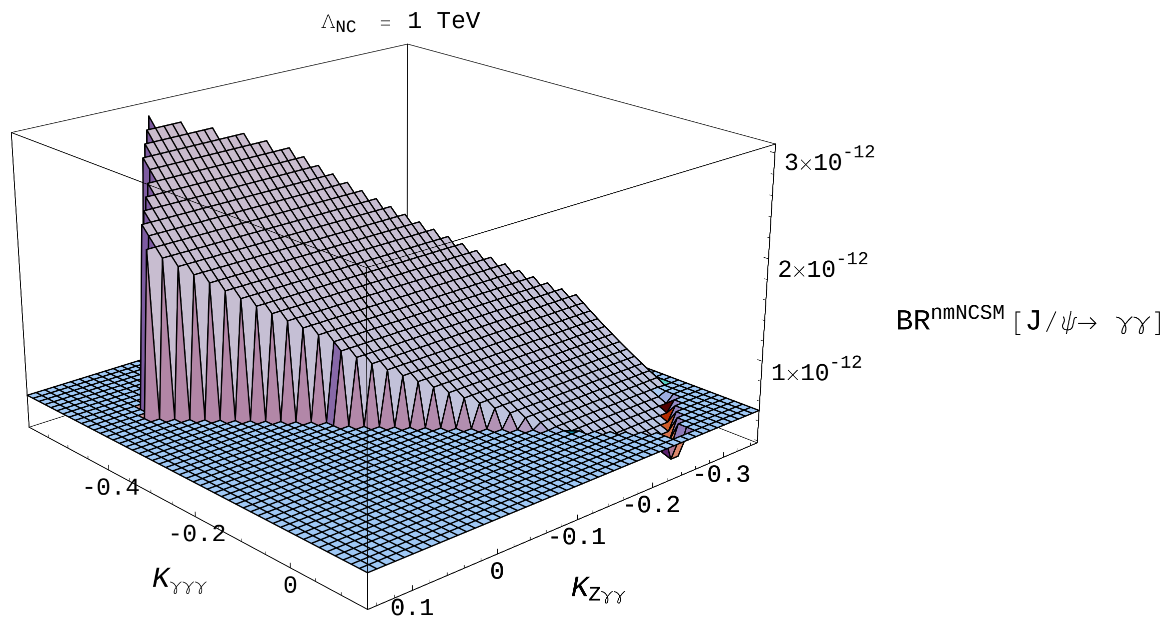
$$c_V^c = \frac{1}{2}\left(1-\frac{8}{3}\sin^2\theta_W\right),\ c_V^b = -\frac{1}{2}\left(1-\frac{4}{3}\sin^2\theta_W\right)$$

$$\Gamma^{\mathrm{nmNCSM}}(\overline{Q}Q_{1^{--}}\to\gamma\gamma) = \frac{4\alpha^2\pi}{3}|\Psi_\Upsilon(0)|^2\frac{M^2}{\Lambda_{\mathrm{NC}}^4}\left[7\vec{E}_\theta^2+3\vec{B}_\theta^2\right]$$

$$\times\left[\frac{e_Q^2}{2}-e_Q\sin 2\theta_W K_{\gamma\gamma\gamma}-\left(\frac{M}{M_Z}\right)^2 c_V^Q K_{Z\gamma\gamma}\right]^2$$



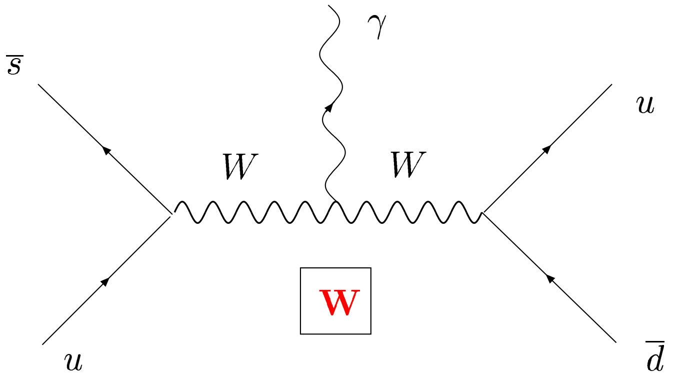
Branching ratio $BR^{\rm nmNCSM}(\Upsilon \to \gamma\gamma)$ as a function of $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants, at the scale of non-commutativity $\Lambda_{\rm NC} = 1$ TeV. The horizontal plane at the value of 4.7×10^{-12} indicates the $BR^{\rm mNCSM}(\Upsilon \to \gamma\gamma)$, which one obtains by setting $K_{\gamma\gamma\gamma} = K_{Z\gamma\gamma} = 0$.



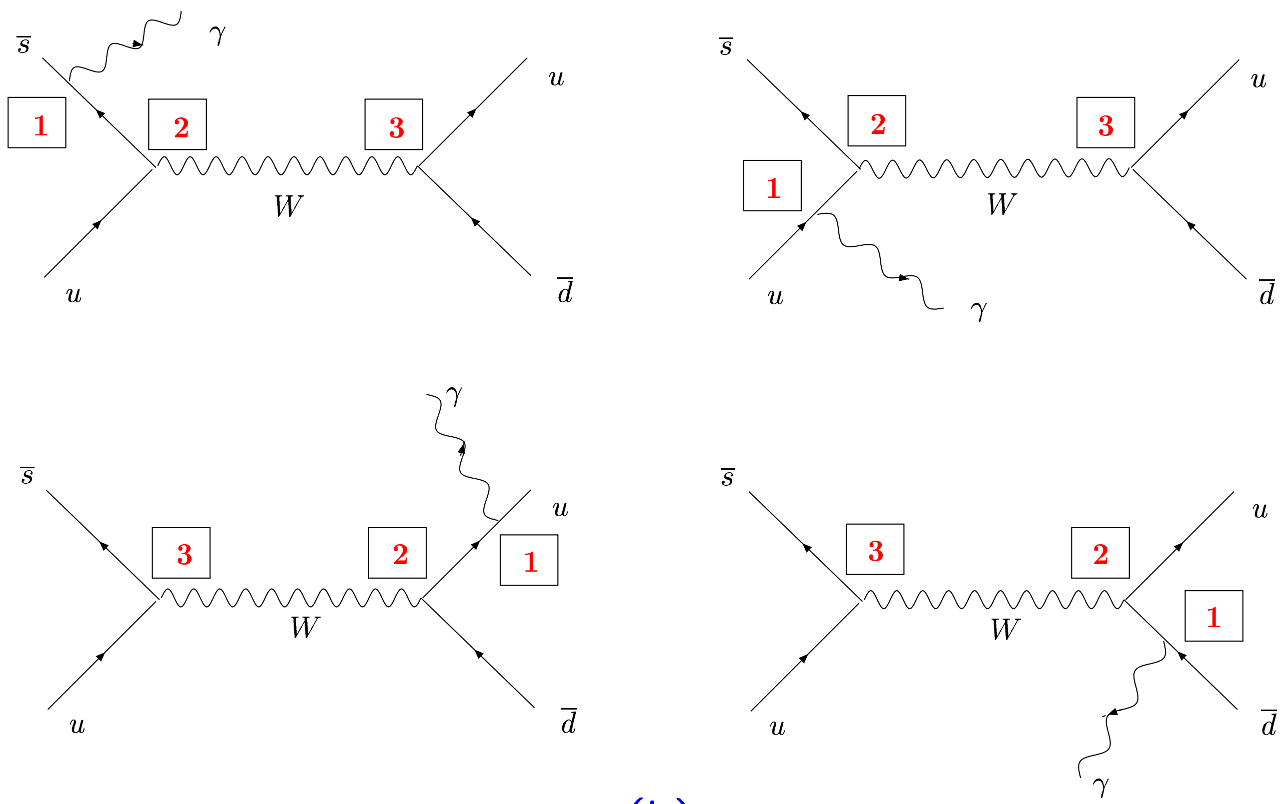
Branching ratio $BR^{\rm nmNCSM}(J/\psi \to \gamma\gamma)$ as a function of $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants, at the scale of non-commutativity $\Lambda_{\rm NC} = 1$ TeV. The horizontal plane at the value of 5.1×10^{-13} indicates the $BR^{\rm mNCSM}(J/\psi \to \gamma\gamma)$, which one obtains by setting $K_{\gamma\gamma\gamma} = K_{Z\gamma\gamma} = 0$.

* CHARGED CURRENT DECAY: $K \to \pi\gamma, \ldots$

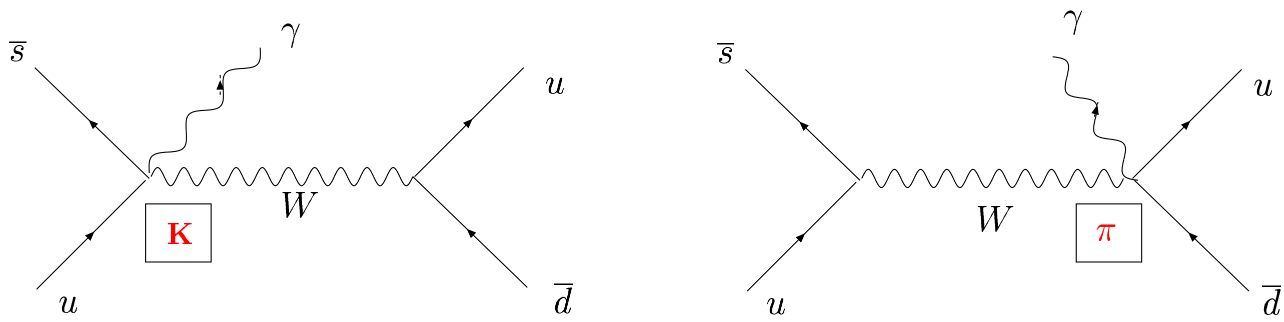
[B. Melic, K. Passek-Kumericki and J.T.; K → pi gamma decay and space-time noncommutativity, Phys. Rev. D **72** (2005) 057502]



(a)



(b)



(c)

Total free quark amplitude : $\mathcal{M} = (\mathcal{M}^{\mathsf{SM}}_{(a+b)} + \mathcal{M}^{\theta}_{(a+b+c)})_\mu\, \varepsilon^\mu(q)\,.$

Hadronization: collinear quarks, the VSA and the PCAC:

$$\langle\pi|J^\dagger_\mu J^\mu|K\rangle = \langle\pi|J^\dagger_\mu|0\rangle\langle 0|J^\mu|K\rangle,\ \ \langle\pi^+(p)|\bar u\gamma_\mu\gamma_5 d|0\rangle = -ip_\mu f_\pi$$

Total amplitude for the $K^+\to\pi^+\gamma$ decay:

$$\begin{aligned}
\mathcal{A}(K^+\to\pi^+\gamma) &= \langle\pi^+(p)|(\mathcal{M}^{\mathsf{SM}}_{(a+b)} + \mathcal{M}^{\theta}_{(a+b+c)})_\mu|K^+(k)\rangle\,\varepsilon^\mu(q)\\
&= \frac{e\,G_F}{4\sqrt{2}}V_{ud}V^\dagger_{us}f_\pi f_K\left(\mathcal{A}^{\mathsf{SM}}_{(a+b)} + i\mathcal{A}^{\theta}_{(a+b+c)}\right)_\mu\varepsilon^\mu(q)\\
\mathcal{A}^{\mathsf{SM}}(K^+\to\pi^+\gamma) &= \frac{e\,G_F}{4\sqrt{2}}V_{ud}V^\dagger_{us}f_\pi f_K\left(\mathcal{A}^{\mathsf{SM}}_{(a+b)}\right)_\mu\varepsilon^\mu(q) = 0
\end{aligned}$$

Gauge and Lorentz invariance in SM satisfied!

$$\begin{aligned}
\left(\mathcal{A}^\theta_{(a)}\right)_\mu &= 2\left[k^2(\theta p)_\mu - p^2(\theta k)_\mu - 2(q\theta k)k_\mu\right]\\
\left(\mathcal{A}^\theta_{(b)}\right)_\mu &= \frac{kp}{kq}(Q_u+Q_s)\Big((q\theta k)k_\mu - (kq)(\theta k)_\mu\Big)\\
&\quad - \frac{kp}{kq}(Q_u+Q_d)\Big((q\theta k)p_\mu - (kq)(\theta p)_\mu\Big)\\
&\quad - R_\pi\,(Q_u-Q_s)(kq)(\theta p)_\mu + i\,R_\pi\,(Q_u+Q_s)\epsilon_{\mu\nu\rho\tau}q^\nu(\theta p)^\rho k^\tau\\
&\quad + R_K\,(Q_u-Q_d)(kq)(\theta k)_\mu + i\,R_K\,(Q_u+Q_d)\epsilon_{\mu\nu\rho\tau}q^\nu(\theta k)^\rho p^\tau\\
\left(\mathcal{A}^\theta_{(c)}\right)_\mu &= (Q_u+Q_d)\Big(p^2(\theta k)_\mu - (kp)(\theta p)_\mu + (q\theta k)p_\mu\Big)\\
&\quad - (Q_u+Q_s)\Big(k^2(\theta p)_\mu - (kp)(\theta k)_\mu - (q\theta k)k_\mu\Big)\\
&\quad + 2(m_dQ_u+m_uQ_d)\frac{p^2(\theta k)_\mu}{m_d+m_u} - 2(m_sQ_u+m_uQ_s)\frac{k^2(\theta p)_\mu}{m_s+m_u}
\end{aligned}$$

$$R_\pi = \frac{p^2}{kq}\frac{m_d-m_u}{m_d+m_u}\,,\quad R_K = \frac{k^2}{kq}\frac{m_s-m_u}{m_s+m_u}$$

VERY IMPORTANT : $k = p+q\,,\ \ Q_u = \frac{2}{3}\,,\ \ Q_d = Q_s = -\frac{1}{3}$

Gauge invariance for $\mathcal{A}^\theta = \left(\mathcal{A}^\theta_{(a+b+c)}\right)_\mu\varepsilon^\mu(q)$ in NCSM satisfied!

Discusion: HADRON SECTOR

The range of the scale of non-commutativity:

$$1 \geq \Lambda_{\mathrm{NC}}/\mathrm{TeV} \geq 0.25$$

Quarkonia decays: $J/\psi \to \gamma\gamma$, $\Upsilon(1S) \to \gamma\gamma$

$$\Gamma^{\mathrm{exp.}}(\Upsilon(1S) \to e^+e^-) = (1.314 \pm 0.029)\,\mathrm{keV}$$
$$\Gamma^{\mathrm{exp.}}_{\mathrm{tot}}(\Upsilon(1S)) = (53.0 \pm 1.5)\,\mathrm{keV}$$
$$\Gamma^{\mathrm{exp.}}(J/\psi \to e^+e^-) = (5.4 \pm 0.15 \pm 0.07)\,\mathrm{keV}$$
$$\Gamma^{\mathrm{exp.}}_{\mathrm{tot}}(J/\psi) = (91.0 \pm 3.2)\,\mathrm{keV}$$

$$5 \times 10^{-12} \lesssim BR^{\mathrm{mNCSM}}(\Upsilon(1S) \to \gamma\gamma) \lesssim 10^{-9}$$
$$5 \times 10^{-13} \lesssim BR^{\mathrm{mNCSM}}(J/\psi \to \gamma\gamma) \lesssim 10^{-10}$$
$$2 \times 10^{-11} \lesssim BR^{\mathrm{nmNCSM}}(\Upsilon(1S) \to \gamma\gamma) \lesssim 4 \times 10^{-9}$$
$$3 \times 10^{-12} \lesssim BR^{\mathrm{nmNCSM}}(J/\psi \to \gamma\gamma) \lesssim 8 \times 10^{-10}$$

Flavor changing decays: $K^+ \to \pi^+\gamma$

$$0.8 \times 10^{-16} \lesssim BR(K^+ \to \pi^+\gamma) \lesssim 2.0 \times 10^{-14}$$

Note the experimental result for similar process

$$BR(K^+ \to \pi^+\nu\bar{\nu}) \simeq 1.6 \times 10^{-10}$$

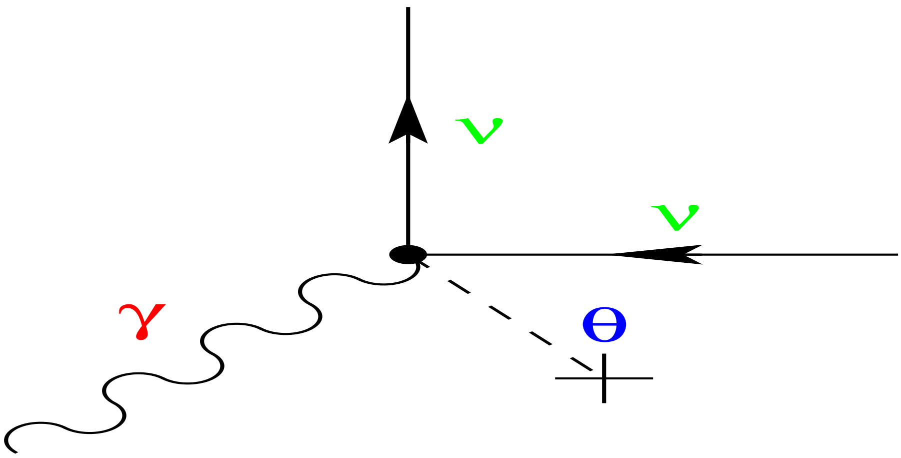
Brookhaven experiments (at 90%CL):

E787 (2002) $\longrightarrow$ $BR(K^+ \to \pi^+\gamma) < 3.6 \times 10^{-7}$

E949 (2005) $\longrightarrow$ $BR(K^+ \to \pi^+\gamma) < 2.3 \times 10^{-9}$

NEUTRINO SECTOR

"Transverse plasmon" decay: $\gamma_{\rm pl} \to \nu\bar{\nu}$ [P. Schupp, J.T., J. Wess and G. Raffelt, "The photon neutrino interaction in non-commutative gauge field theory and astrophysical bounds," Eur. Phys. J. C **36** (2004) 405]



Neutrino-photon interaction introduced via:
$\star$–commutator with covariant derivative

$$\widehat{D}_\mu \widehat{\psi} = \partial_\mu \widehat{\psi} - i\kappa e \left[\widehat{A}_\mu \star \widehat{\psi} - \widehat{\psi} \star \widehat{A}_\mu \right]$$

The action for a neutral fermion that couples, in the adjoint of non-commutative U(1), to an Abelian gauge boson in the NC background is:

$$S = \int d^4x \left(\bar{\widehat{\psi}} \star i\gamma^\mu \widehat{D}_\mu \widehat{\psi} - m \bar{\widehat{\psi}} \star \widehat{\psi} \right)$$

SW map: $\widehat{\psi} = \psi + e\theta^{\nu\rho} A_\rho \partial_\nu \psi + \mathcal{O}(\theta^2)$
$\widehat{A}_\mu = A_\mu + e\theta^{\rho\nu} A_\nu \left[\partial_\rho A_\mu - \frac{1}{2}\partial_\mu A_\rho \right] + \mathcal{O}(\theta^2)$

The gauge invariant action of order θ^1 and $\kappa = 1$

$$S = \int d^4x\, \bar{\psi}\, [(i\gamma^\mu \partial\mu - m) - \frac{e}{2} F_{\mu\nu}(i\theta^{\mu\nu\rho}\partial\rho - \theta^{\mu\nu} m)\Big]\, \psi.$$

Feynman rule for $\gamma(q) \to \nu(k')\bar{\nu}(k)$ vertex:

$$\Gamma^\mu_{\binom{\mathrm{L}}{\mathrm{R}}}(\nu\bar{\nu}\gamma) = ie\frac{1}{2}(1 \mp \gamma_5)\left[(q\theta k)\gamma^\mu + (\not{k} - m_\nu)(\theta q)^\mu - \not{q}(\theta k)^\mu\right].$$

For massless neutrinos the vertex becomes totally symmetric:

$$\Gamma^\mu_{\binom{\mathrm{L}}{\mathrm{R}}}(\nu\bar{\nu}\gamma) = ie\frac{1}{2}(1 \mp \gamma_5)\left(\theta^{\mu\nu}\gamma^\tau + \theta^{\nu\tau}\gamma^\mu + \theta^{\tau\mu}\gamma^\nu\right) k_\nu q_\tau$$

In a stellar plasma, the dispersion relation of photons is identical with that of a massive particle

$$q^2 \equiv \mathrm{E}_\gamma^2 - \mathbf{q}_\gamma^2 = \omega_{\mathrm{pl}}^2$$

ω_{pl} – the plasma frequency.
The plasmon (off-shell photon) decay rate to the left and/or right massive neutrinos

$$\Gamma(\gamma_{\mathrm{pl}} \to \bar{\nu}_{\binom{\mathrm{L}}{\mathrm{R}}}\nu_{\binom{\mathrm{L}}{\mathrm{R}}}) = \frac{\alpha}{48}\frac{\omega_{\mathrm{pl}}^6}{\mathrm{E}_\gamma \Lambda_{\mathrm{NC}}^4}\sqrt{1 - 4\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}}$$
$$\times \left[\left(1 + 2\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2} - 12\frac{m_\nu^4}{\omega_{\mathrm{pl}}^4}\right)\sum_{i=1}^{3}(c^{0i})^2 + 2\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}\left(1 - 4\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}\right)\sum_{\substack{i,j=1\\ i<j}}^{3}(c^{ij})^2\right].$$

In the rest frame of the medium and for massless neutrinos the decay rate is

$$\Gamma_{\mathsf{NC}}(\gamma_{\mathsf{pl}} \to \nu_{\binom{\mathsf{L}}{\mathsf{R}}}\bar{\nu}_{\binom{\mathsf{L}}{\mathsf{R}}}) = \frac{\alpha}{48}\frac{1}{\Lambda_{\mathsf{NC}}^4}\frac{\omega_{\mathsf{pl}}^6}{\mathsf{E}_\gamma}$$

The corresponding SM neutrino-penguin-loop rate:

$$\Gamma_{\mathsf{SM}}\left(\gamma_{\mathsf{pl}} \to \nu_{\mathsf{L}}\bar{\nu}_{\mathsf{L}}\right) = \frac{c_{\mathsf{V}}^2 G_{\mathsf{F}}^2}{48\pi^2\alpha}\frac{\omega_{\mathsf{pl}}^6}{\mathsf{E}_\gamma}.$$

For ν_e: $\quad c_{\mathsf{V}} = \frac{1}{2} + 2\sin^2\Theta_{\mathsf{W}}$

$\nu_\mu,\ \nu_\tau$: $\quad c_{\mathsf{V}} = -\frac{1}{2} + 2\sin^2\Theta_{\mathsf{W}}$

for the SM: $\quad c_{\mathsf{V}}^2 = 0.79.$

$$\mathcal{R} \equiv \frac{\sum_{\mathsf{flavours}}\Gamma_{\mathsf{NC}}\left(\gamma_{\mathsf{pl}} \to \nu_{\mathsf{L}}\bar{\nu}_{\mathsf{L}} + \nu_{\mathsf{R}}\bar{\nu}_{\mathsf{R}}\right)}{\sum_{\mathsf{flavours}}\Gamma_{\mathsf{SM}}(\gamma_{\mathsf{pl}} \to \nu_{\mathsf{L}}\bar{\nu}_{\mathsf{L}})} = \frac{6\pi^2\alpha^2}{c_{\mathsf{V}}^2 G_{\mathsf{F}}^2 \Lambda_{\mathsf{NC}}^4},$$

$$\Lambda_{\mathsf{NC}} = \frac{80.8}{\mathcal{R}^{1/4}}\ (\mathsf{GeV}).$$

A standard globular cluster stars argument:

any new energy-loss mechanism must not exceed the standard neutrino losses by much

$\longrightarrow$ approximate requirement $\mathcal{R} < 1$

$$\longrightarrow \quad \Lambda_{\mathsf{NC}} > \left(\frac{6\pi^2\alpha^2}{c_{\mathsf{V}}^2 G_{\mathsf{F}}^2}\right)^{1/4} \cong 81\ \mathsf{GeV}$$

Neutrino $\star$-dipole moments and $\star$-charge radii: $\mathrm{d}^{\mathrm{el}}_{\mathrm{mag}}$, $\langle r^2_\nu \rangle_{\mathrm{NC}}$

[P. Minkowski, P. Schupp and J.T., "Neutrino dipole moments and charge radii in non-commutative space-time," Eur. Phys. J. C **37** (2004) 123]

$\nu_i \longrightarrow \nu_j + \gamma$ transitions, generated through 1-loop electroweak "neutrino–penguin" diagrams: the exchange of $\ell = e, \mu, \tau$ and weak bosons

$$
\begin{aligned}
J^{\mathrm{eff}}_\mu(\gamma\nu\bar\nu)\epsilon^\mu(q) &= \Big\{F_1(q^2)\bar\nu_j(p')(\gamma_\mu q^2 - q_\mu \not{q})\nu_i(p)_L \\
&- iF_2(q^2)\Big[m_{\nu_j}\bar\nu_j(p')\sigma_{\mu\nu}q^\nu\nu_i(p)_L \\
&+ m_{\nu_i}\bar\nu_j(p')\sigma_{\mu\nu}q^\nu\nu_i(p)_R\Big]\Big\}\epsilon^\mu(q).
\end{aligned}
$$

General decomposition of the second term

$$
\mathrm{T} = -i\epsilon^\mu(q)\bar\nu(p')\left[A(q^2) - B(q^2)\gamma_5\right]\sigma_{\mu\nu}q^\nu\nu(p),
$$

gives the electric and magnetic dipole moments

$$
d^{\mathrm{el}}_{ji} \equiv B(0) = \frac{-e}{M^{*2}}\left(m_{\nu_i} - m_{\nu_j}\right)\sum_{\ell=e,\mu,\tau}\mathrm{U}^\dagger_{jk}\mathrm{U}_{ki}F_2\Big(\frac{m^2_{\ell_k}}{m^2_W}\Big),
$$

$$
\mu_{ji} \equiv A(0) = \frac{-e}{M^{*2}}\left(m_{\nu_i} + m_{\nu_j}\right)\sum_{\ell=e,\mu,\tau}\mathrm{U}^\dagger_{jk}\mathrm{U}_{ki}F_2\Big(\frac{m^2_{\ell_k}}{m^2_W}\Big),
$$

where $i, j, k = 1, 2, 3$ denotes neutrino species, and

$$
F_2\Big(\frac{m^2_{\ell_k}}{m^2_W}\Big) \simeq -\frac{3}{2} + \frac{3}{4}\frac{m^2_{\ell_k}}{m^2_W}, \quad \frac{m^2_{\ell_k}}{m^2_W} \ll 1,
$$

$M^* = 4\pi\, v = 3.1$ TeV;
$v = (\sqrt{2}\, G_F)^{-1/2} = 246$ GeV, VEV for Higgs field.

Dirac neutrino: $i = j$; $\mu_{ii} \equiv \mu_{\nu_i}$. ($m_\nu = 0.05\,\mathrm{eV}$):

$$
\begin{aligned}
\mu_{\nu_i} &= \frac{3e}{M^{*2}} m_{\nu_i} \left[1 - \frac{1}{2} \sum_{\ell = e,\mu,\tau} \frac{m_\ell^2}{m_W^2} |\mathrm{U}_{\ell i}|^2 \right] \\
&\simeq 1.56 \times 10^{-26} [\mathrm{e/eV}] = 0.29 \times 10^{-30}\, [\mathrm{e\ cm}] \\
&= 1.60 \times 10^{-20} \mu_B.
\end{aligned}
$$

in units of [e cm] and Bohr magneton [μ_B], Chirality flip arises only from the neutrino masses.

Majorana neutrino: No particle–antiparticle distinction ($\psi_i = \psi_i^c$); one has to use both charged lepton and antilepton propagators in the loop calculation of neutrino-penguin diagrams $\to$ the first term in F_2 vanishes in the summation over ℓ due to the orthogonality condition of U (GIM cancellation)

$$
\begin{aligned}
d^{\mathrm{el}}_{\bar\nu_j \nu_i} &= \frac{3e}{2M^{*2}} \left(m_{\nu_i} - m_{\nu_j} \right) \sum_{\ell = e,\mu,\tau} \frac{m_{\ell_k}^2}{m_W^2} \mathrm{U}^\dagger_{jk} \mathrm{U}_{ki}, \\
\mu_{\bar\nu_j \nu_i} &= \frac{3e}{2M^{*2}} \left(m_{\nu_i} + m_{\nu_j} \right) \sum_{\ell = e,\mu,\tau} \frac{m_{\ell_k}^2}{m_W^2} \mathrm{U}^\dagger_{jk} \mathrm{U}_{ki}.
\end{aligned}
$$

Transition matrix element T is a complex antisymmetric quantity in lepton-flavor space:

$$\begin{aligned}\mathrm{T_{ji}} &= -i\epsilon^{\mu}\bar{\nu}_j\left[(A_{ji}-A_{ij})-(B_{ji}-B_{ij})\gamma_5\right]\sigma_{\mu\nu}q^{\nu}\nu_i \\ &= -i\epsilon^{\mu}\bar{\nu}_j\left[2\mathrm{i}\mathrm{Im}A_{ji}-2\mathrm{Re}B_{ji}\gamma_5\right]\sigma_{\mu\nu}q^{\nu}\nu_i.\end{aligned}$$

Explicitly clear that for $i=j$, $d^{\mathrm{el}}_{\nu_i}=\mu_{\nu_i}=0$.
The above first term vanishes if the relative CP of ν_i and ν_j is even;
the second term vanishes if it is odd.

$$\begin{aligned}d^{\mathrm{el}}_{\nu_i\nu_j} &= \frac{3e}{2M^{*2}}\left(m_{\nu_i}-m_{\nu_j}\right)\sum_{\ell=e,\mu,\tau}\frac{m^2_{\ell_k}}{m^2_W}\mathrm{Re}\mathrm{U}^{\dagger}_{jk}\mathrm{U}_{ki}, \\ \mu_{\nu_i\nu_j} &= \frac{3e}{2M^{*2}}\left(m_{\nu_i}+m_{\nu_j}\right)\sum_{\ell=e,\mu,\tau}\frac{m^2_{\ell_k}}{m^2_W}\mathrm{i}\,\mathrm{Im}\mathrm{U}^{\dagger}_{jk}\mathrm{U}_{ki},\end{aligned}$$

Majorana case: mixing matrix U is approximatively unitary

$$\sum_{i=1}^{3}\mathrm{U}^{\dagger}_{jk}\mathrm{U}_{ki}=\delta_{ji}-\varepsilon_{ji},$$

where ε is a hermitian nonnegative matrix

$$\begin{aligned}|\varepsilon|=\sqrt{\mathrm{Tr}\,\varepsilon^2} &= \mathcal{O}\left(m_{\nu_{\mathrm{light}}}/m_{\nu_{\mathrm{heavy}}}\right), \\ &\sim 10^{-22}\text{ to }10^{-21}.\end{aligned}$$

The case $|\varepsilon|=0$ is excluded by the very existence of oscillation effects.

The neutrino dipole moments violate lepton number by ± 2 and for a general neutrino mass matrix, they independently violate CP.

The corresponding analytic structure is quite definite, globally referred to as the see–saw mechanism.

Assuming hierarchical structure:
$|m_3 + m_2| \simeq |m_3 - m_2| \simeq |\Delta m^2_{32}|^{1/2} = 0.05$ eV.
Seting: $|\mathrm{Re}\mathrm{U}^{\dagger}_{3\tau}\mathrm{U}_{\tau 2}| \simeq |\mathrm{Im}\mathrm{U}^{\dagger}_{3\tau}\mathrm{U}_{\tau 2}| \leq 0.5$.
The electric and magnetic transition dipole moments denoted as $\left(d^{\mathrm{el}}_{\mathrm{mag}}\right)_{23}$ are

$$\begin{aligned}
\left|\left(d^{\mathrm{el}}_{\mathrm{mag}}\right)_{23}\right| &= \frac{3e}{2M^{*2}}\,\frac{m_\tau^2}{m_W^2}\sqrt{|\Delta m^2_{32}|}\binom{|\mathrm{Re}\mathrm{U}^{\dagger}_{3\tau}\mathrm{U}_{\tau 2}|}{|\mathrm{Im}\mathrm{U}^{\dagger}_{3\tau}\mathrm{U}_{\tau 2}|}, \\
&\lesssim 2.03\times 10^{-30}[\mathrm{e/eV}] = 0.38\times 10^{-34}\,[\mathrm{e\ cm}] \\
&= 2.07\times 10^{-24}\,\mu_B < \left|\left(d^{\mathrm{el}}_{\mathrm{mag}}\right)_{\mathrm{d-quark}}\right|
\end{aligned}$$

To extract an upper limit on the $\star$-gradient interaction we compare the strength $|m_\nu\, e\,\kappa\,\theta\, F|$ with the dipole transition interactions $|F\, d^{\mathrm{el}}_{\mathrm{mag}}|$ for Dirac/Majorana cases. Assuming that contributions from the neutrino-mass extended SM are at least as large as those from noncommutativity, for $\kappa = 1$ we derive the following bound:

$$|\Lambda_{\mathrm{NC}}|^{\mathrm{Dirac}}_{\mathrm{Majorana}} \gtrsim \left|\frac{e\,\kappa\, m_\nu}{\left(d^{\mathrm{el}}_{\mathrm{mag}}\right)^{\mathrm{Dirac}}_{\mathrm{Majorana}}}\right|^{1/2} \simeq \binom{150}{1.80}\ \mathrm{TeV}.$$

This is the main result of our considerations which on the scale of noncommutativity involves only the basic properties of neutrinos and photons.

Radius of the photon–neutrino interaction: Neutrino $\star$-charge radii $r^{*2} = \langle r_\nu^2 \rangle_{\rm NC}$

Noncommutativity can be a source of "transvers plasmon" decay into neutrino–antineutrino pairs. This is to be compared with the same process induced by the neutrino charge radii defined by the axial electromagnetic interaction form factor in the neutrino-mass extended SM:

$$\langle r_\nu^2 \rangle = 6 \left[\frac{\partial F_1(q^2)}{\partial q^2} \right]_{q^2=0} ; \left[F_1(q^2) \right]_{q^2 \to 0} \longrightarrow \frac{q^2}{6} \langle r_\nu^2 \rangle ,$$

which in the limit of massless neutrinos corresponds to

$$\langle r_{\nu_\ell}^2 \rangle \cong \frac{2}{M^{*2}} \left(3 - 2\log \frac{m_\ell^2}{m_W^2} \right) = \frac{G_F}{\sqrt{2}\,\pi^2} \left(\frac{3}{4} + \log \frac{m_W}{m_\ell} \right) .$$

We estimate the charge radii in the SM from by taking $\ell = e$: $\sqrt{|\langle r_{\nu_e}^2 \rangle|_{\rm SM}} \simeq 6.4 \times 10^{-17}$ [cm]. Here we remark that astrophysical estimates give interesting bounds. These calculations should implement all neutrino flavor properties. The so derived bounds may also help in establishing the Majorana nature of light neutrinos.

To estimate the $\star$-charge radii we first evaluate the SM rate induced by the charge radii:

$$\sum_{\ell=e,\mu,\tau} \Gamma_{\rm SM}(\gamma \to \bar\nu_\ell^{\rm L} \nu_\ell^{\rm L}) = \frac{\alpha}{144} \frac{q^6}{E_\gamma} \sum_{\ell=e,\mu,\tau} \left| \langle r_{\nu_\ell}^2 \rangle \right|^2 ,$$

Plasmon at rest $q^2 = E_\gamma^2 = \omega_{\rm pl}^2$.
Average of the plasmon frequencies of red-giant and white-dwarf stars $\omega_{\rm pl} = 15$ keV gives

$$\Gamma^{-1}_{\rm SM}(\gamma \to \bar\nu\nu) \;=\; \left(\frac{1\,\mathrm{keV}}{\omega_{\rm pl}}\right)^5 \times 0.25 \times 10^{13}\ \text{years}$$
$$\simeq\; 3 \times 10^6\ \text{years};$$

(compare with astrophysical observations).

Off-shell photon to massless Majorana neutrinos decay rate in the rest system of medium:

$$\sum_{\ell=e,\mu,\tau} \Gamma_{\rm NC}(\gamma \to \bar\nu_\ell^{\rm L}\nu_\ell^{\rm L}) = \frac{\alpha}{16}\frac{\kappa^2\, q^6}{\mathrm{E}_\gamma \Lambda_{\rm NC}^4}\sum_{i=1}^{3}(c^{0i})^2$$

and $\Gamma_{\rm SM}(\gamma \to \bar\nu_\ell^{\rm L}\nu_\ell^{\rm L}) \gtrsim \Gamma_{\rm NC}(\gamma \to \bar\nu_\ell^{\rm L}\nu_\ell^{\rm L})$ gives the range of noncommutativity via the $\star-$charge radii:

$$r^* = \sqrt{|\langle r_\nu^2\rangle_{\rm NC}|} = \frac{\sqrt{\sqrt{3}\,\kappa}}{\Lambda_{\rm NC}}.$$

For $|\Lambda_{\rm NC}|^{\rm Dirac}_{\rm Majorana} = \binom{1.8}{150}$ TeV and $\kappa = 1$

$$r^*|^{\rm Dirac}_{\rm Majorana} \lesssim \begin{pmatrix} 1.4\times 10^{-17} \\ 1.6\times 10^{-19}\end{pmatrix}[\mathrm{cm}];\quad \begin{pmatrix} \simeq \frac{1}{5}\sqrt{|\langle r_{\nu_e}^2\rangle|_{\rm SM}} \\ \text{unobservable}\end{pmatrix}$$

$\star-$induced charge radii r^* at the $\Lambda_{\rm NC} \gtrsim 150$ TeV, dominated by the neutrino-mass extended standard model physics and are practically unobservable.

Discusion: NEUTRINO SECTOR

* From the energy loss in the globular stellar clusters, requirement $\longrightarrow \mathcal{R} < 1 \quad \longrightarrow$
– The constraint $\Lambda_{\rm NC} > 80$ GeV, represents the lower bound on the NC scale – $\Lambda_{\rm NC}$
* By comparing SM and NCSM neutrino electric and magnetic moments we found

$$|\Lambda_{\rm NC}|^{\rm Dirac}_{\rm Majorana} \gtrsim \left| \frac{e\,\kappa\,m_\nu}{\left(d^{\rm el}_{\rm mag}\right)^{\rm Dirac}_{\rm Majorana}} \right|^{1/2} \simeq \binom{150}{1.80}\ {\rm TeV}.$$

– Above bounds depend on the NC coupling κ and are based on different laboratory.

– In conclusion, we have compared the neutrino mass extended standard model charge radii and electromagnetic dipole moments of neutrinos with their analogs arising from a theory of noncommutative space-time.

– In this way we can "understand" neutrinos as particles which manifest them self as Majorana objects at the very short distances (very high energies).

DISCUSION

Limits on Λ_{NC} from theory and experiment

DECAYS: 1$\to$ 2

* $Z \to \gamma\gamma \quad \Rightarrow \Lambda_{\mathrm{NC}} > 162$ GeV, [Duplančić, J.T. et al.]

* $\gamma_{\mathrm{pl}} \to \nu\bar{\nu} \quad \Rightarrow \Lambda_{\mathrm{NC}} > 81$ GeV, [Schupp, JT, Wess, Raffelt]

* $J/\psi \to \gamma\gamma \quad \Rightarrow \Lambda_{\mathrm{NC}} > 9$ GeV, [Melic, Passek, J.T.]

* $K \to \pi\gamma \quad \Rightarrow \Lambda_{\mathrm{NC}} > 43$ GeV, [Melic, Passek, J.T.]

SCATTERINGS: 2$\to$ 2

* $e^+e^- \to \gamma\gamma \Rightarrow \Lambda_{\mathrm{NC}} > 141$ GeV, [OPAL Coll. (2003)]

* $\gamma\gamma \to \bar{f}f \quad \Rightarrow \Lambda_{\mathrm{NC}} > 200$ GeV, [T. Ohl et al.]

* $\bar{f}f \to Z\gamma \quad \Rightarrow \Lambda_{\mathrm{NC}} > 1$ TeV, [T. Ohl et al.]

NEUTRINO DIPOLE MOMENTS:

* $(\mathrm{d_{mag}})^{\mathrm{Dirac}} \quad \Rightarrow \Lambda_{\mathrm{NC}} > 1.8$ TeV, [Minkowski et al.]

* $(\mathrm{d^{el}_{mag}})^{\mathrm{Majorana}} \Rightarrow \Lambda_{\mathrm{NC}} > 150$ TeV, [Minkowski et al.]

CONCLUSION

* The SM forbidden decay signals are clean due to the absence of the SM contributions.

* Deviations from SM, due to the NC contributions, of the differential cross sections for $2 \longrightarrow 2$ transitions could be significant and potentialy measurable.

* The sensitivity to the noncommutative parameter $\theta^{\mu\nu}$ could be in a range of the next generation of linear colliders with a c.m.e. around a few TeV's.

* Experimental discovery of the SM forbidden decays and/or SM deviations in the scattering processses would indicate physics beyond the SM.

* To determine if above signals are coming from the NC or some other sources would require extra theoretical and experimental work.